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ISODYNE PHOTOELASTICITY AND GRADIENT PHOTOELASTICITY: PHYSICAL AND MATHEMATICAL MODELS, EFFICACY, APPLICATIONS

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Abstract

The term „isodynes" has been proposed by Pindera and Mazurkiewicz to denote a new family of characteristic lines of plane stress fields. These lines carry information on total normal forces acting on related cross sections and yield the distribution and values of related normal and shear stress components. Two families of isodynes related to two characteristic directions yield the values of all three components of a plane stress field and additional redundant information. The concept of photoelastic isodynes is the result of generalization of the theory of a particular kind of scattered light fringes presented by Pindera and Straka.

Distinction should be made between the concept of photoelastic isodynes mentioned above, the concept of elastic isodynes introduced by Pindera, and the concept of generalized isodynes introduced by Pindera and Krasnowski.

Isodyne photoelasticity methods can be applied to determine all three stress components in photoelastic models, and in original machine or structural parts using isodyne coatings.

The term „gradient photoelasticity" has been proposed by Pindera and Hecker to denote a new method of photoelasticity which utilizes relationship between the curvature of light paths in a photoelastic object and the gradients of symmetrical and distortional parts of stress/strain tensors. Utilizing basic mathematical model of photoelastic effect presented by Ramachandran and Ramaseshan, gradient photoelasticity yields the momentary values of absolute and relative photoelastic coefficients, and their dependence on the wavelength of electromagnetic radiation.

Both methods can be applied to determine the values of stress intensity factors for arbitrary cracks, and all stress components in composite structures.

1. Introduction

One of the major tasks of stress analysis is determination of stress components of plane stress field and of surface stress/strain field. The rapid growth of fracture mechanics and the introduction of concept of stress intensity factor developed interest in methods directly yielding values of normal stress components in direction of crack propagation.

Table 1 Characteristics of Physical Models of Contemporary Photoelasticity

BASIC FEATURES OF SOME PARTICULAR METHODS OF PHOTOELASTICITY

Basic Mathematical Model	Alteration of dielectric tensor, representing optical response of material, is a linear, homogeneous function of stress/strain tensor.
Simplified Mathematical Model	Alteration of index tensor, representing phase velocity of radiation in stressed material, is a linear, homogeneous function of stress/strain or strain rate tensors.

GROUP 1: EACH OBJECT IS COMPOSED OF SINGLE MACROSCOPICALLY HOMOGENEOUS MATERIAL — AXES OF STRESS AND STRAIN TENSORS ARE COLLINEAR
stress/strain fields are given by independent boundary conditions.

Method / Feature	TRANSMISSION	SCATTERING	ISODYNE	GRADIENT	SPECIAL (Samples)
Quantity being measured	Modulation of light intensity caused by chromatic interference.	Intensity of light scattered at a point depending on primary or secondary modulation.	Intensity distribution of light scattered at points of a selected plane at constant scattering parameters.	Deflection of light beam passing a stressed object caused by curvature of light path.	a) Reflected light intensity and polarization state. b) Bent light intensity. c) Evanescent light intensity.
Measurement principle	Interference of polarized wave fronts, warped by stress state. Birefringence is accumulated along thickness of object. Incidence: normal and oblique.	State of polarization of beam scattered at points along primary beam.	State of birefringence in a plane. Values of scattering parameters and optical path are constant. Birefringence is accumulated along characteristic distance.	Bending and resolving of light beams by stress gradients. Normal and oblique incidence results in disparate rotations of both wave fronts.	a) Stress reflectivity. b) Mirage effect allows residual stress evaluation c) Evanescent wave (surface wave) carries information on stress birefringence.

Table 1 c.d.

Basic assumptions (conditions, constraints)	Light path is rectlinear. Corresponding rays in both related wave fronts are collinear. Geometric opitcs is applicable.		Known basic mathematical models of interaction between radiation and inhomogeneous, anisotropic bodies, and of processes at boundaries are applicable. Simplified assumptions (e.g. rectilinear propagation) are acceptable only when justified. Transfer function of the total system must be known.		
			Rayleigh model of scattering is applicable.	Curvature of light path is small-relations are practically linear.	Depend on the method.
Corresponding parameters of mathematical models	Axes of birefringence Amount of accumulated spectral birefringence. Mechanical & optical creep compliance, and relaxation modulus. Elastic constants, when applicable.	All three parameters of polarization at a point, characterizing the polarization ellipse.	Observation angle. Aximuthal angle. Total retardation along spections. Length of optical path.	Curvatures of both paths of bent light beams and related axes of polarization.	a) Real and imaginary parts of refractive index. b) Total birefringence along surface section. c) Total birefringence along surface section.
Character of measurements	Point-wise Plane-wise	Point-wise Line-wise	Plane-wise Line-wise Point-wise	Point-wise Line-wise,	Depends on chosen technique
Information yield optically	Two independent relations for plane stress field (3 unknown quantities).		Four independent relations for plane stress field.	Two independent relations for gradients of volumetric and deviatoric parts of stress tensor.	Depends on chosen technique.
Typical object geometry	Plates (disk problems).	3-D objects	Plates (disk problems).	3-D symmetrical objects	Any.
Materials of objects	Solid or liquid dielectrics, transparent in the ultraviolet — microwave band: e.g.: glass, polymers, semiconductors (germanium, silicon), ceramics				Dielectrics. Metals. Semiconductors.
Evaluated quantities	Load-induced stresses or strains fields in solids, or velocity vector fields in liquids.				Surface stresses. Residual stresses.

Table 1 c.d.

GROUP 2: EACH OBJECT IS COMPOSED OF DIFFERENT MATERIALS — AXES OF STRESS AND STRAIN TENSORS ARE COLLINEAR
Stress fields are induced by imposed strain fields, or interacting strain fields.

BASIC METHOD	TRANSMISSION	SCATTERING	ISODYNE	GRADIENT	SPECIAL (Sample)
Samples of Typical Methods	Photoelastic coatings. Plates in bending. Integrated photoelasticity.	Scattering. Speckle.	Isodyne coating. Contact problems. Determination of stress-intensity factors in fracture mechanics. Determination of interface stresses. Studies of elastic-plastic deformation fields.	Determination of stresses in crack regions in fracture mechanics. Determination of all photoelastic coefficients.	Photoelastic studies of propagation of ultrasonic radiation.
Typical features	Elementary relations of mechanics not always applicable.	More advanced mathematical models of optical phenomena are necessary.	Elementary mathematical models of mechanics and optics are not applicable.	Elementary optics and elementary relations of mechanics not always applicable.	Elementary optics not applicable.

However, no convenient method exists to determine reliably and inexpensively all three stress components of plane stress field, related to arbitrary Cartesian coordinate system. The classical methods of transmission photoelasticity yield only two independent pieces of information, whole field-wise: isochromatic and isoclinics. The needed third piece of information can be obtained using numerous auxilliary methods, either analytical, numerical, experimental, or mixed. These methods are often tedious, and usually compound the measurement and evaluation errors.

Very promising is the utilization of scattered light. The classical scattered light techniques of experimental stress analysis are basically point-wise or line-wise techniques. To obtain reliable and accurate results, in a convenient manner, using scattered light techniques, it is necessary to apply rather sophisticated instrumentation [1 - 3]. Information on stress field contained in typical wholefield recordings of scattered radiation intensities, which are obtained by using sheets of light, can not be reliably retrieved without additional pieces of information. — Particularly important are data on values of the observation and azimuthal angles at all scattering points, and information on the transfer function of measurement systems [4 - 6], including data on the actual viscoelastic responses of materials [7 - 8]. Knowledge of time-dependence of the optical creep compliance and the optical relaxation modulus are of major importance.

Classical methods of photoelasticity — the transmitted, and scattered light methods are based on several simplifying assumptions regarding the interaction between radiation and matter, patterns of light propagation, validity of analytical solutions for stress/strain states, distinction between the thin and thick plate problems, etc. A summary of some basic features of typical theoretical and empirical approaches, presented in a manner compatible with the scope and objective of this paper, is given in Table 1. The components (or parameters) of the basic physical and mathematical models of the interaction between deformed bodies and flow of various forms of energy, listed in Table 1, must be understood and discussed within the framework of fundamental physical phenomena.

Experimental and analytical methods of stress analysis utilize the concept of characteristic lines of plane stress field, such as:

— isochromatics
— isopachics
— principal stress trajectories (isostatics)
— maximal shear stress trajectories
— isoclinics
— singular lines (points), etc.

It is shown in this paper that it is useful to introduce a new family of characteristic lines of plane stress field and to call them isodynes.

It is also shown, that it is desirable to depart from the very simplifying assumption that the light propagation in a stressed body is rectilinear. It has been known for a long time that this assumption is not correct and leads to unnecessary errors, because the actual light path can be noticeably curved under influence of inhomogeneity produced by stress state.

Since the curvature of the light path carries information on the stress gradients and on the photoelastic material coefficients, it is worthwhile to depart from the elementary mathematical models of photoelastic effects and to base the theory of photoelastic experiments on more comprehensive physical and mathematical models.

2. Isodyne photoelasticity

2.1 Elastic Isodynes — A New Family of Characteristic Lines of Plane Stress Field. Let us choose an arbitrary direction in a plane stress field, Fig. 1, and call it characteristic direction; a characteristic line is any line collinear with characteristic direction; characteristic section is the length of characteristic line between two chosen points.

It is understood that a plane stress field is characterized by the following condition:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0, \quad \text{where } i, j = 1, 2 \tag{0}$$

Let us define isodynes as geometric loci of points at which total normal force intensity (total normal force per unit thickness) Δp_n, acting on characteristic section between two isodynes is constant and proportional to an increase of the order of isodyne, Δm_s,

$$\Delta p_n = S_s \Delta m_s = \text{const}, \tag{1}$$

and the corresponding normal force, ΔP_n, acting on characteristic section thickness b is:

$$\Delta P_n = b\Delta p_n = bS_s \Delta m_s = \text{const}$$

where S_s is a constant coefficient to be calculated or determined experimentally.

Two-dimensional stress states in elastic plates can be conveniently characterised by Airy stress function, $\phi(x, y)$, which — in the absence of body forces — yields the known expressions for the stress components with respect to a Cartesian coordinate system, (x, y, z):

$$\frac{\partial \Phi(x,y)}{\partial x} = \Phi_x(x,y); \quad \frac{\partial \Phi(x,y)}{\partial y} = \Phi_y(x,y), \tag{2a}$$

$$\sigma_{xx} = \frac{\partial^2 \Phi}{\partial y^2} = \frac{\partial}{\partial y}\Phi y \quad \sigma_{yy} = \frac{\partial^2 \Phi}{\partial x^2} = \frac{\partial}{\partial x}\Phi x, \tag{2b}$$

$$\sigma_{xy} = \sigma_{yx} = -\frac{\partial^2 \Phi}{\partial x \partial y} = -\frac{\partial}{\partial y}\Phi_x = -\frac{\partial}{\partial x}\Phi_y. \tag{2c}$$

There exist particular relations between the stress components σ_{ij}, the intensities p_i of normal forces acting on sections between isodynes, and the functions Φ_x, Φ_y, [9]:

$$\int \sigma_{yy} dx = \int \frac{\partial^2 \Phi}{\partial x^2} dx = \Phi_x + f_{xx}(y) = p_y(x, y_0) = S_s m_{sxx}(x, y_0), \tag{3a}$$

$$\int \sigma_{xx} dy = \int \frac{\partial^2 \Phi}{\partial y^2} dy = \Phi_y + f_{yy}(y) = p_x(x_0, y) = S_s m_{syy}(x_0, y), \tag{3b}$$

ELASTIC AND PHOTOELASTIC ISODYNES: DEFINITIONS, DERIVED QUANTITIES

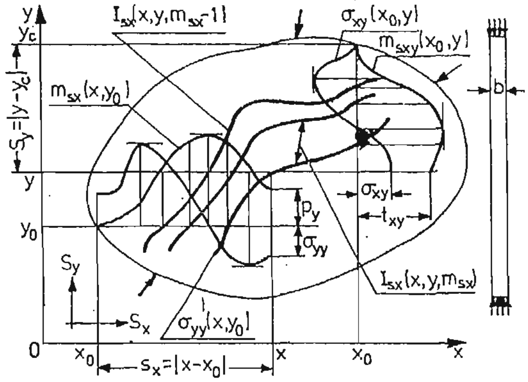

Fig. 1. Concept of plane isodynes

MAJOR FEATURE

ISODYNES ARE RELATED TO CHOSEN CHARACTERISTIC DIRECTIONS:

$I_{sx} = I_{sx}(x, y, m_{sx}) = \text{const.}$; $I_{sy} = I_{sy}(x, y, m_{sy}) = \text{const.}$

CHARACTERISTIC CROSS-SECTIONS THROUGH ISODYNE FIELDS

$m_{sxx} = m_{sxx}(x, y_0) = S_s^{-1} p_{yy}(x, y_0) = (bS_s)^{-1} P_{yy}(x, y_0)$

$m_{sxy} = m_{sxy}(x_0, y) = S_s^{-1} t_{xy}(x_0, y) = (bS_s)^{-1} [T_{xy}(x_0, y) + C(y)]$

$m_{syy} = m_{syy}(x_0, y) = S_s^{-1} P_{xx}(x_0, y) = (bS_s)^{-1} P_{xx}(x_0, y)$

$m_{syx} = m_{syx}(x, y_0) = S_s^{-1} t_{yx}(x, y_0) = (bS_s)^{-1} [T_{yx}(x, y_0) + C(x)]$

CONDITIONS:

— HOOKE'S BODY

— RAMACHANDRAN-RAMASESHAN BODY

— PLANE STRESS FIELD

SYMBOLS:

I_{sx}, I_{sy}: $x-O$ y-ISODYNES

m: ORDER (PARAMETER) OF ISODYNE

S_x: x-CHARACTERISTIC DIRECTION

S_s: ELASTIC ISODYNE FACTOR $\left[\dfrac{\text{FORCE}}{\text{LENGTH}}\right]$

P_y: TOTAL NORMAL FORCE ON SECTION s_x

p_y: NORMAL FORCE INTENSITY ($= b^{-1}P_y$)

T_{xy}: TOTAL SHEAR FORCE ON SECTION s_y

t_{xy}: SHEAR FORCE INTENSITY ($= b^{-1}T_{xy}$)

σ_{ij}: STRESS COMPONENTS ($i, j = x, y$)

f, C: BOUNDARY CONDITIONS FUNCTION

RELATIONS FOR STRESS COMPONENTS

1. $\sigma_{yy}(x, y_0) = S_s \dfrac{d}{dx} m_{sxx}(x, y_0)$

2. $\sigma_{xx}(x_0, y) = S_s \dfrac{d}{dy} m_{syy}(x_0, y)$

3. $\sigma_{xy}(x_0, y) = -S_s \dfrac{d}{dy} m_{sxy}(x_0, y) + f'_{yy}(x)$

4. $\sigma_{yx}(x, y_0) = -S_s \dfrac{d}{dx} m_{syx}(x, y_0) + f'_{xx}(y)$

and

$$\sigma_{yy} = \frac{\partial}{\partial x}\Phi_x = \frac{d}{dx}p_y = S_s\frac{d}{dx}m_{sxx}(x, y_0), \tag{3c}$$

$$\sigma_{xx} = \frac{\partial}{\partial y}\Phi_y = \frac{d}{dy}p_x = S_s\frac{d}{dy}m_{syy}(x, y_0) \tag{3d}$$

where p_y and p_x denote intensities of normal forces acting on cross sections collinear with the corresponding directions x and y, and m_{sxx} and m_{syy} denote cross sections through x- and y- isodyne fields in the x- and y-directions. Followingly, the equation of isodynes can be presented in the form:

$$I_{sx} = I_{sx}(x, y, m_{sx}) = \int \frac{\partial^2\Phi}{\partial x^2}dx = p_y(x, y, m_{sx}) = S_s m_{sx}(x, y) = \text{const}, \tag{4a}$$

$$I_{sy} = I_{sy}(x, y, m_{sy}) = \int \frac{\partial^2\Phi}{\partial y^2}dy = p_x(x, y, m_{sy}) = S_s m_{sy}(x, y) = \text{const}. \tag{4b}$$

The loci of points described by relations (4a) and (4b) can be called „elastic isodynes" because they represent lines of constant values of normal forces acting on corresponding cross-sections. The arbitrary directions x and y, to which elastic isodynes are related, can be conveniently called „characteristic directions", S_x and S_y. The functions Φ_x and Φ_y could be called „generalized isodynes":

$$\Phi_x(x, y, m_{sx}) = I_{sx}(x, y, m_{sx}) - f_{xx}(y) = \text{const}, \tag{5a}$$

$$\Phi_y(x, y, m_{sy}) = I_{sy}(x, y, m_{sy}) - f_{yy}(x) = \text{const}. \tag{5b}$$

Functions $f_{xx}(y)$ and $f_{yy}(x)$ can be determined from the boundary conditions. The generalized isodynes Φ_x and Φ_y are simply related to the corresponding isodyne fields characterized by the isodyne orders m_{sx} and m_{sy}:

$$\Phi_x = S_s m_{sx}(x, y) - f_{xx}(y) = \text{const}, \tag{6a}$$

$$\Phi_y = S_s m_{sy}(x, y) - f_{yy}(x) = \text{const}. \tag{6b}$$

Relations (6a, b) can be used to determine the shear stress components σ_{xy} and σ_{yx}, and their intensities t_{xy} and t_{yx}.

$$\sigma_{xy} = -\frac{\partial}{\partial y}\Phi_x = -\frac{\partial}{\partial y}m_{sx}(x, y) + \frac{d}{dy}f_{xx}(y) = -S_s\frac{d}{dy}m_{sxy}(x, y) +$$

$$+ f'_{xx}(y) = \frac{d}{dy}t_{xy}(x, y), \tag{7a}$$

$$\sigma_{yx} = -\frac{\partial}{\partial x}\Phi_y = -\frac{\partial}{\partial x}m_{sy}(x, y) + \frac{d}{dx}f_{yy}(x) = -S_s\frac{d}{dx}m_{syx}(x, y) +$$

$$+ f'_{yy}(y) = \frac{d}{dx}t_{yx}(x, y), \tag{7b}$$

$$t_{xy} = t_{xy}(x, y, m_{sxy}) = \int \sigma_{xy}dy = -\int \frac{\partial\Phi_x}{\partial y}dy = -S_s m_{sxy}(x, y) + f_{xx}(y) + C_1, \tag{8a}$$

$$t_{yx} = t_{yx}(x, y, m_{syx}) = \int \sigma_{yx}dx = -\int \frac{\partial\Phi_y}{dx}dx = -S_s m_{syx}(x, y) + f_{yy}(x) + C_2 \tag{8b}$$

Obviously,

$$T_{xy} = bt_{xy} \quad \text{and} \quad T_{yx} = bt_{yx}. \tag{9a,b}$$

At any chosen point $\sigma_{xy} = \sigma_{yx}$, thus

$$S_s\left[\frac{d}{dy} m_{sxx}(x, y) - \frac{d}{dx} m_{syx}(x, y)\right] = f'_{xx}(y) - f'_{yy}(x). \tag{10}$$

Relations (10) can be used to check the accuracy of determination of the functions $f'_{xx}(y)$ and $f'_{yy}(x)$.

According to relations (3a, b) and (7a, b) two families of isodynes related to two different — preferably mutually perpendicular — directions, for instance, $I_{sx}(x, y)$ and $I_{sy}(x, y)$ or shortly (x, y) — isodynes, yield four independent pieces of information on the values of the normal and shear stress components, σ_{xx}, σ_{yy}, σ_{xy} and σ_{yx}, where $\sigma_{xy} = \sigma_{yx}$. In additional, the fields of (x, y) — isodynes are related by the condition

$$\sigma_{xx} + \sigma_{yy} = \sigma_1 + \sigma_2$$

thus the equation of isopachics may be presented in the form:

$$\frac{\partial}{\partial x}\Phi_x + \frac{\partial}{\partial y}\Phi_y = S_s\left(\frac{d}{dx} m_{sxx} + \frac{d}{dy} m_{syy}\right) = S_s m_i = \text{const}, \tag{11}$$

where m_i denotes the order of isopachics and S_s denotes the elastic coefficient of isopachics.

It is convenient to use the term „characteristic lines of plane stress fields" to denote a set of functions characterizing the stress fields, such as the isostatics, isoclinics, isochromatics, isopachics etc. [10], [11]. Within such a framework the elastic isodynes represent a new family of the characteristic lines of plane stress fields.

The four independent pieces of information given by (x, y)-isodynes describe three independent stress components, σ_{xx}, σ_{yy}, $\sigma_{xy} = \sigma_{yx}$; one piece of information is redundant and can be used to increase the accuracy of experimental evaluation of stress components Various known techniques of differentiation of isodyne fields can be applied or adapted, to determine the quantities of interest.

2.2 Photoelastic Isodynes. It has been shown [12] that it is possible to produce a particular whole field scattered light intensity modulations related to chosen characteristic direction x, when the following conditions are satisfied:

a) primary beam is co-planar with the (σ_1, σ_2)-plane;
b) angle between polarization axis of primary beam and (σ_1, σ_2)-plane is $\pi/4$.
c) at each point of the scattering plane:
   - observation angle is equal to $\pi/2$;
   - azimuthal angles are equal to 0 or $\pi/2$, respectively;
   - optical paths between corresponding points in the object and the image planes are the same.

The light intensity modulation can be described using the concept of constant intensity lines, related to chosen characteristic directions x and y:

$$I_{s1,2}/kI_0 = (I_s)_n = F_{s1,2}(x, y) = \text{const}, \tag{12}$$

where the subscripts 1, 2 denote light beams S_1 and S_2 scattered under the azimuthal angles 0 and $\pi/2$, respectively, Fig. 2.f

TRANSMISSION PHOTOELASTICY. ISODYNE PHOTOELASTICITY DEFINITIONS. SAMPLAS OF RECORDINGS PROBLEM: STRESS DETERMINATION IN ARBITRARY SECTIONS

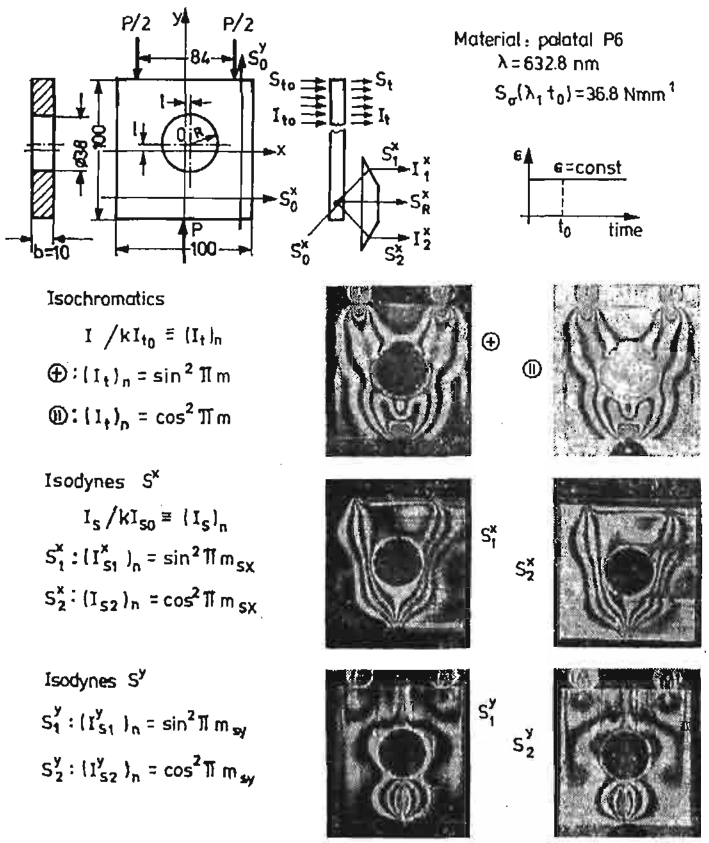

Fig. 2. Square plate with an unsymmetric hole loaded by three forces: scheme of experiment, and isochromatics and isodyne fields

It is easy to show that the intensities of scattered beams S_1 and S_2 depend on the state of polarization at scattering points and there-fore carry information on the magnitude of the relative retardation produced along the path of primary beam S_0:

$$(I_{s1}^x)_n = \sin^2 \pi m_{sx} = \sin^2 \frac{\psi_{sx}}{2} = 0.5(1-\cos 2\pi m_{sx}) = 0.5(1-\cos\psi_{sx}) = F_{s1}^x(x, y), \tag{13}$$

$$(I_{s2}^x)_n = \cos^2 \pi m_{sx} = \cos^2 \frac{\psi_{sx}}{2} = 0.5\,(1+\cos 2\pi m_{sx}) = 0.5(1+\cos\psi_{sx}) = F_{s2}^x(x, y), \tag{14}$$

CHARACTERISTIC LINES OF PLANE STRESS FIELD BY ISODYNE PHOTOELASTICITY:

— LINES OF CONSTANT NORMAL FORCE: $P_y(x, y)$ = constant

— DISTRIBUTION OF NORMAL FORCE IN A CROSS-SECTION: $P_y = P_{y(x,c)}$

— DISTRIBUTION OF NORMAL STRESS IN A CROSS-SECTION : $\sigma_y = \sigma_y(x, c)$

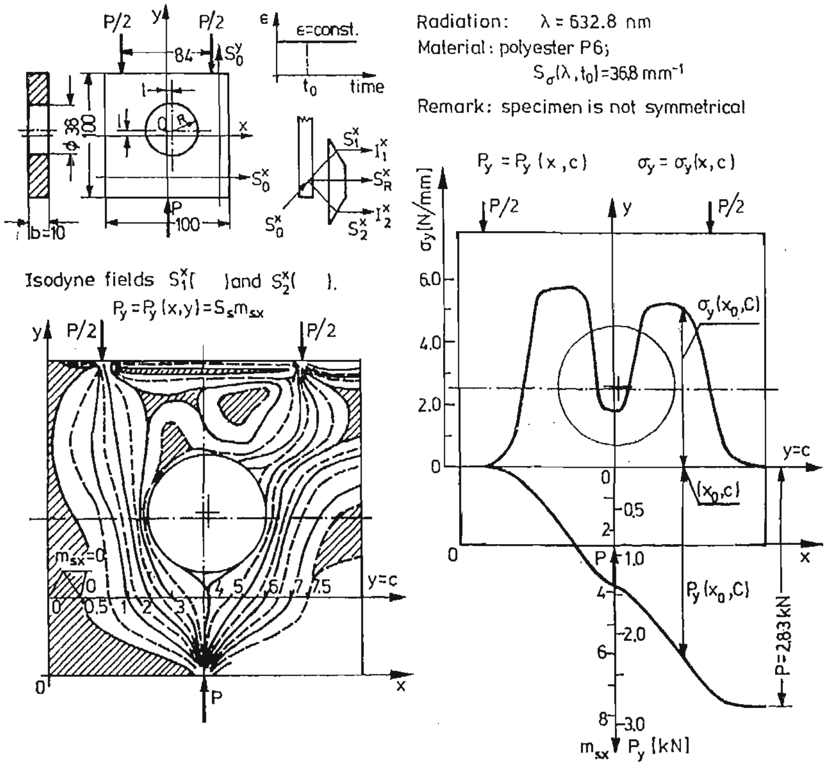

Fig. 3. Square plate with an unsymmetric hole loaded by three forces: isodyne stress analysis in selected cross-sections

where the rate of wavefront separation is given by

$$dR_x = \lambda dm_{sx} = \frac{1}{2\pi}\lambda d\psi_{sx} = (n_1' - n_3')dx = C_\sigma(\sigma_1 - \sigma_3)dx = C_\sigma C_y dx, \tag{15}$$

or

$$\sigma_y = \frac{\lambda}{C_\sigma}\frac{dm_{sx}}{dx} = S_\sigma\frac{dm_{sx}}{dx} = \frac{1}{2\pi}S_\sigma\frac{d\psi_{sx}}{dx}, \tag{16}$$

and

$$P_y(0, x) = \int_0^x b\sigma_y dx = bS_\sigma m_{sx} = S_s m_{sx}. \tag{17}$$

Obviously, the elastic and photoelastic isodynes are formally identical,

$$\begin{aligned}(I_{sx})\text{elastic} &\equiv (I_{sx})\text{photoelastic},\\ (I_{sy})\text{elastic} &\equiv (I_{sy})\text{photoelastic}.\end{aligned} \tag{18}$$

When the boundary conditions are such that the functions $f_{xx}(y)$, $f_{yy}(x)$, C_1 and C_2 vanish then the elastic and photoelastic isodynes are identical with the first derivatives of Airy stress function.

More information on the theory and technique of isodyne photoelasticity is given in References [6, 9, 12 - 26, 31].

2.3 Examples of Applications. Isodyne photoelasticity allows to determine — reliably, easily, and inexpensively — all components of plane stress fields in chosen cross sections. This method is particularly suitable to collect information on stress fields needed to evaluate stress intensity factors, three-dimensional stress states in regions of notches and cracks, interlaminar stresses in composite structures, residual stresses, thermal stresses, etc.

Figures 2 and 3 illustrate the theory, technique, and applicability of the isodyne photoelasticity.

It appears that the isodyne photoelasticity is particularly suitable to determine ranges of applicability of analytical methods of stress analysis.

Isodyne photoelasticity makes it possible to develop new techniques for determination of surface stress components in engineering prototypes, The related method of isodyne coatings is presented in [19].

3. Gradient photoelasticity

3.1 Basic Relations. It has been known for a very long time that the path of light propagating through inhomogeneous bodies is not rectilinear — light path is curved. A particular example of this phenomenon is the well known mirage.

It has been observed that a noticeable mirage effect is produced by stress state in photoelastic objects [21 - 23]. This effect limits the resolution of photoelastic measurements [21], and obscures isochromatic fields in regions of high birefringence gradients [22], Fig. 4. However, this effect can be utilized to measure particular residual stress states [23]. It has been shown recently that the optical anisotropy caused by an inhomogeneous stress state produces a noticeable separation of each curved light beam; this effect can be used to determine the gradients of some linear functions of stress components and the values of absolute stress-optic coefficients [24 - 27].

It is common to present the influence of the optical inhomogeneity of a body on the geometry of the path of light propagating in this body by the simplified relation [28]:

$$\bar{K} = \frac{1}{\varrho_1}\bar{\nu} = \frac{1}{n}\left(\operatorname{grad} n - \frac{dn}{ds}\bar{s}\right) = \operatorname{grad} \ln n - \frac{1}{n}\frac{dn}{ds}\bar{s} \tag{19}$$

The most general phenomenological mathematical model of the stress/strain produced optical anisotropy presents an alteration of an optical parameter as a linear homogeneous function of the components of the stress or strain tensors, [29]. The relations presented in [29], together with the relation (19), lead to the following relationships between the curvatures of both ray components propagating with the velocities v_1, v_2 and the stress

LIGHT PROPAGATION THROUGH A SOLID BODY:
INFLUENCE OF INHOMOGENEITY GRADIENT, AND ANISOTROPY GRADIENT CAUSED BY STRAIN/STRESS GRADIENTS

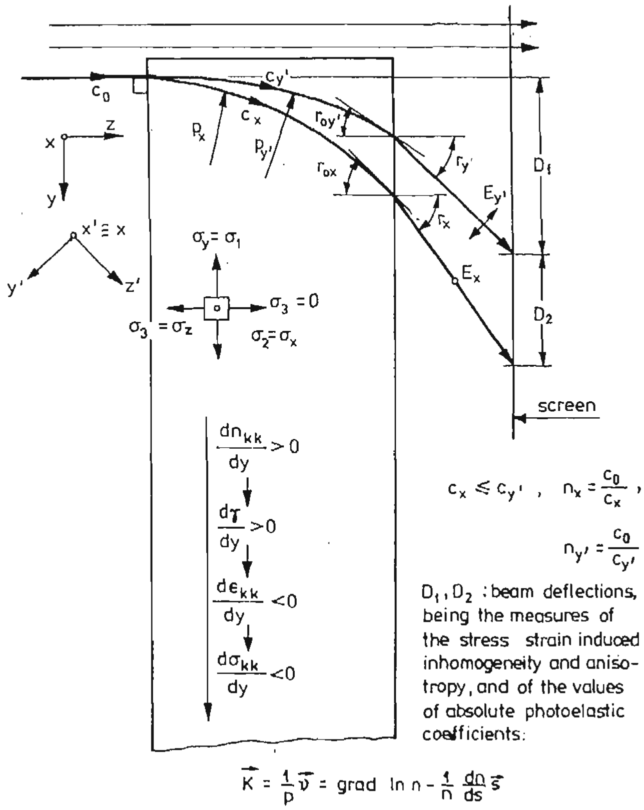

Fig. 4. Paths of light in a body which became inhomogeneous, optically and mechanically, under influence of an inhomogeneous stress field: influence of gradients of the sum and differences of principial stresses

tensor components:

$$\bar{K}_1 = \frac{1}{\varrho_1}\cdot\bar{\nu} = \frac{1}{n_1}\left(\mathrm{grad}\,n_1 - \frac{dn_1}{ds}\bar{s}\right), \tag{20}$$

where

$$n_1 = n_0 + C_1\sigma_1 + C_2(\sigma_2+\sigma_3), \tag{21}$$

and

$$\bar{K}_2 = \frac{1}{\varrho_2}\bar{\nu} = \frac{1}{n_2}\left(\mathrm{grad}\,n_2 - \frac{dn_2}{ds}\bar{s}\right), \tag{22}$$

where

$$n_2 = n_0 + C_1\sigma_2 + C_2(\sigma_3+\sigma_1). \tag{23}$$

These equations can be used as a fundation of a mathematical model relating deflections of a light beam traversing a body to gradients of the sum and difference of principal stresses.

LIGHT PROPAGATION THROUGH SOLID BODY, DEFLECTIONS IN FUNCTION OF MEAN STRESS GRADIENTS

$$\frac{1}{2}(\Delta D_x+\Delta D_y) = f_s\left(\frac{dS}{dy}\right), \quad S = \frac{\sigma_x+\sigma_y}{2}$$

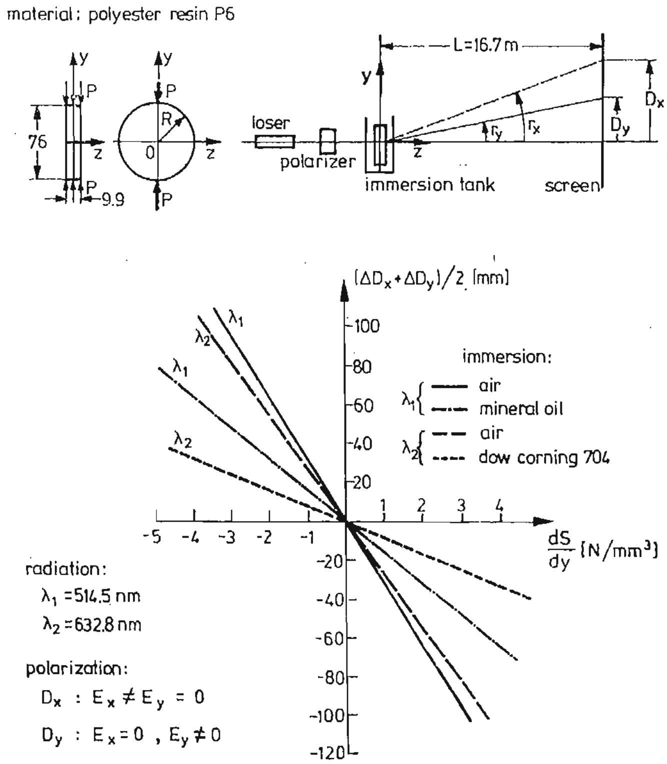

Fig. 5. Gradient photoelasticity: example of the dependence of light deflection on gradient of the sum of principial stresses

This model, when applied to plane stress states, yields simple linear equations between the deflections, stress gradients and values of the absolute and relative stress-optic coefficients C_1, C_2 and $C_\sigma = C_1 - C_2$ where, of course,

$$C_1, C_2, C_\sigma = C(\lambda) \tag{24}$$

Transmission photoelasticity yields:

$$C_\sigma = C_1 - C_2 = \frac{n_1 - n_2}{\sigma_1 - \sigma_2} \tag{25}$$

3.2 Examples of Mirage Effect Applications. Typical form of the mirage effect produced in a photoelastic object is presented in Fig. 4, [22]. The magnitude of the mirage effect depends on the orientation of polarization axis, Fig. 4. In a general case, the impinging unpolarized beam is resolved into two polarized beams, curvature of which is approximated by relations (20) and (22).

Using measurement system depicted in Fig. 5 it is easy to derive relations between deflections D_1, D_2, D_1-D_2, Fig. 4, and corresponding gradients of the symmetrical and deviatoric parts of a stress tensor. Typical results are presented in Fig. 5. Obviously, the magnitude of effect depends on the wavelength of radiation.

Gradient photoelasticity yields directly data on values of gradients of the sum and difference of principal stresses, which are of particular interest in regions of high stress gradients, e.g., in regions of notches or crack tips.

An analysis of the method of integrated photoelasticity from the point of view of gradient photoelasticity is given in [30].

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Р е з ю м е

ИЗОДИННАЯ И ГРАДИЕНТНАЯ ФОТОУПРУГОСТЬ. ФИЗИЧЕСКИЕ И МАТЕМАТИЧЕСКИЕ МОДЕЛИ ВОЗМОЖНОСТИ И ПРИМЕНЕНИЯ

Под термином изодины предложенным Пиндером и Мазуркевичем понимаем новое семейство линий характерных плоскому напряженному состоянию. Из этих линий возникают информации относительно целой нормальной силы, действующей в определенном сечении модели.

Два семейства изодин определенные для двух характерных направлений делают возможным определить значения всех трех составляющих плоского напряженного состояния. Замысел изодин является результатом предложенного Пиндером и Старком обобщения теории рассеяного света.

Метод изодин можно применять как для анализа фотоупрутих моделей так и используя метод фотоупругости слоя к действительным деталям машин.

Термин градиентная фотоупругость предположенный Пиндером и Хекером обозначает новый фотоупругий метод, использующий зависимость искривления луча света в фотоупругом объекте от градиентов сферической и наклонной составляющих тензора напряжений или деформации. Используя основную математическую модель фотоупругого эффекта, предположенную Рамахандраном и Рамаишаном, градиентная фотоупругость ведет непосредственно к абсолютным и относительным значениям фотоупругих коэффициентов в зависимости от длины волны электомагнитного излучения.

Оба метода можна применить, примерно, к определению коэффициента интенсивности напряжения или составляющих напряжения в конструкциях из композитов.

S t r e s z c z e n i e

ELASTOOPTYKA IZODYNOWA I GRADIENTOWA. MODELE FIZYCZNE I MATEMATYCZNE, MOŻLIWOŚCI I ZASTOSOWANIE.

Termin izodyna został zaproponowany przez Pinderę i Mazurkiewicza i oznacza nową rodzinę linii charakterystycznych dla płaskiego stanu naprężenia. Linie te dostarczają informacji o całkowitej sile normalnej, działającej na określonym przekroju modelu. Dwie rodziny izodyn wyznaczone dla dwóch charakterystycznych kierunków pozwalają wyznaczyć wartości wszystkich trzech składowych płaskiego stanu naprężenia. Pomysł izodyn jest rezultatem prezentowanego przez Pinderę i Starkę uogólnienia teorii światła rozpraszanego.

Metoda izodyn może być stosowana dla modeli elastooptycznych jak również rzeczywistych elementów maszyn wykorzystując metodę warstwy elastooptycznej.

Termin elastooptyka gradientowa został zaproponowany przez Pinderę i Heckera dla oznaczenia nowej metody elastooptycznej, wykorzystującej zależności pomiędzy zakrzywieniem promieni światła w obiekcie elastooptycznym a gradientami składowej kulistej i składowej skośnej tensora naprężenia lub odkształcenia. Wykorzystując podstawowy model matematyczny efektu elastooptycznego przedstawiony przez Ramachandrana i Ramaseshana, elastooptyka gradientowa daje natychmiast wartości absolutnych i względnych współczynników elastooptycznych w zależności od długości fali promieniowania elektromagnetycznego.

Obydwie metody mogą być stosowane np. do wyznaczenia współczynnika intensywności naprężeń a również do wyznaczenia składowych naprężenia w konstrukcjach z kompozytów.

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