MECHANIKA
TEORETYCZNA
I STOSOWANA
2/3, 21 (1983)

ONE-DIMENSIONAL CONTINUOUS MODEL OF LATTICE TYPE SURFACE STRUCTURES

ROMAN NAGÓRSKI

Politechnika Warszawska

1. Introduction

The equations of a one-dimensional continuous model of lattice-type structures with densely packed and regularly spaced lattice of elements are discussed in the paper. The equations are obtained by applying the concept of a continuum with internal constraints [1] to the equations of surface-type fibrous medium of Cosserats' type [2] which is a continuous, two-dimensional model of a structure [3].

Considerable costs of the numerical computations of the discret and discretized systems and the known difficulties with founding the solutions to the boundary-value problems are related to the partial equations in two dimensions. Therefore the construction of the one-dimensional model seems to be justified.

The aim of this paper is formulate the equations describing the one-dimensional model of a static problem of the linear (infinitesimal) theory of elastic structures with kinematic-type ideal constraints in their integrable form and with the regular basic surface of the medium. An example of a grid on a cylindrical surface with a circular cross-section and axial-circumferencial lattice-type prismatic bars is also presented.

The proposed constraint equations represent certain generalization of the hyphotesis of flat cross-sections. We assume that the cross-sections perpendicular to the axis of the medium surface independently of the translations and rotations, can be subjected also to the homogeneus deformations in their plane.

The generalization of the forementioned approach which includes the cases of vibrations and stability as well as more general kinematic and kinetic constraints imposed on structures was also developed by the author, however exceeds the scope of this paper

2. Equations of a surface-type fibrous medium with kinematic internal constraints

The equilibrium equations and the static boundary conditions for linear surface-type fibrous medium of Cosserats' type with internal constraints can be presented as [1] - [3]:

$$
\begin{aligned}
&p^{\beta\alpha}|_{\beta}-b^{\alpha}_{\beta}p^{\beta}+q^{\alpha}+r^{\alpha} = 0, \qquad p^{\alpha}|_{\alpha}+b_{\alpha\beta}p^{\alpha\beta}+q+r = 0,\\
&m^{\beta\alpha}|_{\beta}-b^{\alpha}_{\beta}m^{\beta}+e^{\alpha}_{\beta}p^{\beta}+h^{\alpha}+s^{\alpha} = 0, \qquad m^{\alpha}|_{\alpha}+b_{\alpha\beta}m^{\alpha\beta}+e_{\alpha\beta}p^{\alpha\beta}+h+s = 0,
\end{aligned}
\tag{2.1}
$$

and

$$p^{\beta\alpha}n_\beta = \overset{*}{p}{}^\alpha+\varrho^\alpha, \quad p^\beta n_\beta = \overset{*}{p}+\varrho, \quad m^{\beta\alpha}n_\beta = \overset{*}{m}{}^\alpha+\sigma^\alpha, \quad m^\beta n_\beta = \overset{*}{m}+\sigma, \tag{2.2}$$

where $p^{\alpha\beta}$, p^α and $m^{\alpha\beta}$, m^α are components of the cross-sectional forces and moments, q^α, q, h^α, h and $\overset{*}{p}{}^\alpha, \overset{*}{p}, \overset{*}{m}{}^\alpha, \overset{*}{m}$ are components of external surface and boundary load, r^α, r, s^α, s and ϱ^α, ϱ, σ^α, σ are surface and boundary reactions of constraints, $g_{\alpha\beta}$, $b_{\alpha\beta}$, $e_{\alpha\beta}$ denote components of the metric and curvature tensors as well as those of Ricci's pseudotensor of the medium surface π, n_β are components of the unit vector normal to a boundary $\partial\pi$ and tangent to π, $(...)|$ stends for the surface covariant derivative ($\alpha, \beta = 1, 2$).

It is assumed that the constraints are ideal, i.e.

$$\int\limits_{\pi} (r^\alpha\delta v_\alpha+r\delta v+s^\alpha\delta\vartheta_\alpha+s\delta\vartheta)d\pi+\int\limits_{\partial\pi}(\varrho^\alpha\delta v_\alpha+\varrho\delta v+\sigma^\alpha\delta\vartheta_\alpha+\sigma\delta\vartheta)d(\partial\pi) = 0, \tag{2.3}$$

for any variations $\delta v_\alpha, \ldots, \delta\vartheta$ of components of the displacement vector v_α, v and those of the rotation vector ϑ_α, ϑ compatible with constraints in their integrable form

$$[v_\alpha, v, \vartheta_\alpha, \vartheta](u^\beta) = \sum_{K=1}^{N}[v_{\alpha K}, v_K, \vartheta_{\alpha K}, \vartheta_K](u^\beta)\psi_K(u^1), \tag{2.4}$$

where $v_{\alpha K}, v_K, \vartheta_{\alpha K}, \vartheta_K$ are known, sufficiently regular functions of coordinates (u^β) on the surface π, while ψ_K are the unknown generalized displacements. It is also assumed that surface is generated by one-parameter family of any contours, provided that these contours have no common points and are piecewise smooth $\Gamma(u^1)$ $(u^1 \in \langle u^1_1, u^1_2\rangle)$ and can by defined by means of u^2 coordinate. Another assumption is, that if $\Gamma(u^1)$ is an open contour $(\partial\Gamma(u^1) \neq \emptyset)$ then for the part $\partial\pi$ different from $\Gamma(u^1_\alpha)$ the static boundary conditions are given. The boundary conditions on $\partial\pi = \Gamma(u^1_\alpha)$ $(\Gamma(u^1_1) \neq \Gamma(u^1_2))$ can be static or kinematic compatible with constraints (2.4). Eqs. (2.4) can be relatively easily generalized to the case in which the components of the state of displacements are the functions of the derivatives of ψ_K with respect to u^1. In such a case the form of the relevant equations and formulae becomes more complex.

The geometric relations can be formulated as follows [3]:

$$\begin{aligned} \gamma_{\alpha\beta} &= v_\beta|_\alpha - b_{\alpha\beta}v - e_{\alpha\beta}\vartheta, & \gamma_\alpha &= v|_\alpha + b^\beta_\alpha v_\beta + e_\alpha{}^\beta\vartheta_\beta, \\ \varkappa_{\alpha\beta} &= \vartheta_\beta|_\alpha - b_{\alpha\beta}\vartheta, & \varkappa_\alpha &= \vartheta|_\alpha + b^\beta\vartheta_\beta, \end{aligned} \tag{2.5}$$

while the constitutive equations can be defined from the formulae

$$p^{\alpha\beta} = \frac{\partial e}{\partial\gamma_{\alpha\beta}}, \quad p^\alpha = \frac{\partial e}{\partial\gamma_\alpha}, \quad m^{\alpha\beta} = \frac{\partial e}{\partial\varkappa_{\alpha\beta}}, \quad m^\alpha = \frac{\partial e}{\partial\varkappa_\alpha} \tag{2.6}$$

where e is the elastic potential defined as follows

$$e = \frac{1}{2}(A^{\alpha\beta\xi\eta}\gamma_{\alpha\beta}\gamma_{\xi\eta}+A^{\alpha\xi}\gamma_\alpha\gamma_\xi+B^{\alpha\beta\xi\eta}\varkappa_{\alpha\beta}\varkappa_{\xi\eta}+B^{\alpha\xi}\varkappa_{\alpha\xi}), \tag{2.7}$$

where $A^{\alpha\beta\xi\eta}, \ldots, B^{\alpha\xi}$ are elastic rigidity tensors.

If there is known a continuous lattice of Δ family of fibres on the surface then the coordinates of the state of strain of the fibres are defined as follows [3],

$$\begin{aligned} \gamma_\Delta &= \gamma_{\alpha\beta}t^\alpha_\Delta t^\beta_\Delta, & \tilde{\gamma}_\Delta &= \gamma_{\alpha\beta}t^\alpha_\Delta \tilde{t}^\beta_\Delta, & \check{\gamma}_\Delta &= \gamma_\alpha t^\alpha_\Delta, \\ \varkappa_\Delta &= \varkappa_{\alpha\beta}t^\alpha_\Delta t^\beta_\Delta, & \tilde{\varkappa}_\Delta &= \varkappa_{\alpha\beta}t^\alpha_\Delta \tilde{t}^\beta_\Delta, & \check{\varkappa}_\Delta &= \varkappa_\alpha t^\alpha_\Delta, \end{aligned} \tag{2.8}$$

where $t^\alpha_\Delta, \tilde{t}^\alpha_\Delta$ are the components of a field of versors which are tangent and perpendicular to the curves from the Δ family ($\Delta = \mathrm{I}, \mathrm{II}, \ldots$).

The internal stress densities in the Δ fibres can be described using the following formulae

$$
\begin{aligned}
p_\Delta &= R_\Delta \gamma_\Delta, & \tilde{p}_\Delta &= \tilde{R}_\Delta \tilde{\gamma}_\Delta, & \check{p}_\Delta &= \check{R}_\Delta \check{\gamma}, \\
m_\Delta &= S_\Delta \varkappa_\Delta, & \tilde{m}_\Delta &= \tilde{S}_\Delta \tilde{\varkappa}_\Delta, & \check{m}_\Delta &= \check{S} \check{\varkappa}_\Delta,
\end{aligned}
\tag{2.9}
$$

where $R_\Delta, \ldots, S_\Delta$ are measures of the elastic rigidity, and

$$
\begin{aligned}
p^{\alpha\beta} &= \sum_\Delta (p_\Delta t^\alpha_\Delta t^\beta_\Delta + \tilde{p}_\Delta t^\alpha_\Delta \tilde{t}^\beta_\Delta), & p_\alpha &= \sum_\Delta \check{p}_\Delta t^\alpha_\Delta, \\
m^{\alpha\beta} &= \sum_\Delta (m_\Delta t^\alpha_\Delta t^\beta_\Delta + \tilde{m}_\Delta t^\alpha_\Delta \tilde{t}^\beta_\Delta), & m^\alpha &= \sum_\Delta \check{m}_\Delta t^\alpha_\Delta.
\end{aligned}
\tag{2.10}
$$

Substituting (2.8) to (2.9) and then to (2.10) and combining the obtained result with (2.6), (2.7) we arrive at [3]

$$
\begin{aligned}
A^{\alpha\beta\xi\eta} &= \sum_\Delta t^\alpha_\Delta t^\xi_\Delta (t^\beta t^\eta R + \tilde{t}^\beta_\Delta \tilde{t}^\eta_\Delta \tilde{R}_\Delta), & A^{\alpha\xi} &= \sum_\Delta t^\alpha t^\xi_\Delta \check{R}, \\
B^{\alpha\beta\xi\eta} &= \sum_\Delta t^\alpha t^\xi_\Delta (t^\beta t^\eta_\Delta S_\Delta + \tilde{t}^\beta_\Delta \tilde{t}^\eta \tilde{S}_\Delta), & B^{\alpha\xi} &= \sum_\Delta t^\alpha_\Delta t^\xi_\Delta \check{S}_\Delta,
\end{aligned}
\tag{2.11}
$$

When the fibrous medium is a continuous model of a surface grid ($\Delta = \mathrm{I}, \mathrm{II}$ or $\Delta = \mathrm{I}, \mathrm{II}, \mathrm{III}$) then

$$
p_\Delta = \frac{P_\Delta}{\tilde{l}_\Delta}, \quad \tilde{p}_\Delta = \frac{\tilde{P}_\Delta}{\tilde{l}_\Delta}, \quad \check{p} = \frac{\check{P}_\Delta}{\tilde{l}_\Delta}, \quad m_\Delta = \frac{M_\Delta}{\tilde{l}_\Delta}, \quad \tilde{m} = \frac{\tilde{M}_\Delta}{\tilde{l}_\Delta}, \quad \check{m}_\Delta = \frac{\check{M}_\Delta}{\tilde{l}_\Delta},
\tag{2.12}
$$

where $P_\Delta, \tilde{P}_\Delta, \check{P}_\Delta$ are respectively longitudinal forces and shear tangent and normal to π, $M_\Delta, \tilde{M}_\Delta, \check{M}_\Delta$ are respectively torques and couples tangent and normal to π in the middle cross-sections of the bars of Δ family, and $\tilde{l}_\Delta$ is a distance between adjacent curves of a discret lattice of bars axes of the structure. Moreover

$$
\begin{aligned}
R &= \frac{E_\Delta A_\Delta}{\tilde{l}_\Delta}, & \tilde{R}_\Delta &= \frac{12 E_\Delta \check{J}}{\tilde{l}_\Delta l^2_\Delta}, & \check{R}_\Delta &= \frac{12 E \tilde{J}_\Delta}{\tilde{l}_\Delta l^2_\Delta}, \\
S_\Delta &= \frac{G_\Delta J_\Delta}{\tilde{l}_\Delta}, & \tilde{S}_\Delta &= \frac{E_\Delta \tilde{J}_\Delta}{\tilde{l}_\Delta}, & \check{S}_\Delta &= \frac{E_\Delta \check{J}_\Delta}{\tilde{l}_\Delta},
\end{aligned}
\tag{2.13}
$$

where $E_\Delta, G_\Delta, l_\Delta, A_\Delta, J_\Delta, \tilde{J}_\Delta, \check{J}_\Delta$ are the Young moduls, the torsional modulus, the length, the cross-section surface area, the polar and principal moments respectively of the cross-sections of bars from the Δ family [3].

3. Equations of the one-dimensional continuous model

Eliminating from (2.1) - (2.3) the components of the constraint reactions and using (2.4) a generalized equilibrium equations and boundary conditions, i.e. Lagrange-type equations of the second kind [1] are obtained

$$(3.1)\qquad \begin{aligned} &\Psi'_K+\Phi_K+F_K = 0, \quad u^1 \in (u^1_1, u^1_2) \quad ((\ldots)' = d(\ldots)/du^1),\\ &\Psi_K = G_{K\alpha} \quad \text{or} \quad \psi_K = \psi_{K\alpha}, \quad u^1 = u^1_\alpha (K = 1, 2, \ldots, N), \end{aligned}$$

where Ψ_K, Φ_K are the generalized internal forces, F_K, $G_{K\alpha}$ the external forces, $\psi_{K\alpha}$ the generalized boundary displacements

$$(3.2)\qquad \begin{aligned} \Psi_K &= \int\limits_{\Gamma(u^1)} (p^{1\alpha}v_{\alpha K}+p^1 v_K+m^{1\alpha}\vartheta_{\alpha K}+m^1\vartheta_K)\frac{\sqrt{g}}{\sqrt{g_{22}}}\,d\Gamma,\\ \Phi_K &= \int\limits_{\Gamma(u^1)} (p^{\alpha\beta}\gamma_{\alpha\beta K}+p^\alpha\gamma_{\alpha K}+m^{\alpha\beta}\varkappa_{\alpha\beta K}+m^\alpha\varkappa_{\alpha K})\frac{\sqrt{g}}{\sqrt{g_{22}}}\,d\Gamma,\\ F_K &= \int\limits_{\Gamma(u^1)} (q^\alpha v_{\alpha K}+q v_K+h^\alpha\vartheta_{\alpha K}+h\vartheta_K)\frac{\sqrt{g}}{\sqrt{g_{22}}}\,d\Gamma+\\ &\quad +\sum (\overset{*}{p}{}^\alpha v_{\alpha K}+\overset{*}{p}v_K+\overset{*}{m}{}^\alpha\vartheta_{\alpha K}+\overset{*}{m}\vartheta_K)L,\\ G_{K\beta} &= (-1)^\beta \int\limits_{\Gamma(u^1)} (\overset{*}{p}{}^\alpha v_{\alpha K}+\overset{*}{p}v_K+\overset{*}{m}{}^\alpha\vartheta_{\alpha K}+\overset{*}{m}\vartheta_K)\frac{\sqrt{g}}{\sqrt{g_{22}}}\,d\Gamma, \end{aligned}$$

while

$$(3.3)\qquad \begin{aligned} &\gamma_{\alpha\beta K} = v_{\beta K}|_\alpha - b_{\alpha\beta}v_K - e_{\alpha\beta}\vartheta_K, \quad &&\gamma_{\alpha K} = v_K|_\alpha + b^\beta_\alpha v_{\beta K}+e^\beta_\alpha\vartheta_K,\\ &\varkappa_{\alpha\beta K} = \vartheta_{\beta K}|_\alpha - b_{\alpha\beta}\vartheta_K, &&\varkappa_{\alpha K} = \vartheta_K|_\alpha + b^\beta_\alpha\vartheta_{\beta K} \end{aligned}$$

and $Ldu^1 = d(\partial\pi)$ on the part of $\partial\pi$ which different then $\Gamma(u^1_\alpha)$

$$(3.4)\qquad L(u^1) = \sqrt{g_{11}+2g_{12}\frac{du^2}{du^1}+g_{22}\left(\frac{du^2}{du^1}\right)^2}.$$

Substituting RHS of Eqs (2.4) to Eqs (2.5) and then to the constitutive equations derived from Eqs (2.6), (2.7) we obtain the components of the strain and stress states as the functions of the generalized displacements ψ_K and their derivatives ψ'_K. After substituting these functions in formulae $(3.2)_{1,2}$ we arrive at the constitutive equations of one-dimensional model

$$(3.5)\qquad \Psi_K = \sum_{L=1}^{N} (\overset{1}{\Psi}_{KL}\psi_L+\overset{2}{\Psi}_{KL}\psi'_L), \qquad \Phi_K = \sum_{L=1}^{N} (\overset{1}{\Phi}_{KL}\psi_L+\overset{2}{\Phi}_{KL}\psi'_L),$$

where $\overset{\alpha}{\Psi}_{KL}$, $\overset{\alpha}{\Phi}_{KL}$ ($\alpha = 1, 2$; $K, L = 1, 2, \ldots, N$) are generalized elastic rigidities

$$(3.6)\qquad \begin{aligned} \overset{1}{\Psi}_{KL} &= \int\limits_{\Gamma(u^1)} (A^{1\beta\xi\eta}\gamma_{\xi\eta L}v_{\beta\varkappa}+A^{1\xi}\gamma_{\xi L}v_K+B^{1\beta\xi\eta}\varkappa_{\xi\eta L}\vartheta_{\beta K}+B^{1\xi}\varkappa_{\xi L}\vartheta_K)\frac{\sqrt{g}}{\sqrt{g_{22}}}\,d\Gamma,\\ \overset{2}{\Psi}_{KL} &= \int\limits_{\Gamma(u^1)} (A^{1\beta 1\eta}v_{\eta L}v_{\beta\varkappa}+A^{11}v_L v_K+B^{1\beta 1\eta}\vartheta_{\eta L}\vartheta_{\beta K}+B^{11}\vartheta_L\vartheta_K)\frac{\sqrt{g}}{\sqrt{g_{22}}}\,d\Gamma,\\ \overset{1}{\Phi}_{KL} &= \int\limits_{\Gamma(u^1)} (A^{\alpha\beta\xi\eta}\gamma_{\eta\xi L}\gamma_{\alpha\beta K}+A^{\alpha\xi}\gamma_{\xi L}\gamma_{\alpha K}+B^{\alpha\beta\xi\eta}\varkappa_{\xi\eta L}\varkappa_{\alpha\beta K}+B^{\alpha\xi}\varkappa_{\xi L}\varkappa_{\alpha K})\frac{\sqrt{g}}{\sqrt{g_{22}}}\,d\Gamma, \end{aligned}$$

$$\text{(3.6) [cont.]} \quad \overset{2}{\Phi}_{KL} = \int\limits_{\Gamma(u^1)} (A^{\alpha\beta 1\eta} v_{,\eta L} \gamma_{\alpha\beta K} + A^{\alpha 1} v_L \gamma_{\alpha K} + B^{\alpha\beta 1\eta} \vartheta_{,\eta L} \varkappa_{\alpha\beta K} + B^{\alpha 1} \vartheta_L \varkappa_{\alpha K}) \frac{\sqrt{g}}{\sqrt{g_{22}}} d\Gamma.$$

Substituting RHS of Eqs (3.5) into Eqs (3.1) a system of the governing equations describing the model is obtained. This is a system of the ordinary linear differential equations and the boundary conditions. After solving the problem the components of the states of displacement, strain and stress in the medium can be obtained from Eqs (2.4) - (2.7). The constraint reactions, which can characterise the accuracy of the one-dimensional model [4] may be obtained from Eqs (2.1), (2.2). Using Eqs (2.4), (2.8), (2.9), (2.12) the displacements and rotations of structural nodes as well as the forces, couples and torques in the cross-sections of bars can be determined.

4. Cylindrical grid

A surface-type grid designed on a cylindrical surface and made of the two families of prismatic bars which represent a regular and dense axially-circumferential lattice will be considered in this section (see Fig. 1).

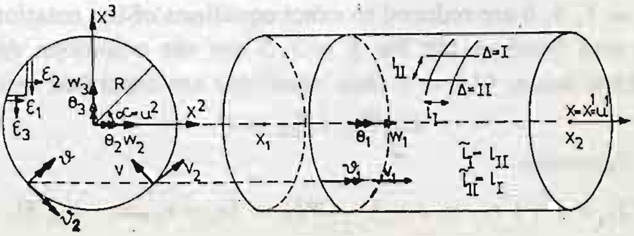

Fig. 1

In this case

$$t_I^1 = t_I^2 = t_{II}^2 = -\tilde{t}_{II}^1 = 1, \quad \tilde{t}_I^2 = \tilde{t}_I^1 = t_{II}^1 = \tilde{t}_{II}^2 = 0. \tag{4.1}$$

Using Eqs (4.1), (2.8), (2.10), (2.11) the governing relations of the cylindrical grid can be obtained easily.

Let us take into account the following form of the constraint equations (2.4) (see Fig. 1)

$$\begin{aligned}
v_1 &= w_1 + R(\Theta_2 \sin\alpha - \Theta_3 \cos\alpha),\\
v_2 &= -w_2 \sin\alpha + w_3 \cos\alpha + R\left[\Theta_1 + \varepsilon_1 \cos 2\alpha - \frac{1}{2}(\varepsilon_2 - \varepsilon_3)\sin 2\alpha\right],\\
v &= -w_2 \cos\alpha - w_3 \sin\alpha - R(\varkappa_1 \sin 2\alpha + \varkappa_2 \cos^2\alpha + \varkappa_3 \sin^2\alpha),\\
\vartheta_1 &= \Theta_1 - \Theta + \xi_1 \cos 2\alpha - \frac{1}{2}\xi \sin 2\alpha,\\
\vartheta_2 &= -\Theta_2 \sin\alpha + \Theta_3 \cos\alpha - R(\varkappa_1 \sin 2\alpha + \varkappa_2 \cos^2\alpha + \varkappa_3 \sin^2\alpha),\\
\vartheta &= -\Theta_2 \cos\alpha + \Theta_3 \sin\alpha - \frac{R}{2}\left[\lambda + \varkappa_1 \cos 2\alpha - \frac{1}{2}(\varkappa_2 - \varkappa_3)\sin 2\alpha\right],
\end{aligned} \tag{4.2}$$

where $\psi^T = [w_1, w_2, \ldots, \lambda]$ are the generalized displacements, which are unknown functions the argument $u^1 = x$ ($u^2 = \alpha$), while $\psi_1^T = [w_1]$ is the parameter of extension, $\psi_2^T = [w_2, \Theta_3]$ and $\psi_3^T = [w_3, \Theta_2]$ the bending parameters, $\psi_4^T = [\Theta_1, \Theta, \lambda]$ the parameters of torsion, $\psi_5^T = [\varepsilon_1, \zeta_1, \varkappa_1]$ the parameters of homogeneous shape deformation of the cross-section $x = \text{const}$, $\psi_6^T = [\varepsilon_2, \varepsilon_3, \varkappa_2, \varkappa_3, \xi]$ the parameters of homogeneous linear deformation of this cross-section. It is assumed that the cross-section of the structure is subjected to a rigid displacement and rotation defined by displacements w_i and rotations Θ_i and to a homogeneous deformation in its plane described by $\varepsilon_i (i = 1, 2, 3)$. The remaining parameters describe the "free" rotations ϑ_α, ϑ [3]. The conditions $\gamma_\alpha = 0$, $\gamma_{12} - \gamma_{21} = 0$ lead to the classical of the Kirchhoff-Love's theory of shells with continuous structure and to the Bernoulli-Timoshenko's flat cross-section hipotheses with adequate constraints imposed on parameters $\Theta_2, \Theta_3, \Theta, \lambda, \zeta_1, \zeta, \varkappa_1, \varkappa_2, \varkappa_3$.

Applying the procedure described in Sec. 3 we obtain a system of equations

$$\mathbf{L}_k \boldsymbol{\Psi}_k + \mathbf{F}_k = \mathbf{0}, \quad x \in (x_1, x_2); \quad \boldsymbol{\alpha}_k \boldsymbol{\Psi}_k = \mathbf{G}_{k\alpha} \text{ lub } \boldsymbol{\Psi}_k = \boldsymbol{\Psi}_{k\alpha}, \quad x = x_\alpha, \tag{4.3}$$

with the matrices of the ordinary differential operators $\mathbf{L}_k$ and $\boldsymbol{\alpha}_k$ with derivatives at most of the second and first order, respectively, and with the rigidity dependent coefficients R_Δ, $\tilde{R}_\Delta$, ..., $\check{S}_\Delta$ (see (2.13)).

Eqs (4.3) for $k = 1, 4, 6$ are reduced to exact equations of the rotationally-symmetrical extension, torsion and bending [5], for $k = 2, 3$ are the equations describing bending of a Timoshenko-type beam. If $k = 6$ then equations are separated into two system

$$\mathbf{L}_{6\alpha} \boldsymbol{\Psi}_{6\alpha} + \mathbf{F}_{6\alpha} = 0 \tag{4.4}$$

for the unknown functions

$$\boldsymbol{\Psi}_{61}^T = [\varepsilon_2 + \varepsilon_3, \varkappa_2 + \varkappa_3], \quad \boldsymbol{\Psi}_{62}^T = [\varepsilon_2 - \varepsilon_3, \varkappa_2 - \varkappa_3, \xi]. \tag{4.5}$$

References

1. C. WOŹNIAK, *Constrained continuous media I.* General theory, Bull. Acad. Polon. Sci., Série. Techn., 21, 3, 1973
2. C. WOŹNIAK, *Theory of fibrous media. I. II.*, Arch. of Mech. 17, 5-6, 1965
3. C. WOŹNIAK, *Lattice-type surface structures*, PWN, Warsaw 1970 (in polish)
4. C. WOŹNIAK, *On the tolerance approach to solid mechanics*, Bull. Acad. Polon. Sci., Série Sci. Techn. (in print)
5. R. NAGÓRSKI, *Bending of cylindrical lattice shell under rotationally symmetric load*, AIL, 24, 4, 1978 (in polish)

Резюме

УРАВНЕНИЯ ОДНОРАЗМЕРНОЙ СПЛОШНОЙ МОДЕЛИ СЕТЧАТЫХ ПОВЕРХНОСТНЫХ КОНСТРУКЦИЙ

В данной работе выведены уравнения одноразмерной и сплошной модели плотных и регулярных сетчатых поверхностных конструкций. Эти уравнения получено, применяя идеи континуум с внутренними связями и уравнения волокнистой поверхностной среды типа Коссерат. Рассмотрено случай статики по линейной теории, интегрированные связи кинематического типа и стержневые конструкции. Рассмотрено также пример цилиндрической системы типа ростверка.

S t r e s z c z e n i e

JEDNOWYMIAROWY MODEL CIĄGŁY SIATKOWYCH DŹWIGARÓW POWIERZCHNIOWYCH

Przedmiotem referatu są równania jednowymiarowego modelu ciągłego sprężystych siatkowych dźwigarów powierzchniowych o gęstej regularnej siatce elementów. Równania te uzyskano stosując koncepcję kontinuum z więzami wewnętrznymi do równań powierzchniowego ośrodka włóknistego typu Cosseratów, będącego ciągłym dwuwymiarowym modelem dźwigara. W komunikacie ograniczono rozważania do przypadku statyki, teorii liniowej, idealnych więzów całkowalnych typu kinematycznego dla konstrukcji o powierzchni podstawowej w postaci jednoparametrowej rodziny konturów. Przykładowo rozpatrzono ruszt cylindryczny.

Praca została złożona w Redakcji dnia 5 stycznia 1983 roku