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EQUATIONS OF THE SHELIS WITH INCLUSIONS ALONG ONE OF THE PARAMETER LINES

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1. Basic equations of shells

In the paper of Cz. WOŹNIAK [1], [2] bases of analytical mechanics of material continuum have been given and among other things equations of motion and boundary conditions have been formulated for shells as a body with internal constraints.

For static problems, taking the function of motion defined by the equation

$$\chi(X, t) = \psi(Z, t)+d(Z, t)Y, \quad \begin{array}{l} X \in B_R = \Pi \times F \\ Z = (Z^1, Z^2) \in \Pi, \qquad Y \in F \end{array} \tag{1}$$

the equations of motion have the form [1]

$$\begin{array}{l} H^{kK}{}_{,K}+h^k+f^k = 0, \\ M^{kK}{}_{,K}+m^k+e^k = 0, \end{array} \quad Z \in \Pi \tag{2}$$

and the boundary conditions are defined by the following relations [1]

$$\begin{array}{l} H^{kK}n_K = p^k+\mu^\varrho \dfrac{\partial \beta_\varrho}{\partial \psi_k} - \dfrac{d}{ds}\left(\mu^\varrho \dfrac{\partial \beta_\varrho}{\partial \psi_{k,s}}\right), \\ M^{kK}n_K = c^k+\mu^\varrho \dfrac{\partial \beta^\varrho}{\partial d_k} - \dfrac{d}{ds}\left(\mu^\varrho \dfrac{\partial \beta^\varrho}{\partial d_{k,s}}\right). \end{array} \quad Z \in \partial\Pi \tag{3}$$

The functions ψ and d may be dependent on each other and satisfy certain conditions [1]

$$\begin{array}{ll} \alpha_\nu(Z, \psi(Z, t), d(Z, t), \overline{\nabla}\psi(Z, t), \overline{\nabla}d(Z, t)) = 0, & Z \in \Pi \\ \beta_\varrho(Z, \psi(Z, t), d(Z, t), \partial\psi(Z, t), \partial d(Z, t)) = 0, & Z \in \partial\Pi. \end{array} \tag{4}$$

2. Kinetic contact conditions for shells with inclusions

In order to take into account the influence of the inclusion on the motion of the shell, we also handle the inclusion as material continuum [3].

Let us assume that the notion of inclusion from the reference configuration I_R to the actual configuration is described by the function $x = \tilde{\chi}(X, t)$, $X = (X^V) \in I_R$.

In accordance with the analytical mechanics of material continuum, the field of external body loads $\tilde{b}$ and the field of external surface loads $\tilde{p}_R$ and the field of internal contact forces $\tilde{t}_R \equiv \tilde{T}_R n_R$ (T_R is the first Piola-Kirchhoff tensor of stress) should satisfy the principle of virtual work [2]

$$\int_{I_R} \tilde{\varrho}_R(\tilde{b}-\ddot{\tilde{\chi}}) \cdot \delta\tilde{\chi} dv_R + \oint_{\partial I_R} \tilde{p}_R \cdot \delta\tilde{\chi} d\sigma_R = \int_{I_R} \tilde{T}_R \cdot \nabla(\delta\tilde{\chi}) dv_R, \tag{5}$$

where $\tilde{\varrho}_R$ is the mass density of material of the inclusion in the reference configuration

We put that the field of external surface loads affecting the boundary of inclusion ∂I_R or its part is given by the field of internal forces of the shell

$$\tilde{p}^k = \bar{T}^{kV} n_V, \tag{6}$$

where $n_R = (n)_V$ is a unit normal vector in the place of contact of the shell with the boundary of inclusion. Assuming that the inclusion is closely connected with the shell, the functions of motion for the shell χ and the inclusion $\tilde{\chi}$ have the same values on the common surface $S_R \equiv \partial B_R \cap \partial I_R$.

Let the motion of arbitrary particle of inclusion be given in the following form:

$$\tilde{\chi}(X, t) = \varphi(Z^1, t) + d_A(Z^1, t) Z^A, \quad Z^1 \in \tilde{\Pi}, \quad Z = (Z^2, Z^3) \in \tilde{F}, \tag{7}$$

where $I_R = \tilde{\Pi} \times \tilde{F}$ is the region occupied by the inclusion in the reference configuration; $\tilde{\Pi}$ is the set of material particles being on the axis of inclusion, $\tilde{F}$ — the set of particles being in the cross-section of inclusion.

Putting that the dimensions of the cross-section of inclusion are $h \times l (Z^2 \in (-0{,}5l; 0{,}5l)$, $Z^3 \in (-0{,}5h; 0{,}5h)$ and taking into account the condition (6) we obtain

$$\begin{aligned}
\Big[\int_{-0,5h}^{0,5h} \tilde{p}^k dZ^3\Big]_{-0,5l}^{0,5l} &= \Big[\int_{-0,5h}^{0,5h} \bar{T}^{kL} n_L dZ^3\Big]_{-0,5l}^{0,5l} = [H^{kl}] n_L,\\
\Big[\int_{-0,5h}^{0,5h} \tilde{p}^k Z^2 dZ^3\Big]_{-0,5l}^{0,5l} &= \Big[Z^2 \int_{-0,5h}^{0,5h} \bar{T}^{kL} n_L dZ^3\Big]_{-0,5l}^{0,5l} = [Z^2 H^{kl}] n_L,\\
\Big[\int_{-0,5h}^{0,5h} \tilde{p}^k Z^3 dZ^3\Big]_{-0,5l}^{0,5l} &= \Big[\int_{-0,5h}^{0,5h} \bar{T}^{kL} Z^3 n_L dZ^3\Big]_{-0,5l}^{0,5l} = [M^{kL}] n_L,
\end{aligned} \tag{8}$$

The integrals in the relations (8) denote the jumps of the internal resultant forces in the section of the shell oriented by a unit vector n_P while crossing the axis of the inclusion.

Using the principle of virtual work (5) and putting the relations (7), (8) and applying to (5) the du Bois-Reymonda lemma we obtain the equations ot motion for the inclusion in the following form

$$\begin{aligned}
[H^{kL}] n_L &= -(\tilde{H}^{k1}{}_{,1} + \tilde{f}^k),\\
[H^{kL} Z^2] n_L &= -(\tilde{H}^{2k1}{}_{,1} + \tilde{h}^{2k} + \tilde{f}^{2k}),\\
[M^{kL}] n_L &= -(\tilde{H}^{3k1}{}_{,1} + \tilde{h}^{3k} + \tilde{f}^{3k}),
\end{aligned} \tag{9}$$

where

$$\tilde{H}^{k1} \equiv \int\limits_{-0,5l}^{0,5l} \int\limits_{-0,5h}^{0,5h} \tilde{T}^{k1} dZ^3 dZ^2,$$

$$\tilde{H}^{Ak1} \equiv \int\limits_{-0,5l}^{0,5l} \int\limits_{-0,5h}^{0,5h} \tilde{T}^{k1} Z^A dZ^3 dZ^2,$$

$$\tilde{h}^{Ak} \equiv \int\limits_{-0,5l}^{0,5l} \int\limits_{-0,5h}^{0,5h} \tilde{T}^{kA} dZ^3 dZ^2, \tag{10}$$

$$\tilde{f}^{k} \equiv \Big[\int\limits_{-0,5l}^{0,5l} \tilde{p}^k dZ^2 \Big]_{-0,5h}^{0,5h},$$

$$\tilde{f}^{Ak} \equiv \Big[\int\limits_{-0,5l}^{0,5l} \tilde{p}^k Z^A dZ^2 \Big]_{-0,5h}^{0,5h}.$$

The equations (9) are kinetic constant conditions for the shells in the place of the occurence of the inclusion and they include its influence on the motion of the shell. We determine the generalized internal forces $\tilde{H}^{k1}$, $\tilde{H}^{Ak1}$ assuming the appropriate form of constitutive equations and for the inclusion being a hyperelastic body $\tilde{T}^{kV} = \tilde{\varrho}_R \dfrac{\partial \tilde{\sigma}}{\partial \tilde{\chi}_{k,V}}$; $\tilde{\sigma}$ is the strain energy function.

Equations of motion (2), boundary conditions (3) and kinetic contact conditions (9) describe the boundary value problem being discussed.

3. Cylindrical shells with ring inclusions loaded axially symmetrically

We write the equations formulated above in the cylindrical coordinates R, X^1, X^2 assuming that the considered values do not depend on variable X^1 (fig. 1).

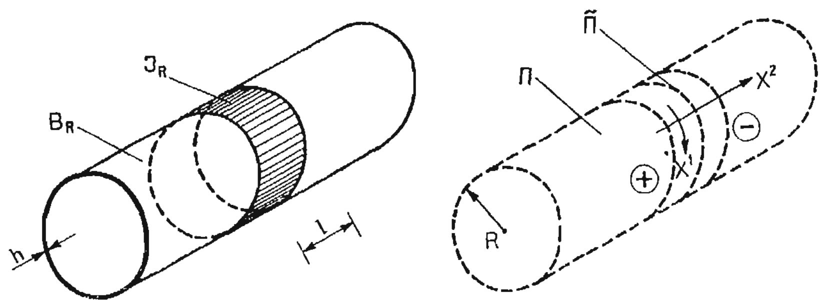

Fig. 1

We put that the shell and the inclusion are made of isotropic material. The equations given below will refer to the case in which the constraints (4) are determined

$$\alpha_1 = d^V d_V - 1 = 0 \tag{11}$$

which by the linearization leads to $d^3 = d_3 = 1$. The constraints assumed so describe the Reissner theory of shells.

From the equations of motion given by eq. (2) we obtain four differential equations for cylindrical shells loaded axially symmetrically

$$\begin{bmatrix} -D^{2222} & 0 & -\bar{D}^{2222} \\ 0 & D^{3232} & 0 \\ -\bar{D}^{2222} & 0 & \bar{\bar{D}}^{2222} \end{bmatrix} \begin{bmatrix} \bar{\psi}_{2,22} \\ \bar{\psi}_{3,22} \\ \bar{d}_{2,22} \end{bmatrix} + \begin{bmatrix} 0 & -D^{223} & 0 \\ -D^{223} & 0 & D^{322} \\ 0 & D^{322} & 0 \end{bmatrix} \begin{bmatrix} \bar{\psi}_{2,2} \\ \bar{\psi}_{3,2} \\ \bar{d}_{2,2} \end{bmatrix} +$$
$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{R}D^{113} & 0 \\ 0 & 0 & D^{22} \end{bmatrix} \begin{bmatrix} \bar{\psi}_2 \\ \bar{\psi}_3 \\ \bar{d}_2 \end{bmatrix} + \begin{bmatrix} -f^2 \\ f^3 \\ -e^2 \end{bmatrix} = 0, \tag{12}$$

and

$$2\lambda_1 - 'D^{223}\bar{\psi}_{2,2} + \bar{D}^{3232}\bar{\psi}_{3,2} + (\bar{D}^{322} - '\bar{D}^{223})\bar{d}_{2,2} - D^{33}\bar{\psi}_3 + e^3 = 0$$

where λ^1 is Lagrange multipler which corresponds to eq. (11). In the above system equations the components of function of motion are denoted in the following manner

$$\bar{\psi}_V \equiv \psi_V - \mathring{\psi}_V, \quad \bar{d}_V \equiv d_V - \mathring{d}_V, \tag{13}$$

where $\mathring{\psi} = (0, X^2, R)$, $\mathring{d} = (0, 0, 1)$. The factors at the unknown components of function of motion depend on Lamé constants μ, λ and thickness h and radius R of the cylindrical shell.

$$\begin{aligned} 'D^{223} &= \frac{6R}{h^2}\,'\bar{D}^{223} = RD^{223} = h\lambda, \\ D^{3232} &= D^{322} = D^{22} = \frac{12R}{h^2}\bar{D}^{3232} = \frac{12R}{h^2}\bar{D}^{322} = h\mu, \\ D^{2222} &= \frac{12}{h^2}\bar{\bar{D}}^{2222} = 12R\bar{D}^{2222} = h(\lambda+2\mu), \\ D^{113} &= \left[1+\frac{1}{3}\left(\frac{h}{2R}\right)^2 + \frac{1}{5}\left(\frac{h}{2R}\right)^4 + \ldots\right]\frac{h}{R}(\lambda+2\mu) \\ D^{33} &= -\left[\frac{1}{3}\left(\frac{h}{2R}\right)^2 + \frac{1}{5}\left(\frac{h}{2R}\right)^4 + \ldots\right]\frac{h}{R}(\lambda+2\mu) + \frac{h}{R}\lambda. \end{aligned} \tag{14}$$

The kinetic contact conditions for shell (9) should be reduced to the form including the components of the function of motion of shell ψ and d only. Taking into account the equality of the deformation functions of the shell and of the inclusion on the common surface S_R we obtain the mentioned below relations between components

$$\bar{\psi}_V = \frac{1}{2}(\bar{\varphi}_V^+ + \bar{\psi}_V^-), \quad \bar{d}_{2V} = -\frac{1}{l}(\bar{\psi}_V^+ - \bar{\psi}_V^-), \quad \bar{d}_{3V} = \frac{1}{2}(\bar{d}_V^+ + \bar{d}_V^-). \tag{15}$$

We determine with sing „+" the components of the vector of motion for the part of shell which is oriented by the positive unit normal vector $\boldsymbol{n}_R$ while crossing the axis of the inclusion and with sing „−" those for the one oriented by the vector $-\boldsymbol{n}_R$. The kinetic contact conditions by the application of the relations (15) have the form

$$
\begin{bmatrix}
D^{2222} & 0 & \bar{D}^{2222} & -D^{2222} & 0 & -\bar{D}^{2222} \\
0 & D^{3232} & 0 & 0 & D^{3232} & 0 \\
-\frac{l}{2}D^{2222} & 0 & -\frac{l}{2}\bar{D}^{2222} & -\frac{l}{2}D^{2222} & 0 & -\frac{l}{2}\bar{D}^{2222} \\
0 & -\frac{l}{2}D^{3232} & 0 & 0 & -\frac{l}{2}D^{3232} & 0 \\
\bar{D}^{2222} & 0 & \bar{\bar{D}}^{2222} & -\bar{D}^{2222} & 0 & -\bar{\bar{D}}^{2222} \\
0 & \bar{D}^{3232} & 0 & 0 & -\bar{D}^{3232} & 0
\end{bmatrix}
\begin{bmatrix}
\bar{\psi}^{+}_{2,2} \\ \bar{\psi}^{+}_{3,2} \\ \bar{d}^{+}_{2,2} \\ \bar{\psi}^{-}_{2,2} \\ \bar{\psi}^{-}_{3,2} \\ \bar{d}^{-}_{2,2}
\end{bmatrix}
+
$$

$$
+\begin{bmatrix}
0 & D^{223} & 0 & 0 & -D^{223} & 0 \\
\frac{1}{l}L^{322} & -\frac{1}{2R}L^{113} & D^{322} & -\frac{1}{l}L^{322} & -\frac{1}{2R}L^{113} & -D^{322} \\
\frac{1}{l}L^{2222} & -\frac{l}{2}D^{223}-\frac{1}{2}L^{322} & 0 & -\frac{1}{l}L^{2222} & -\frac{l}{2}D^{223}-\frac{1}{2}L^{322} & 0 \\
0 & \frac{1}{l}L^{2323} & -\frac{l}{2}D^{322}-\frac{1}{2}L^{3232} & 0 & -\frac{1}{l}L^{2323} & -\frac{l}{2}D^{322}-\frac{1}{2}L^{3223} \\
0 & \frac{1}{l}L^{2332} & -\frac{1}{2}L^{3232} & 0 & -\frac{1}{l}L^{2332} & -\frac{1}{2}L^{2332} \\
\frac{1}{l}L^{2233} & -\frac{1}{2}L^{333} & \bar{D}^{322} & -\frac{1}{l}L^{2233} & -\frac{1}{2}L^{333} & -\bar{D}^{322}
\end{bmatrix}
\begin{bmatrix}
\psi^{+}_{2} \\ \psi^{+}_{3} \\ \bar{d}^{+}_{2} \\ \psi^{-}_{2} \\ \psi^{-}_{3} \\ \bar{d}^{-}_{2}
\end{bmatrix}
+
\begin{bmatrix}
\tilde{f}^{2} \\ \tilde{f}^{3} \\ \tilde{f}^{22} \\ \tilde{f}^{23} \\ \tilde{f}^{32} \\ \tilde{f}^{33}
\end{bmatrix}
= [0] \qquad (16)
$$

where

$$
\tilde{f}^{2} = \Big[\int_{-0,5l}^{0,5l} \tilde{p}^{2}dZ^{2}\Big]_{-0,5h}^{0,5h}, \quad \tilde{f}^{3} = \Big[\int_{-0,5l}^{0,5l} \tilde{p}^{3}dZ^{2}\Big]_{-0,5h}^{0,5h},
$$

$$
\tilde{f}^{22} = \Big[\int_{-0,5l}^{0,5l} \tilde{p}^{2}Z^{2}dZ^{2}\Big]_{-0,5h}^{0,5h}, \quad \tilde{f}^{23} = \Big[\int_{-0,5l}^{0,5l} \tilde{p}^{3}Z^{2}dZ^{2}\Big]_{-0,5h}^{0,5h}, \qquad (17)
$$

$$
\tilde{f}^{32} = \Big[\int_{-0,5l}^{0,5l} \tilde{p}^{2}Z^{3}dZ^{2}\Big]_{-0,5h}^{0,5h}, \quad \tilde{f}^{33} = \Big[\int_{-0,5l}^{0,5l} \tilde{p}^{3}Z^{3}dZ^{2}\Big]_{-0,5h}^{0,5h},
$$

and

$$
\begin{aligned}
L^{113} &= \frac{lh}{R}\left[1+\frac{1}{3}\left(\frac{h}{2R}\right)^2+\frac{1}{5}\left(\frac{h}{2R}\right)^4+\dots\right](\tilde{\lambda}+2\tilde{\mu}),\\
L^{333} &= \frac{lh}{R}\left[\frac{1}{3}\left(\frac{h}{2R}\right)^2+\frac{1}{5}\left(\frac{h}{2R}\right)^4+\dots\right](\tilde{\lambda}+2\tilde{\mu})+\frac{lh}{R}\tilde{\lambda},\\
L^{2323} &= \frac{l^3h}{12R^2}\left[1+\frac{1}{3}\left(\frac{h}{2R}\right)^2+\frac{1}{5}\left(\frac{h}{2R}\right)^4+\dots\right]+lh\tilde{\mu},\\
L^{2222} &= lh(\tilde{\lambda}+2\tilde{\mu}),\\
L^{2233} &= RL^{322} = lh\tilde{\lambda},\\
L^{3232} &= L^{2332} = L^{3223} = lh\tilde{\mu},
\end{aligned}
\tag{18}
$$

$\tilde{\mu}$, $\tilde{\lambda}$ — Lamé constants for material of the inclusion:

The first three equations (12) together with kinetic contact conditions (16) and suitable boundary conditions let us calculate the components of the function of motion of the shell and the internal resultant forces in the shell and the inclusion and also the reaction forces of constraints loading the shell which secure the deformation consistent with the assumed constraints. The internal resultant forces may be calculated from relations given in [3] and for the shell being considered they have the following form

$$
\begin{aligned}
M^{11} &= \frac{\lambda h^3}{12}d_{2,2}-(\lambda+2\mu)h\left[\frac{1}{3}\left(\frac{h}{2R}\right)^2+\frac{1}{5}\left(\frac{h}{2R}\right)^4+\dots\right]\overline{\psi}_3,\\
M^{22} &= \frac{(\lambda+2\mu)h^3}{12R}\overline{\psi}_{2,2}+\frac{(\lambda+2\mu)h^3}{12}\overline{d}_{2,2},\\
M^{32} &= \frac{\mu h^3}{12R}(\overline{\psi}_{3,2}+\overline{d}_2),\\
H^{11} &= \lambda h\overline{\psi}_{2,2}+\frac{(\lambda+2\mu)h}{R}\left[1+\frac{1}{3}\left(\frac{h}{2R}\right)^2+\frac{1}{5}\left(\frac{h}{2R}\right)^4+\dots\right]\overline{\psi}_3\\
H^{22} &= (\lambda+2\mu)h\overline{\psi}_{2,2}+\frac{(\lambda+2\mu)h^3}{12R}\overline{d}_{2,2}+\frac{\lambda h}{R}\overline{\psi}_3,\\
H^{32} &= \mu h\overline{\psi}_{3,2}.\\
M^{12} &= M^{21} = M^{31} = H^{12} = H^{21} = H^{31} = 0.
\end{aligned}
\tag{19}
$$

The internal forces in the inclusion which are different from zero are determined by the relations

$$
\begin{aligned}
\tilde{H}^{11} &= \frac{(\tilde{\lambda}+2\tilde{\mu})lh}{2R}\left[1+\frac{1}{3}\left(\frac{h}{2R}\right)^2\frac{1}{5}\left(\frac{h}{2R}\right)^4+\dots\right](\overline{\psi}_3^{+}+\overline{\psi}_3^{-})-\tilde{\lambda}h(\overline{\psi}_2^{+}-\overline{\psi}_2^{-}),\\
\tilde{H}^{31} &= -\frac{(\tilde{\lambda}+2\tilde{\mu})lh}{2}\left[\frac{1}{3}\left(\frac{h}{2R}\right)^2+\frac{1}{5}\left(\frac{h}{2R}\right)^4+\dots\right](\overline{\psi}_3^{+}+\overline{\psi}_3^{-}).
\end{aligned}
\tag{20}
$$

The reaction forces of constraints for the shell may be calculated from [1] and for this shell they have the form

$$\bar{r}^1 = 0$$

$$\bar{r}^2 = -\left(\frac{\lambda+\mu}{R}\bar{\psi}_{3,2}+\frac{\mu}{R}\bar{d}_2\right)\left[1-\frac{Y}{R}+\left(\frac{Y}{R}\right)^2-\left(\frac{Y}{R}\right)^3+\ldots\right]-(\lambda+2\mu)(\bar{\psi}_{2,2}+Y\bar{d}_{2,2}),$$

$$\bar{r}^3 = -(\lambda+\mu)\bar{d}_{2,2}-\mu\bar{\psi}_{3,22}+\frac{\lambda+2\mu}{R^2}\left[1-2\frac{Y}{R}+3\left(\frac{Y}{R}\right)^2-4\left(\frac{Y}{R}\right)^3+\ldots\right]\bar{\psi}_3,$$

$$\bar{s}^1 = 0,$$

$$\bar{s}^2 = \mu(\bar{d}_2+\bar{\psi}_{3,2})-p^2,$$

$$\bar{s}^3 = \frac{\lambda}{h}\left[1+\frac{h}{2R}+\left(\frac{h}{2R}\right)^2+\left(\frac{h}{2R}\right)^3+\ldots\right]\bar{\psi}_3+\lambda\left(\bar{\psi}_{2,2}-\frac{h}{2}\bar{d}_{2,2}\right)-p^3, \quad \text{for} \quad Y=-\frac{h}{2},$$

$$\bar{s}^3 = \frac{\lambda}{h}\left[1-\frac{h}{2R}+\left(\frac{h}{2R}\right)^2-\left(\frac{h}{2R}\right)^3+\ldots\right]\bar{\psi}_3+\lambda\left(\bar{\psi}_{2,2}+\frac{h}{2}\bar{d}_{2,2}\right)-p^3, \quad \text{for} \quad Y=\frac{h}{2}.$$

4. Example

Let us consider a cylindrical shell of length L in which two rings have been placed. The Young's modulus for the material of rings is 20% greater than that for the material of the shell but the Poisson's ratios are the same. The shell is loaded a uniform load p along its length (fig. 2) and simple-supported at its ends. It is assumed that the thickness of shell is $h = 0{,}04$ m, the radius of the middle surface of shell $R = 0{,}60$ m and the width of a ring $l = 0{,}03$ m.

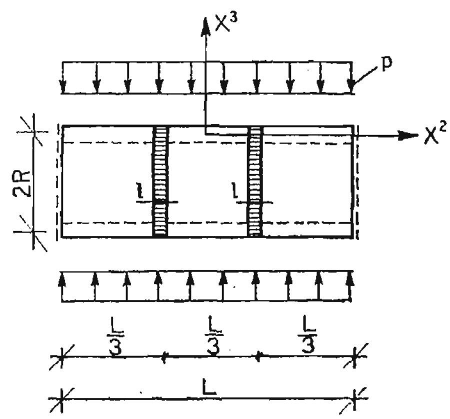

Fig. 2

Solving this example, the region occupied by shell in the reference configuration has been assumed as the sum of three regions $B_R^{\mathrm{I}}, B_R^{\mathrm{II}}, B_R^{\mathrm{III}}$ (fig. 3).

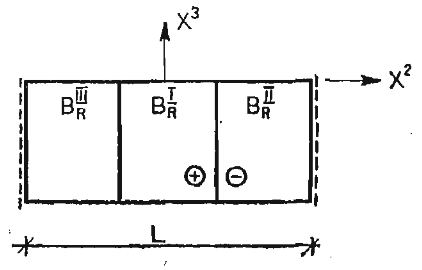

Fig. 3

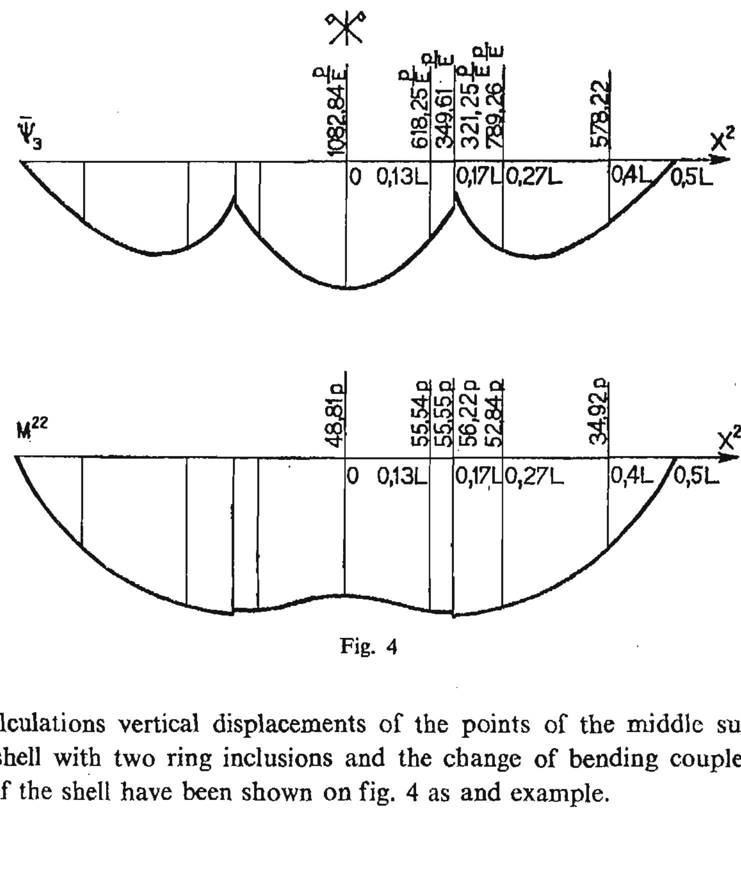

Fig. 4

After calculations vertical displacements of the points of the middle surface of the cylindrical shell with two ring inclusions and the change of bending couple M^{22} along the length of the shell have been shown on fig. 4 as and example.

Reference

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Резюме

УРАВНЕНИЕ ОБОЛОЧЕК С ИНКЛЮЗИЯМИ ВДОЛЬ ОДНОГО СЕМЕЙСТВА ПАРАМЕТРИЧЕСКИХ ЛИНИЙ

Исходя из уравнений теории сплошных сред с связями сформулированной Ч. Возьняком [1, 2] предлагаем математическую модель оболочек из материала о скачкообразных свойствах. Области занятые материалом о свойствах разных от свойств основново материала названы инклюзиями.

Решение проблемы с скачкообразными неоднородностями сведено к решению проблемы для однородного материала с некоторыми связями для состояния напряжения. Получены уравнения можна применить для вычисления оболочек с гибкими инклюзиями вдоль одного семейства параметрических линий на срединной поверхности оболочки.

Streszczenie

RÓWNANIE POWŁOK Z INKLUZJAMI WZDŁUŻ JEDNEJ RODZINY LINII PARAMETRYCZNYCH

Korzystając z równań teorii ośrodków ciągłych z więzami sformułowanej przez Cz. Woźniaka [1, 2], skonstruowano matematyczny model powłok ze skokowymi nieciągłościami własności materiałowych. Obszary zajęte przez materiał mający różne własności od materiału podstawowego nazwano inkluzjami. W sformułowanym modelu rozwiązanie problemu ze skokowymi niejednorodnościami sprowadzone zostało do rozwiązania problemu jak dla materiału jednorodnego, lecz z pewnymi więzami dla stanu naprężenia.

Otrzymane równania mogą być zastosowane do obliczenia powłok z wiotkimi inkluzjami wzdłuż jednej rodziny linii parametrycznych na środkowej powierzchni powłoki.

Praca została złożona w Redakcji dnia 2 lutego 1983 roku