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RECENT TRENDS IN EXPERIMENTAL ANALYSIS

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1. Introduction

According to the fast developments in the past and still at present in computer techniques in hardware and especially in software with new numerical methods, e.g. the finite-element methods, the question is discussed worldwide, whether methods of experimental stress analysis are still necessary or not. It is argued that new generations of computers are developed with increasing capacity and apparently unlimited possibilities, to be adapted to almost every mechanical and structural problem. But despite these discussions methods of experimental stress analysis are introduced and applied to a larger extent than at any time before in research institutes as well as in the industry. They become more and more important just because of the tremendous extension and involvement of computer analysis. Experimental stress analysis is not only used to determine material behaviour and to predict the stress and strain state in structures or in parts of structures. The, results of mathematical analysis and the validity of assumptions and suppositions, which generally are necessary to enable mathematical analysis, are checked experimentally. Methods of experimental stress analysis are introduced into systems control, for permanent supervisory operating systems like ships, airplanes, pressure vessels, dams, bridges, tall buildings, reactor plants and other industrial plants etc. Methods of experimental stress analysis are involved in product assembly, manufacture inspection and quality control.

There is a wider field of applications nowadays. Beyond the "classical" fields of static, dynamic and stability problems such problems are investigated experimentally, where even highly sophisticated numerical methods do not lead towards reliable results, if — for example — large deformations, load cycles, impact loads, wave propagation in solids, problems of fracture mechanics, fatigue problems are regarded. It also becomes more important to consider non-linear elastic, plastic, viscoelastic and viscoplastic as well as nonhomogeneous and non-isotropic response of material on the stress and strain state in structures. And until now there is not known any way to determine and to describe the response of material without experimental analysis.

2. Recent developments in experimental methods

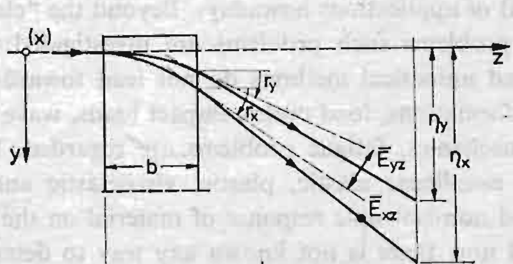
Generally speaking, there are no important and basic new developments in the principles of experimental methods, but to a large extent specific improvements of these methods [1].

Most common and applied are strain gage techniques, especially in industrial laboratories and in systems control. New types of strain gages are available to be used in the range of extremely low and of high temperatures as well as in elastic-plastic strain analysis. Higher precision and reliability is provided with advanced technologies of measuring devices. Semi-conductor techniques, large scale integrated circuits and microprocessors are introduced. Inductive and piezoelectric transducers are demagnified and can be adapted to different problems in experimental analysis.

Moiré techniques are involved in research laboratories mainly. New fields of application are opened because finer grids up to approx. some 1000 lines/cm are available, the application in high temperature ranges is possible as well as application to three-dimensional surfaces to determine their spatial deformations. These techniques are used to analyze elastic-plastic and viscoelastic strain states and dynamic processes, vibration problems as well as pulse effects.

Beside numerous attempts to transmit the principles of photoelasticity to photoplasticity and photoviscoelasticity, strong efforts are made to improve the three-dimensional photoelastic analysis in frozen stress techniques as well as in scattered light techniques. For a higher precision in the interpretation of observed optical phenomena, some activities should be mentioned to get better knowledge in non-linear optical response of model materials and in the effects of light propagation and radiation. The path of light transmitting through an inhomogeneous medium is curvilinear. The curvature increases with the increasing strain/stress gradient [2]. An approximated solution to determine the light propagation through solid body caused by strain/stress gradient is given in [3]. (fig. 1). Sometimes the principle axes of the stress state are rotating along the light path through the medium [4]. In such cases a more general solution can be found by introducing a finite mathematical solution [5] (fig. 2) to determine the stress state by the observed optical phenomena as the birefringence δ and the characteristic directions φ.

Regarding the methods of holography and speckle interferometry, no new fundamental developments have taken place in the methods themselves, but in numerous modifications of experimental setups, in evaluation methods of the originally obtained optical informations, and in specific forms of application. In basic research as well as in the industrial



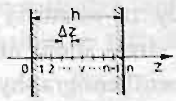
$$r_x \approx \left.\frac{dv_x}{dz}\right|_{z=b} \approx \frac{1}{n_0 L}\eta_x \; ; \quad r_y \approx \left.\frac{dv_y}{dz}\right|_{z=b} \approx \frac{1}{n_0 L}\eta_y$$

$$(C_1+C_2)\frac{\partial}{\partial y}(\sigma_x+\sigma_y) = \frac{1}{bL}(\eta_x+\eta_y)$$

$$(C_1-C_2)\frac{\partial}{\partial y}(\sigma_y-\sigma_x) = \frac{1}{bL}(\eta_y-\eta_x)$$

Fig. 1

$$\frac{d^2\{E\}}{dz^2} = -\frac{\omega^2}{c^2}[\varepsilon]\{E\} = k[\varepsilon]\{E\}$$



$$\{E_n\} = [U_n][U_{n-1}]\cdots[U_\nu]\cdots[U_1]\{E_0\}$$

$$[U_\nu] = \frac{1}{2}\Delta A_\nu[\bar{u}_\nu]$$

$$\{E_0\} = \{E_{x0};0\} \Longrightarrow E_{yn} = \frac{1}{2^n}A_n \cdot \bar{u}_{21} \cdot E_{x0}$$

$$A_n = \Delta A_n \cdot \Delta A_{n-1} \cdots \Delta A_2 \cdot \Delta A_1$$

$$\bar{u}_{21} = \sum_{\varkappa=1}^{2^n} [G_{21\varkappa} \cdot \exp(\sum_\nu \lambda_{1\nu}\Delta z + \sum_\mu \lambda_{3\mu}\Delta z)] \; ; \quad \nu\mu \in \{1/n\};\ \nu \neq \mu;\ \nu + \mu = n$$

$$\boxed{J = E_{yn}E^*_{yn} = \left(\frac{1}{2^n}A_n\right)^2 \bar{u}_{21}\bar{u}^*_{21}E_{x0}E^*_{x0}}$$

Fig. 2

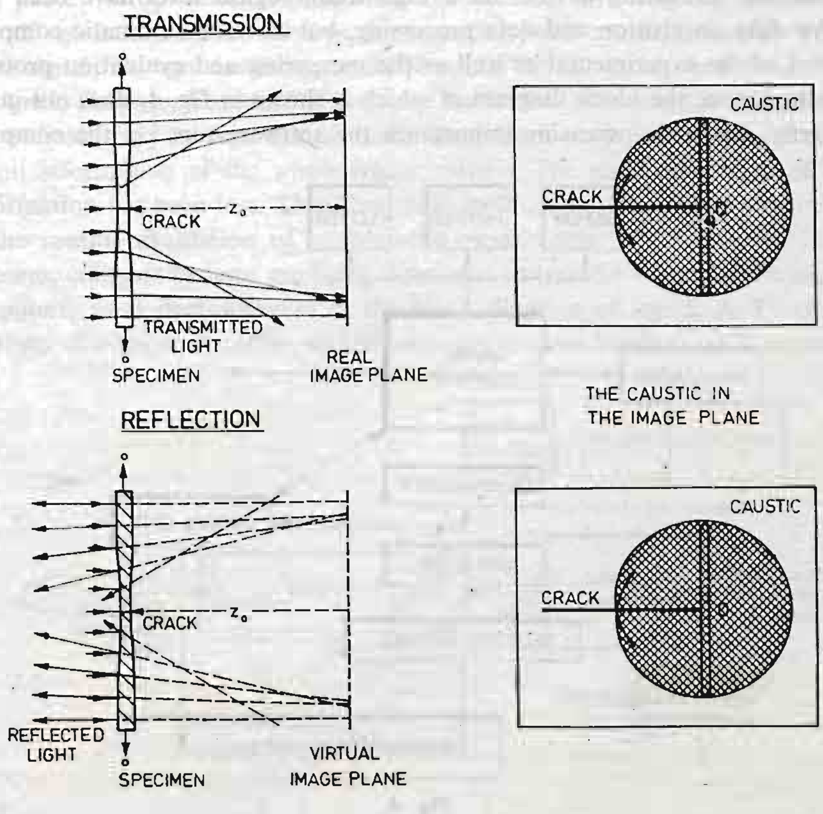


Fig. 3

field, even in quality control more advantage is taken of these methods, since complete experimental devices are available as well as new photographic material with higher resolving power, as for example thermo-film. Interferometric methods are very convenient to analyze the effects of impact loading and pulse propagation in solids.

Especially in fracture mechanics, the method of caustics has been brought to a state of development recently to intensify the practical application. This method allows to analyze singularities in stress states and areas of high stress concentration [6], (fig. 3).

The technique of roenthenography has reached a state of perfection now, so that the available goniometers are used in the industry for manufacture inspection and systems control as well as in laboratories to measure the stresses, especially the residual stresses on surfaces of structures or structural elements.

3. Recent developments in data acquisition and data processing

Based on the already known principles of experimental methods fast progress has been made in the development of measuring systems combined with digital computers of higher or less capacity, which can run on-line, computer-controlled.

Multiposition measuring devices on a high technological level have been produced not only for data acquisition and data processing, but also for automatic computer-operated control of the experimental as well as the measuring and evaluation process. Such an automatic system, the block diagram of which is shown in fig. 4, does not include the hardware only, but with increasing importance the software too, i.e. the computer pro-

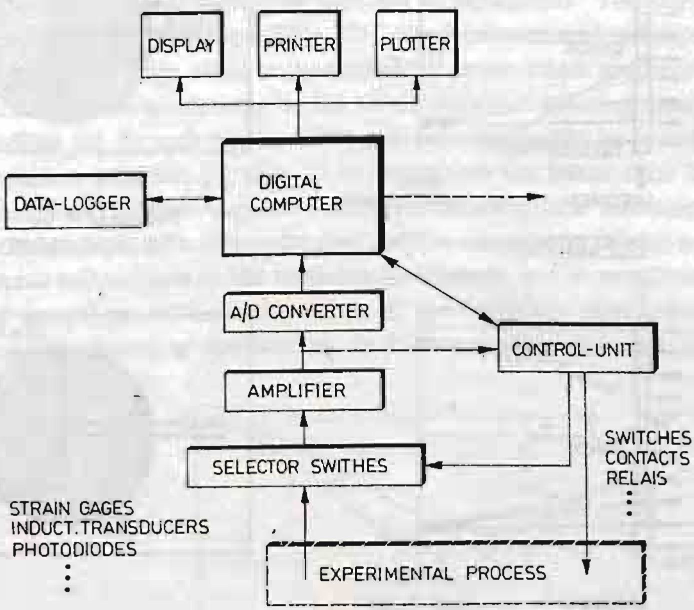


Fig. 4

grams for control, transmission, converting and processing up to the final output informations. Mechanical events, such as strains, deformations, temperature, light intensity, are picked up as electrical signals at some 100 measuring points, which are controlled automatically by selector switches. After amplifying, the signals are converted into digital signals and then transmitted to a digital computer where further processing takes place.

The final data or even the unprocessed data may be stored in a data logger to be recalled if needed. The output of the final results will be done with standard peripheral processors, such as displays, plotters or printers. It is also possible to transmit the information to a larger central processor.

Most remarkable is the closed control circuit to control the measuring process by the experimental events, or vice versa. The operational capability of such systems strongly depends on the software. They are organized to put in the data conveniently as to require minimum computer time for transmission and processing. All data should be reproducible accurately at any time during experiments.

Recent progress in diminution of electronic measuring devices has led to a rapid decrease of costs of the hardware. Proper plug-compatible interfaces allow direct connection via cable sets to standard peripheral devices and to a variety of computers. All timing, signal levels and pinouts are adaptable to those specified by computer manufactures.

Since the early sixties, numerous methods and apparatus have been developed for automatic pickup of optical data pointwise. The order of birefringence and the principal directions of refraction, the position of the regarded point are measured by transmitting the optical signal, i.e. light intensity, to an electrical signal, which then is converted into a digital signal. This may be recorded or stored for further processing in a small computer. But such a device may be regarded as insufficient, as it only permits partial automation and no full information of the whole fringe pattern. The great advantage of such full--field information has been lost. Therefore these methods have not been accepted in practice for the routine evaluation of photoelastic experiments.

At present, complex systems are being developed to transfer visual information directly into computers, as is demonstrated in the block diagram of fig. 5. A TV-camera uses a large variety of one-inch vidicons to detect images at wave length from X-ray to infrared.

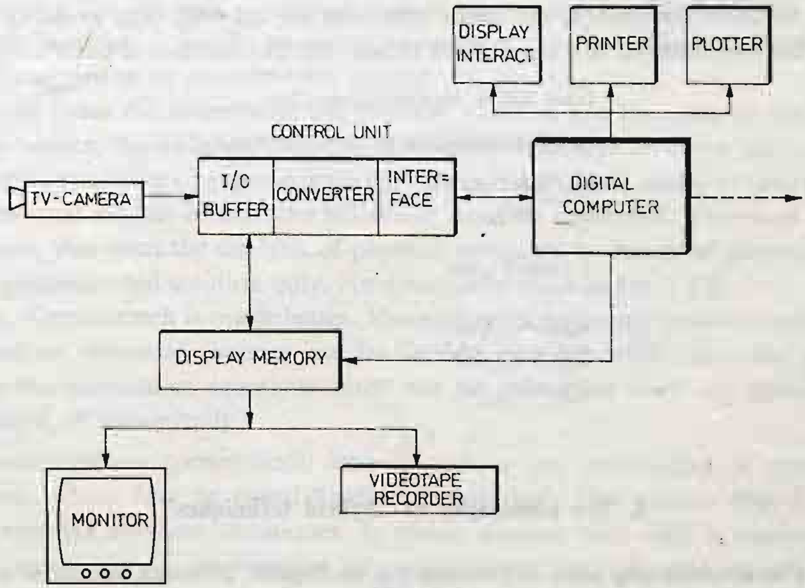


Fig. 5

Total fields of interference fringes, as produced by photoelasticity, holographic or other interferometric methods, by Moiré techniques, can be transformed into a composite analogous video-signal, which may be viewed on a monitor or recorded on standard videotape equipment.

Also, digital signals can be produced by an A/D-converter, which then are transmitted to a computer. The computer processes and analyzes the input data to produce the wanted information. The results may be displayed, printed or plotted. The transformed optical signals also may be stored in a display memory because of availability of data in TV-standards. Resolution up to 1024 × 1024 lines provides more than one million data points with 256 gray levels, which can be sampled to permit highly detailed examination of the field of view. Special attention is to be paid for the scanning of the isoclinics in the automatic process; as yet, a sufficient solution has not been discovered.

This system can also be used in connection with pulsed holographic interferometry, if the optical data is taken from the monitor after the high speed event has been recorded by the holographic camera system.

As the recent developments of methods and the increased resolving power of transducers, receivers, photographic and other recording materials and tools quarantee a higher accuracy of the measured values by reducing the errors to an extremely low level, the evaluation methods have to be improved in order not to lose the advantages of such progress. On the other hand, data processing by means of computers demands and allows more exact numerical evaluation methods. The amount of data available can be quite considerable. Contrary to mathematical solutions, the experimental analysis — similar to numerical methods — yields a set of data in discrete points, the distances of which may be shorter or longer, depending on the involved method and the equipment. The measured values are subject to random errors. So as to fit the "surfaces" to these data, spline functions are introduced for smoothing and balancing (fig. 6). If the basic measured values are to be differentiated, the spline functions are derived, thus avoiding error propagation. But one must be very careful not to run into the problem of overfitting.

Cubic Spline Approximation

$$\tilde{k}_{xi}(x-x_i) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3$$

$$\sum_{i=0}^{n} \left(\frac{\tilde{k}_x(x_i) - k_{xi}}{dk_{xi}} \right)^2 \leq \varepsilon \ , \varepsilon \geq 0 \ ;$$

$$\int_{x_0}^{x_i} (\tilde{k}_x(x))^2 = \min$$

$$\tilde{k}_{xi}(x_i) = \tilde{k}_{x,i-1}(x_i)$$

$$\tilde{k}'_{xi}(x_i) = \tilde{k}'_{x,i-1}(x_i)$$

$$\tilde{k}''_{xi}(x_i) = \tilde{k}''_{x,i-1}(x_i)$$

Fig. 6

4. The philosophy of „hybrid techniques"

In order to describe any kind of process, e.g. biological, physical, social, or economical processes, models must be developed. Generally this means that such processes or events must be transformed into an operational form by verbal descriptions, mathematical algorithms or by picturing and even by mapping one process on to another one. Then the reliability and accuracy of results of any investigation or of any analysis of a considered process strongly depends on the comprehensiveness of this "modelling". The model must describe the reality exactly or as exact as possible and in such a way, that the numerous parameters which have considerable influence on the real event are included in the model.

Despite the tremendous progress in scientific as well as in technological developments, there are still limitations in scientific capacity and in the technical possibilities. Therefore, so far it is not possible to model the real event exactly. All attempts are leading towards approximation processes only, and they are restricted to subsystems.

In stress analysis, the physical process in a real structure must be described in order to predict the response of this structure under random loading conditions to determine for example the stresses, strains, deformations, or the safety against failure, considering the environmental conditions as well as the material behaviour. Generally, it is impossible to formulate a "mathematical model", i.e. to derive "true" constitutive equations according to the real event or, if this should nevertheless be possible, to solve these constitutive equations. Assumptions and simplifications must be introduced on very different levels to find an operational model. This leads to an approach of the reality only. Very often the certainty and admissibility of such approaches are unknown or can hardly be estimated. Therefore the results obtained through mathematical analysis are uncreatin as well, despite introducing advanced numerical methods and computers with high capacity.

It seems to be possible, however, to use more realistic physical models of the regarded processes for experimental analysis. As such physical models the real structure, parts of the structure, a scaled-down replica, or an analogous physical process may be considered. The different reactions under the qiven load conditions may then be observed. But in stress analysis, the observed phenomena are in most cases not identical with the final informations. Therefore it is also necessary to introduce mathematical models for evaluation and transmission of experimental results.

As in many cases the material of the physical model is not the same as the material of the real structure, the different response of material must also be taken into consideration. For the transmission of experimental data from the physical model to the real event, proper operational models of material behaviour must be developed. Therefore it should be pointed out, that even the analysis of physical processes by means of physical models leads to an approximated solution only. An example is given in fig. 7 [7].

However, the approach is much better. More complex and more realistic mathematical models based on advanced theories can be derived and introduced into the analyzing processes as the constitutive equations must not be solved, or they are solved experimentally instead of numerically.

These considerations consequently lead towards a new philosophy of experimental stress analysis, which may be called "hybrid techniques". The pattern (fig. 8) demonstrates the meaning of these techniques. It makes obvious that only a combination of theory and experiment, a combination of mathematical and physical models yields a better knowledge of structure reactions and most reliable results in stress analysis. As

Transmission of results (plate in bending)

perturbation parameter: $\varepsilon = \mu - \mu_m$ (m denotes „model")

general solution: $w(x,y,\mu) = \frac{1-\mu^2}{1-\mu_m^2} \sum_{j=0}^{\infty} \varepsilon^j w(x,y,\mu_m);$

$$\nabla^2 \nabla^2 w_0 = -\frac{p(x,y)}{B_m};$$

$$\nabla^2 \nabla^2 w_j = 0 \; ; \; j \in \mathbb{N},$$

$\Longrightarrow$ internal forces

$$M_x = -B_m \sum_{j=0}^{\infty} \varepsilon^j [w_{j,xx} + (\mu_m + \varepsilon) w_{j,yy}],$$

$$M_y = -B_m \sum_{j=0}^{\infty} \varepsilon^j [w_{j,yy} + (\mu_m + \varepsilon) w_{j,xx}],$$

$$M_{xy} = -B_m (1 - \mu_m - \varepsilon) \sum_{j=0}^{\infty} \varepsilon^j w_{j,xy} \; ;$$

Fig. 7

Mathematical Model	Physical Model
$[P]=[K]\cdot[U]$	$[W]=[D]\cdot[L]$
$[P]=\begin{bmatrix}[P_1]\\ [P_2]\end{bmatrix}$ matrix of forces	$[W]=\begin{bmatrix}[U_1]\\ [P_2]\end{bmatrix}$ matrix of measured data
$[U]=\begin{bmatrix}[U_1]\\ [U_2]\end{bmatrix}$ matrix of displacements	$[L]=\begin{bmatrix}[P_1]\\ [U_2]\end{bmatrix}$ matrix of external loads and given displacements
$[K]$ stiffness matrix	$[D]$ model matrix
input data	unit state of loads and displace ments
$[P_1], [U_2]$	$[\bar{L}]=[I] \Longrightarrow [\bar{W}]=[D]\cdot[I]=[D]$
$\begin{bmatrix}[U_1]\\ [P_2]\end{bmatrix}=[G]\cdot\begin{bmatrix}[P_1]\\ [U_2]\end{bmatrix}$	$\begin{bmatrix}[U_1]\\ [P_2]\end{bmatrix}=[\bar{W}]\cdot\begin{bmatrix}[P_1]\\ [U_2]\end{bmatrix}$

if mathematical model $\hat{=}$ physical model

$$[G] \hat{=} [\bar{W}] \hat{=} [D]$$

Photoviscoelasticity

experiment $\cdot \delta(t), \varepsilon_{22}(t) \Longrightarrow$ evaluation $= C_N(t) = \frac{1}{\sigma_{11}} \cdot \frac{\delta(t)}{d(t)} = \frac{b_0}{P_0(1+\varepsilon_{22}(t))}$

optical creep function

$\Downarrow$

mathematical model: $= \quad C_N^*(t) = \frac{1}{C_N(0)} \left[1 - \int_0^t C_N^*(\tau) \cdot \dot{C}_N(t-\tau)\, d\tau \right]$

optical relaxation function

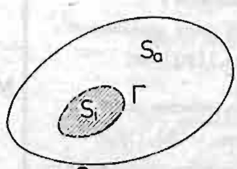
$\Downarrow$

subroutine RELAX

Fig. 8

it is shown in fig. 8, under the assumption of linear elastic behaviour of material (as a mathematical model!) the mechanical process can be formulated generally by a mathematical as well as a physical model. Both these models describe the relations between external and internal forces (or stresses) and deformations (or strains) respectively. Normalizing the load conditions, the experimental analysis yields the stiffness matrix as the normalized model matrix of the physical model. Thus, the numerical simulation of the elastic continuum is replaced by a physical model, which is closer to reality than any mathematical model could be, and the most extensive and expensive part of the numerical calculation, even using computer methods, is avoided.

Considering a subdomain S_i of the whole system S (fig. 9), one may assume that the realistic mathematical modelling of this subdomain is not possible or at least very difficult. Then this subdomain S_i will be imaged by a physical model. For given external loadings and unit boundary conditions, the data $[W_i]_\Gamma$ along Γ will be taken by measu-



domaine S_i:
measured data of boundary values: $[W_i]_\Gamma$

domaine S_a:
mathematical model: $[W_a]_\Gamma = [G]_{\Gamma_0} [L_a]_0 + [G]_\Gamma [L]_\Gamma$

$$[W_i]_\Gamma + [W_a]_\Gamma = 0$$

$$\Longrightarrow [L]_\Gamma = -[G]_\Gamma^{-1} \left([W_i]_\Gamma + [G]_{\Gamma_0} [L_a]_0 \right)$$

$$\Longrightarrow [W_a] = [G] \begin{bmatrix} [L_a] \\ [L]_\Gamma \end{bmatrix}$$

Fig. 9

Circular Plate on a yielding Subgrade

mathematical model: extended plate theory (large deflection)

$$\left.\begin{aligned} &B\nabla^2\nabla^2 w = p_a - p + h \cdot L(w,F) - w_{,rr} \cdot \Phi - \frac{h}{2} \cdot \Phi_{,rr} \\ &\nabla^2\nabla^2 F = \frac{E}{2} L(w,w) + \frac{1}{h} \left(\frac{1}{r} \Phi_{,r} - \mu \cdot \Phi_{,rr} \right) \\ &\qquad\qquad \Downarrow \\ &[D] \cdot (m) = -(p_a) + (p) - [n_r] \cdot (w'') - [n_\varphi] \cdot (w') + \frac{h}{2} (\Phi'') \\ &[D] \cdot \left((n_r) + (n_\varphi) + (\Phi) \right) = E \cdot h \cdot [w'] \cdot (w'') + (\Phi') - \mu \cdot (\Phi'') \end{aligned}\right\} M_S$$

L: diff. operator, F: stress function

$$\Phi = -\int_{(r)} t(\bar{r})\, d\bar{r}$$

Fig. 10

rements. The mathematical model of S_a yields $[W_a]_\Gamma$ caused by the unit geometrical and statical boundary loadings and the external loads on S_a. The conditions of equilibrium and/or compatibility yield the matrix $[L]_\Gamma$ of the effective boundary condition along Γ and furthermore the solution of the system S.

As an example, a thin plate on a yielding subgrade under large deflection is considered [8], [9]. Fig. 10 shows the constitutive equations of the fourth order, describing an advanced geometrically nonlinear plate theory as the mathematical model M_s. Obviously it seems to be very defficult to solve these equations numerically, especially as the boundary conditions are unknown. The physical model M_{se} (fig. 11), however, yields experimental data, and after evaluation of these data gives informations about the deflec-

Circular Plate on a yielding Subgrade

physical model a) Ligtenberg's Moiré Method

measured data := (w')

evaluation (by spline approximation):=

$(w'') \Longrightarrow (m_r), (m_\varphi), (m)$

b) Photoelasticity (reflection polariscop)

measured data := $(\delta), (\psi_\sigma)$

evaluation (by spline approximation):=

$(\bar{n}_r) = (n_r) + (\Phi), \ (\bar{n}_\varphi) = (n_\varphi) + (\Phi)$

} M_{SE}

$\Longrightarrow$ results M_{SE} into constitut. eq's of M_S:

$$(t) = [T^{-1}] \cdot \left[[D] \cdot \left((\bar{n}_r) + (\bar{n}_\varphi) \right) + E \cdot h \cdot [w'](w'') \right]$$

$$(p) = (p_a) - B[D] \cdot \left((w') + (w'') \right) + [n_r] \cdot (w'') + [n_\varphi] \cdot (w')$$

Fig. 11

tion surface w and its derivations respectively as well as of the membrane stress state. These results are introduced into the constitutive equations of the mathematical model M_s and yield the final solution of the regarded problem. The block diagram demonstrates the procedure generally (fig. 12).

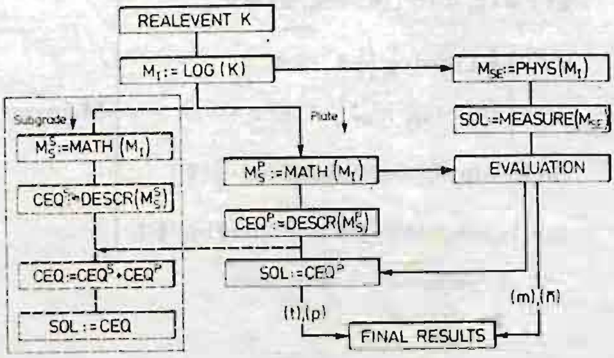


Fig. 12

Another example of the hybrid technique application is demonstrated in fig. 13 and 14. In order to study the influence of viscoelastic response of material on structures by photoviscoelastic experiments, a model of the material behaviour must be formulated. The mechanical and the optical creep function can be determined easily with proper testing methods. However, to determine the inverse functions, mathematical methods (Laplace's transform) are necessary. For the evaluation of stress-optical data, i.e. the order

Photoviscoelasticity

experiment $:= \delta(t), \varepsilon_{22}(t) \Longrightarrow$ evaluation $= C_N(t) = \frac{1}{\sigma_{11}} \cdot \frac{\delta(t)}{d(t)} = \frac{b_0}{P_0(1+\varepsilon_{22}(t))}$

optical creep function

$\Downarrow$

mathematical model: $= \quad C_N^*(t) = \frac{1}{C_N(0)}\left[1 - \int_0^t C_N^*(\tau)\cdot \dot{C}_N(t-\tau)\,d\tau\right]$

optical relaxation function

$\Downarrow$

subroutine RELAX

Fig. 13

Photoviscoelasticity

experimental data: $= \delta(t),\ \psi_N(t)$ in discrete points

$\Downarrow$

mathematical model: $= \quad \sigma_{12}(t) = \frac{1}{2}\left[C_N^*(t)\cdot g(0) + \int_0^t C_N^*(t-\tau)\cdot \dot{g}(\tau)\,d\tau\right]$

$$\sigma_{11}(t) - \sigma_{22}(t) = C_N^*(t)\cdot \bar{g}(0) + \int_0^t C_N^*(t-\tau)\cdot \dot{\bar{g}}(\tau)\,d\tau$$

$\Downarrow$

$g(t) = 2N_{12}(t) = \delta(t)\cdot \sin 2\psi_N(t)$

$\bar{g}(t) = N_{11}(t) - N_{22}(t) = \delta(t)\cdot \cos 2\psi_N(t)$

N_{ij}: components of refraction tensor

routine VISCOP: $= \sigma_{ij}(t),\ \psi_\sigma(t)$

Fig. 14

of birefringence $\delta(t)$ and the angle of isoclinics $\psi_N(t)$, a mathematical model must then be derived, and furthermore advanced computer methods are needed. This example demonstrates very clearly the recent trend in experimental analysis to obtain more precise informations of the real structure behaviour by combining mathematical and physical models in a feed-back process, using all recent developments in measuring equipment, data acquisition equipment, and automatic data processing and computer methods.

But one should have in mind that the physical model together with the complete experimental setup is to be regarded as one system, which includes all measuring devices and the necessary software. Beginning with the manufacturing of the model itself and model material, the light source (in photoelasticity, holography, Moiré technique, etc.) or the manufacturing of strain gages and their application, transducers, filters, amplifiers, the digital equipment, recording materials and recording devices have considerable influence on the results of measurement. And although recent technical developments guarantee an accuracy of the measured values, which is some potentials higher than ever before, the accumulated error caused by transition conditions and the superimposing of noise effects may be within the range of the measured quantities. And despite the rapid speed in the measuring and acquisition process in the range of 1/10 μs, the time expenditure to adjust and to calibrate the whole system correctly increases faster than the automatic measuring process can be accelerated.

Furthermore one must keep in mind that the whole system becomes sort of a "black box". Doubtless to say that the danger exists for the investigator to lose contact and control about what happens on the way from the phenomena to be observed to the final output signal. One must be afraid that exact knowledge will be displaced by believing in the infallible correctness of highly sophisticated and complex systems. The results in experimental analysis are not naturally better and more reliable, the better and more complex the measuring systems used are. Therefore it is still important to use the conventional simple experimental equipments in dependency on the wanted information and the required precision.

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