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SOME PRACTICAL PROBLEMS OF DISPLACEMENT AND STRAIN MEASUREMENT BY INCOHERENT SUPERPOSITION OF INTERFEROGRAMS

PIOTR WESOŁOWSKI

Institute of Physics
Technical University
Budapest

In this paper the suitability of using the regulated pathlengh interferometer due to automatization of 3-D displacement and surface strain measurement as well as problems faced in dealing with the accuracy of the measurements are examined. Data needed for determining all three components of displacement vector are stored on single plate. The displacement and deformation of surface point of a disc subjected to diametral compression was measured to investigate the effectiveness of this technique.

Introduction

The quantitative investigation of displacement vector fields by holographic interferometry requires recording in at least three different interferograms. The conventional multiple hologram method [3] gives different perspectives for views through each of the plates.

Considering the automatization of the evaluation of holograms (the first problem-automatization of fringe counting) the important problem is, what kind of interferometer should be used. In this experiment such an interferometer was chosen, which records independent interferograms simultaneously by incoherent superposition of holograms [6]. The most important property from a point of view of automatic fringe inspection is an accurate identification of corresponding object points on each of interferograms; this is because of the same observational direction for all the interferograms.

Regarding the methods of evaluation of the displacement vector, the authors [2] examining the Haines and Hildebrand method, the Bronch-Bruevich single hologram method as well as the improved version of the latter (Dhir-Sikora method), have arrived at a conclusion that by comparing theoretical and experimental results, the most suitable and with the smallest error is the Dhir-Sikora's. This method uses at least four observation points to obtain an overdetermined set of linear simultaneous equations relating the fringe shifts with three unknown components of displacement and obtains the result by the least square principle [11]. The discrete values of displacement components are smoothed out by cubics splines, after this strains are computed.

The most important properties of regulated pathlength interferometer

The recording of different interferograms on a single plate for evaluation of 3-D displacements is made simultaneously. Independency of interferograms is a result of the absence of correlation between beams not belonging together. The uncorrelation can be realized by producing an optical pathlength difference among corresponding beams, which is larger than the coherence length of the laser used in the experiment. In order to be able to reconstruct interferograms separately there must be present so much reference beams as illumination beams.

Holographic displacement measurement

Let us investigate the problem in Descartes orthogonal coordinates:

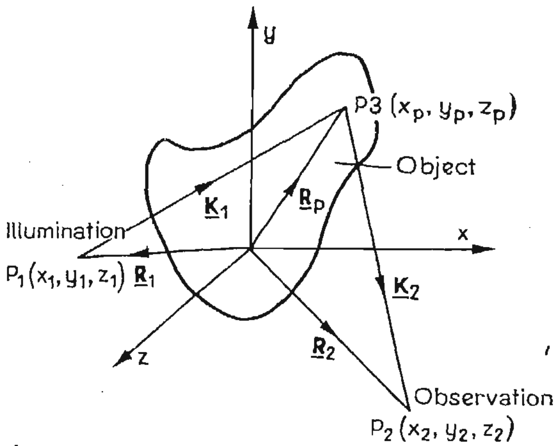

Fig. 1

A displacement vector:

$$\underline{L} = L_x\underline{i}+L_y\underline{j}+L_z\underline{k}. \tag{1}$$

P_3 is a point on the object corresponding to a space vector $\underline{R}_p$, for the object illuminated from a point source defined by $\underline{R}_1$, the reconstructed image can be observed from a point described by $\underline{R}_2$.

Let us define the illumination vector $\underline{K}_1$ and the observation vector $\underline{K}_2$:

$$\underline{R}_p = x_p\underline{i}+y_p\underline{j}+z_p\underline{k}, \tag{2}$$

$$\underline{K}_1 = k\frac{\underline{R}_p-\underline{R}_1}{[\underline{R}_p-\underline{R}_1]} = k\frac{(x_p-x_1)\underline{i}+(y_p-y_1)\underline{j}+(z_p-z_1)\underline{k}}{[(x_p-x_1)^2+(y_p-y_1)^2+(z_p-z_1)^2]^{1/2}} = k\cdot\hat{\underline{K}}_1, \tag{3}$$

$$\underline{K}_2 = k\frac{\underline{R}_2-\underline{R}_p}{|\underline{R}_2-\underline{R}_p|} = k\frac{(x_2-x_p)\underline{i}+(y_2-y_p)\underline{j}+(z_2-z_p)\underline{k}}{[(x_2-x_p)^2+(y_2-y_p)^2+(z_2-z_p)^2]^{1/2}} = k\cdot\hat{\underline{K}}_2 \tag{4}$$

where $\hat{\underline{K}}_1$, $\hat{\underline{K}}_2$ are the unit illumination and observation vectors, respectively, and

$$|\underline{K}_1| = |\underline{K}_2| = k = \frac{2\pi}{\lambda}. \tag{5}$$

is the magnitude of these vectors, with the wavelength λ of the laser light.
The phase shift due to observations of the virtual image from different directions can be written as:

$$\delta = \underline{\underline{K}} \cdot \underline{L} = 2\pi \underline{N}. \tag{6}$$

where

$$2\pi \underline{N} = \underline{\Omega} \tag{7}$$

is fringe locus function.
$\underline{\underline{K}}$ matrix consist of m different sensitivity vectors:

$$\underline{\underline{K}} = \begin{bmatrix} \underline{K}^1 \\ \underline{K}^2 \\ \underline{K}^3 \\ \vdots \\ \underline{K}^m \end{bmatrix} \quad \text{where} \quad \begin{aligned} \underline{K}^1 &= \underline{K}_2^1 - \underline{K}_1^1, \\ \underline{K}^2 &= \underline{K}_2^2 - \underline{K}_1^2, \\ &\vdots \\ \underline{K}^m &= \underline{K}_2^m - \underline{K}_1^m, \end{aligned} \tag{8}$$

and $m \geqq 3$, means number of observations.
Independent on how carefully and how many observations are made, there will be always some errors $\underline{E}$.

$$\underline{E} = \underline{\underline{K}} \cdot \underline{L} - \underline{\Omega}. \tag{9}$$

The purpose of the analysis is to mininalize all of the errors squared, that is:

$$\underline{E}^2 = (\underline{\underline{K}} \cdot \underline{L} - \underline{\Omega})^2. \tag{10}$$

In order to minimize the square of error the partial derivates of Eq. (10) must be zero:

$$\frac{\partial}{\partial L_i} \left\{ \sum_{m=1}^{r} [(\underline{K}_2^m - \underline{K}_1^m) \cdot \underline{L} - \underline{\Omega}^m]^2 \right\} = 0, \tag{11}$$

where $i = x, y, z$

$m = 1, 2, 3, \ldots, r$ is number of all observations.

The solution of Eq. (11) in matrix form is:

$$\underline{L} = (\underline{\underline{K}}^T \cdot \underline{\underline{K}})^{-1} \cdot \underline{\underline{K}}^T \cdot \underline{N} \cdot 2\pi \tag{12}$$

where $\underline{\underline{K}}^T$ is transposed matrix of $\underline{\underline{K}}$.

Experiment

As a model there was used a plexiglass disc with diameter 90 mm, thickness $h = 10$ mm ($E = 3.2 \cdot 10^4$ N/m^2, $\mu = 0.385$) subjected to diametral compression [4].

The optical elements used in experiment: M-mirrors, BS-beamsplitters, COLL.-collimators, PL-microscope lens with pinholes, H-holographic plate (Agfa Geavert 8E75), O-object, L-He-Ne laser with output power of 50 mW.

In the measurement the „zerofringe" method was applied by using an elastic strip,

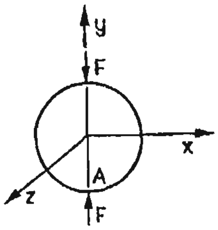

Fig. 2

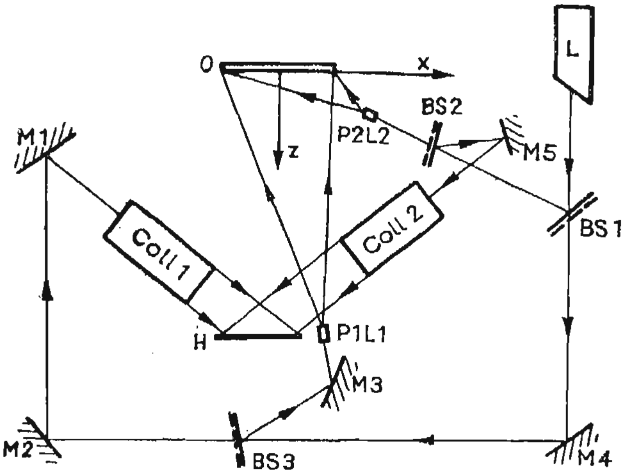

Fig.3

which was stuck between the loading framework and the holographic slab. Two collimators (Carl Zeiss Jena make) were used for producing two plane wave references (Ø 50 mm).

The basic principle of building of holographic interferometers is to maximalise it's sensitivity on the smallest displacement component and to decrease it's sensitivity on the largest displacement component. The illumination points (P1L1, P2L2) were chosen accordingly to sensitivity of the L_z component and L_x, L_y components, respectively. The difference of the optical pathlengths between corresponding beams was $\Delta l = 1820$ mm.

Evaluation of interferograms

Interferograms were recorded in double-exposure method. They were evaluated along horizontal as well as vertical diameter. In the reconstruction process the holographic plate was illuminated by original reference beams; the two independent interferograms were photographed through six observational points (Fig. 4)

Typical fringe pattern belonging to different references are shown on Fig. 5. The numbers of the fringe order were determined semiautomatically on the basis of photographs with interference patterns. In each of investigated points twelve sets of fringe order data were given (six different observational point belong to one reference).

The determination of the number of the fringe order looks as follows: the observer marks the geometrical centre on each fringe (black and white) along the investigated line and then with the aid of a drawing digitizer puts the geometrical data and integer fringe number to a computer connected via interface card. The program based on the least square principle chooses an appropriate polynomial and evaluates fractional fringe order numbers at the required points.

Displacement components along the horizontal diameter were determined using the Eq. 12. Displacement components: L_x, L_y, L_z along the horizontal diameter were computed in 17 points and are shown on Fig. 6. Displacement components along the verical diameter are shown on Fig. 7. For further derivatives $\varepsilon_y = \dfrac{\partial(L_y)}{\partial y}$ only the central range of $y \in$

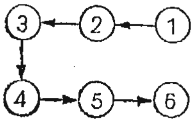

Fig. 4

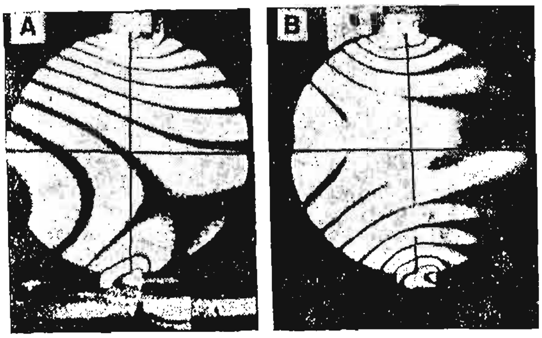

Fig. 5

$\in (-28.125; 28.125)$ is considered because of boundary disturbances due to applied forces. From the Fig. 7 can be seen, that point $A(0; -45; 0)$ practically didn't move so the orthogonal coordinates can be reduced at that point. The rigid body motions are expressed in rotations $\Theta_{x'}$, $\Theta_{y'}$, $\Theta_{z'}$.

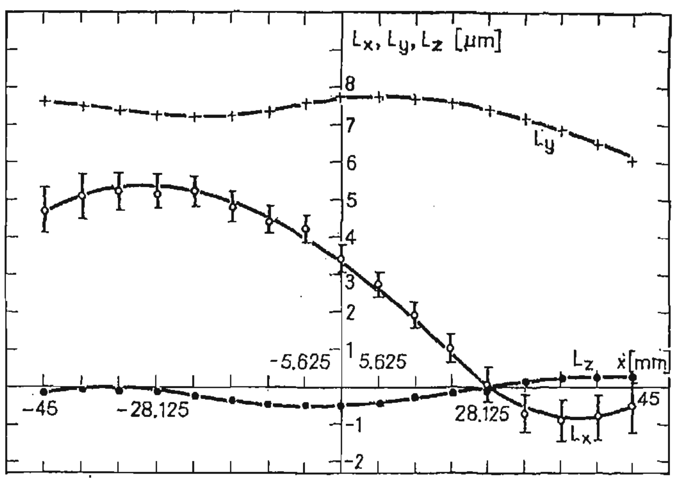

Fig. 6

From the plots of Fig. 6 and Fig. 7: $\Theta_{x'} = 3.67 \cdot 10^{-5}$(rad), $\Theta_{y'} = 6.1 \cdot 10^{-6}$ (rad), $\Theta_{z'} = 3.15 \cdot 10^{-5}$ (rad).

To obtain the value of deformation the values of $L_x(x, y = \text{const})$, $L_y(x, y = \text{const})$, $L_x(x = \text{const}, y)$, $L_y(x = \text{const}, y)$ at discrete points of the surface are needed; afterwards there is a need to smooth the data by fitting an appriopriate curve to it. Smoothed spline functions [1], [8] are appropriate choice for fitting the displacement data as the objective of subsequent differentation. They are attractive for three reasons: their definition is based on the theory of mechanical deformation, second derivates are understood a priori, and their application to interferometric determination of strain has been studied [13]. The values ε_x evaluated along horizontal diameter are shown on Fig. 8 and the values of ε_y are shown on Fig. 9.

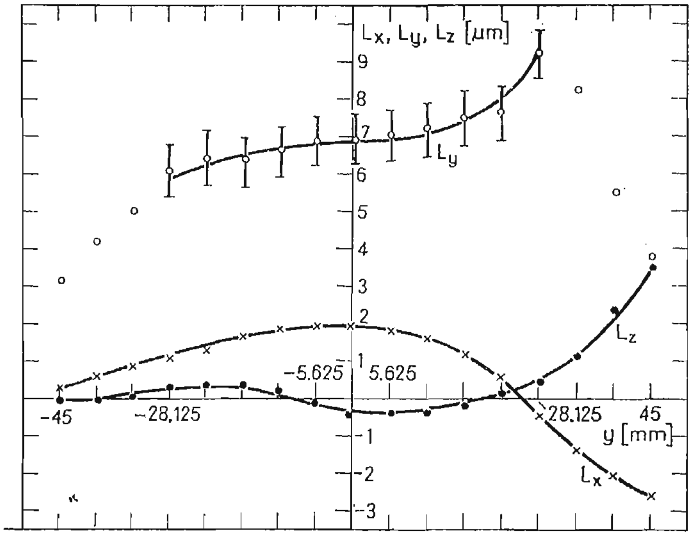

Fig. 7

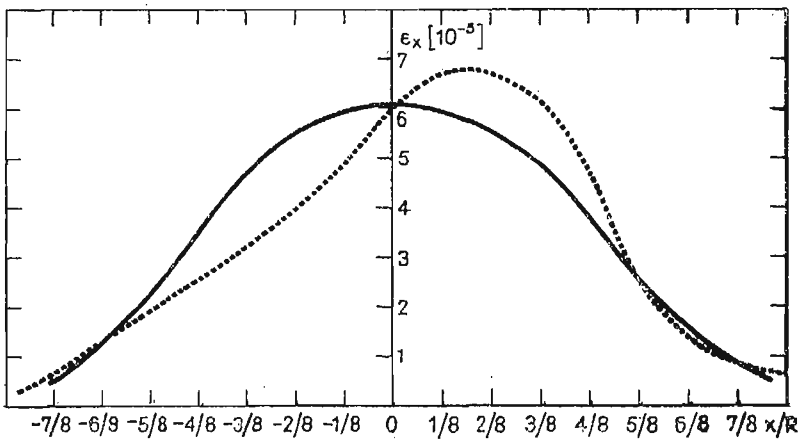

Fig. 8

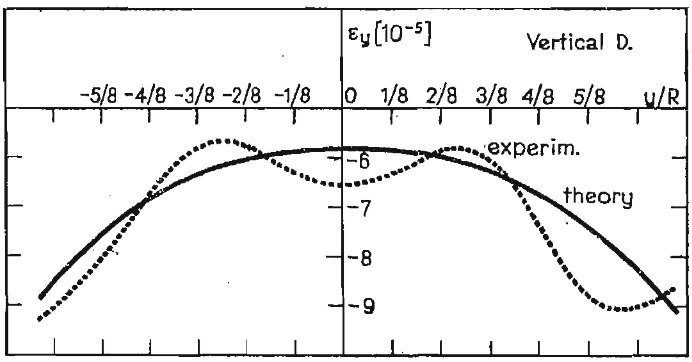

Fig. 9

Discussion about accuracy of the measurement

There are two main error sources:

a) errors associated with measuring the system geometry

b) innacurate determination of fringe order values.

Two inequalities corresponding to these errors are as follows (8):

$$\frac{\|\Delta \underline{L}\|}{\|\underline{L}\|} \leq \text{cond}\,(\underline{\underline{K}}) \cdot \frac{\|\Delta \underline{\underline{K}}\|}{\|\underline{\underline{K}}\|}, \qquad \frac{\|\Delta \underline{L}\|}{\|\underline{L}\|} \leq \text{cond}\,(\underline{\underline{K}}) \cdot \frac{\|\Delta \underline{N}\|}{\|\underline{N}\|}, \tag{13), (14}$$

where

$$\text{cond}\,(\underline{\underline{K}}) = \|\underline{\underline{K}}\| \cdot \|\underline{\underline{K}}^{-1}\|, \tag{15}$$

is the value characteristic of the sensitivity of interferometer, and

$$\|\underline{\underline{K}}\| = (\max \cdot \text{eigchvalue of } \underline{\underline{K}}^T \cdot \underline{\underline{K}})^{1/2}, \tag{16}$$

where $\underline{\underline{K}}^T$ is transpose matrix of $\underline{\underline{K}}$.

For quick determination of the error, which is found by evaluation of displacement vector components another way was proposed [12]:

$$\delta L_k = \sqrt{\left(\frac{1}{2}\,\Delta\varphi_k \cdot \text{tg}\,\frac{\varphi_k}{2}\right)^2 + \left(\frac{\Delta N_{ij}(k)}{N_{ij}(k)}\right)^2} \tag{17}$$

where φ_k is an angle between $(-\underline{K}_1^k)$ and $\underline{K}_2^k$ vectors and $\Delta \underline{N}_{ij}(k)$ is an absolute error occured by reading fringe order values.

Here:

$$\left|\begin{aligned} \Delta\varphi_k &= 1^\circ \text{ (0.17 rad)} \\ N_{ij}(k) &= 0.25 \\ \varphi_{k_{max}} &= 62.3^\circ \end{aligned}\right|$$

So depending on the fringe order, the accuracy changes in range of $8\% \leqq \delta L_k \leqq 13.5\%$; for the central part of disc $\delta L_k \leqq 9\%$. Obviously, a lower fringe order results in higher error. This error is transported into deformation countings. But, knowing the character of displacement by choosing a sufficient „smooth factor" [1] the spline functions can be better fit to discrete points of displacement. However, if the character of displacement is unknown, there is a custom to give for smooth factor value of standard deviation [13].

Recapitulation Suggestions

The difference between theoretical and experimental results can be explained at first by small size of the hologram plate (6×9 cm), which caused too small shifts of fringe order by changing observational points. The second main reason considered, could be found in the model loading system, which didn't load the disc pointwise as it is assumed in theory. Small rotations were observed, but their influence on the strains are negligible. By examining small models it would be worth testing the usefulness of reflection holograms in evaluating displacement field. Using reflection hologram in setup similar to proposed (only the hologram plate is closer to model), large fringe order shifts can be obtained which results in higher accuracy.

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Резюме

НЕКОТОРЫЕ ПРАКТИЧЕСКИЕ ПРОБЛЕМЫ ИЗМЕРЕНИЙ ПЕРЕМЕЩЕНИЙ И ДЕФОРМАЦИЙ ПРИ ИНКОГЕРЕНТНОЙ СУПЕРПОЗИЦИИ ИНТЕРФЕРОГРАММ

Информация о механических характеристиках объектов в экспeрументальной механике получается из экспериментов по определению полей деформаций и напряжений. Для таких исследований могут быть использованы методы голографической интерферометрии. В этой работе показано определение поля перемещения точек поверхности объекта используя интерферометр с регулированой оптикой путем, который дает возможность автоматической инспекции интерферограмм. Дискретное поле перемещения аппроксимировано с помощью сплайн-функций третьего порядка и определено деформацию для диаметрально сжатого диска.

Streszczenie

PEWNE PROBLEMY PRAKTYCZNE POMIARÓW PRZEMIESZCZEŃ I ODKSZTAŁCEŃ PRZY NIEKOHERENTNEJ SUPERPOZYCJI INTERFEROGRAMÓW

Pokazano przykład zastosowania interferometrii holograficznej do pomiaru przemieszczeń i odkształceń za pomocą interferometru z dwiema wzajemnie niespójnymi wiązkami odniesienia. Możliwość obserwacji tego samego punktu badanego ciała na różnych interferogramach — zmieniając wiązki odniesienia pozwala na automatyzację odczytu prążka bez konieczności identyfikacji punktów powierzchni. Przeprowadzony eksperyment ma na celu zbadanie efektywności tej techniki.

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