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GROUP THEORETIC TECHNIQUE FOR THE SIMILARITY SOLUTION OF A NON-LINEAR ELASTIC ROD SUBJECTED TO VELOCITY IMPACT

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1. Introduction

One of the first applications of the dimensional group of transformations to the similarity solutions of problems in fluid mechanics is found in Birkhoff's Hydrodynamics [1]. The work was further extended for applications to partial differential equations by Morgan and Michal [2, 3]. Moran and Gaggioli [4] applied it to a system of partial differential equations arising in Fluid Mechanics taking into account the auxiliary conditions. Moran and Marshek [5] made use of the matrices of exponents of the parameters of a group of transformations to determine the similarity variables of a system of partial differential equations along with their auxiliary conditions. Seshadri and Singh [6] made use of the similarity characteristic relationship at the wave front to reduce a hyperbolic partial differential equation into an ordinary differential boundary value problem in the case of wave propagation in nonlinear elastic rods. Frydrychowicz and Singh [7] applied multiparameter dimensional group of transformations to the analysis of quasilinear partial differential equations of order two in two variables. In this paper, the technique is applied to the study of wave propagation in a nonlinear elastic rod subjected to time dependent velocity impact.

A multiparameter dimensional group of transformations is widely applicable to a variety of non-linear dynamical problems in fluids and solids. This approach leads to the determination of similarity transformations which in the case of unidirectional wave propagation leads to a similarity representation consisting of an ordinary differential equation and the associated auxiliary conditions. Making use of the similarity characteristic relationship [6, 7], the wave front can be located in the transformed space. It turns out that in the case of a nonlinear elastic rod when the similarity characteristic relationship is satisfied, the kinematical condition of compatibility and the balance law of linear momentum are also identically satisfied at the wave front, [8, 9, 10]. For general non-linear case the location of the wave front in the transformed space is given implicity and depends on the slope of the unknown similarity function. However, in the case of a constant velocity impact the location of the wave front is obtained explicitly. The same result holds for time dependent velocity-impact and linearly elastic case. A solution of similarity representation is obtained by assuming the parameter of the material nonlinearity, q, to be close to unity. The solution of general nonlinear case is obtained by numerical approach. A similar problem was treated by D. B. Taulbee et al [11] as a special case in their study of wave propagation in a non-linear viscoelastic rod. However, there was no application of group theoretic approach, and their results were obtained only for odd positive integral values of parameter of non-linearity, q. Also the location of the wave front in the transformed space was assumed fixed and the similarity variable was taken to be unity thereat which holds true only in special cases. In general, under this assumption the kinematical condition of compatibility across the wave front is not satisfied. Furthermore in the treatment of their special case the transformation for $\gamma = 0$ is not the similarity transformation since the variable t is no longer present in the similarity variable. In this paper the application of the continuous multi-parameter dimensional groups of transformations gives the similarity representation formally, the location of the wave front and the boundary conditions are obtained precisely nd the problem can be solved for any positive value of the parameter of nonlinearity, q.

2. Basic Equations

For a non-linear elastic rod the governing equations are:

$$\frac{\partial \sigma}{\partial x} = -\varrho \frac{\partial v}{\partial t}, \text{ equation of motion} \tag{1a}$$

$$\frac{\partial e}{\partial t} = -\frac{\partial v}{\partial x}, \text{ compatibility relation} \tag{1b}$$

$$-\frac{\partial v}{\partial x} = \frac{\partial}{\partial t}\left[\left(\frac{\sigma}{\mu}\right)^{q}\right], \text{ constitutive law for a nonlinear elastic material} \tag{1c}$$

where

$$e = -\frac{\partial u}{\partial x}, \quad v = \frac{\partial u}{\partial t}, \tag{1d,e}$$

$$x \geqslant 0, \quad t \geqslant 0, \quad q > 0.$$

The boundary conditions for a time dependent velocity impact applied in the direction of positive x-axis are assumed to be in the form ,

$$\frac{\partial u}{\partial t}(x = 0, t) = V_c t^{\delta}, \quad t > 0, \tag{2a}$$

and

$$u(x \geqslant x_w(t), t) = 0, \quad t > 0, \tag{2b}$$

The initial conditions are

$$u(x, t = 0) = 0, \quad x > 0, \tag{3a}$$

$$\frac{\partial u}{\partial t}(x, t = 0), \qquad x > 0. \tag{3b}$$

In the above equations x is the axial coordinate, t is the time, σ is the normal stress, u is the displacement along the x axis, v is the particle velocity, e is the strain, ϱ is the mass density, μ is the modulus of elasticity, q is the material parameter of nonlinearity, δ and V_c are parameters of the velocity impact, $x_w(t)$ locates the wave front at any time t. Compressive stress is assumed to be positive.

It may be pointed out that the initial conditions (3a, b) are a consequence of (2b), as at $t = 0$ the wave front in (2b) is coincident with the origin and (3a, b) follow. Thus, conditions (3a) and (3b) are redundant and as a consequence only the conditions (2a, b) need to be taken into account in the formulation of the similarity representation.

3. Determination of four parameter group of transformations G_4^B and Derivation of similarity transformations

In order to determine the 4-parameter dimensional group of transformations G_4^B (for dimensional group of transformations see, for instance, [5], [7], [12], [13], [14], [15]) under which the system of equations (1) together with auxiliary conditions (2), (3) are invariant we introduce the following 8-parameter group of transformations:

$$G_8^B: \begin{cases} S_5^B: \begin{cases} \bar{x} = A_x x, \bar{t} = A_t t; & \text{independent variables} \\ \bar{\mu} = A_\mu \mu, \bar{\varrho}\bar{q} = A_\varrho(\varrho q), \bar{V}_c = A_{V_c} V_c; & \text{physical variables} \end{cases} \\ \bar{u} = A_u u, \bar{v} = A_v v, \bar{\sigma} = A_\sigma \sigma; \quad \text{dependent variables} \end{cases} \tag{4}$$

Where A_x, A_t, A_μ, A_ϱ, A_{V_c}, A_u, A_v, A_σ are eight nondimensional parameters introduced to characterize the eight parameter dimensional group of transformations. In order to check the invariance of differential forms involved in the basic equations, the group G_8^B may be enlarged by including the following transformations

$$\left(\overline{\frac{\partial \sigma}{\partial x}}\right) = \frac{\partial \bar{\sigma}}{\partial \bar{x}} = A_x^{-1} A_\sigma \frac{\partial \sigma}{\partial x}, \tag{5a}$$

$$\left(\overline{\frac{\partial v}{\partial t}}\right) = \frac{\partial \bar{v}}{\partial \bar{t}} = A_t^{-1} A_v \frac{\partial v}{\partial t}, \tag{5b}$$

$$\left(\overline{\frac{\partial u}{\partial t}}\right) = \frac{\partial \bar{u}}{\partial \bar{t}} = A_t^{-1} A_u \frac{\partial u}{\partial t}, \tag{5c}$$

$$\left(\overline{\frac{\partial v}{\partial x}}\right) = \frac{\partial \bar{v}}{\partial \bar{x}} = A_x^{-1} A_v \frac{\partial v}{\partial x}, \tag{5d}$$

$$\overline{\frac{\partial}{\partial t}\left[\frac{\sigma}{\mu}\right]^q} = \frac{\partial}{\partial \bar{t}}\left[\frac{\bar{\sigma}}{\bar{\mu}}\right]^q = A_t^{-1} A_\sigma^q A_\mu^{-q} \frac{\partial}{\partial t}\left(\frac{\sigma}{\mu}\right)^q. \tag{5e}$$

Making use of the transformations (4) and (5) the differential form of (1a)

$$\left\{\frac{\partial \sigma}{\partial x} + \varrho \frac{\partial v}{\partial t}\right\}, \tag{6a}$$

assumes the form

$$\left\{[A_x]^{-1} A_\sigma \frac{\partial \sigma}{\partial x} + A_\varrho [A_t]^{-1} A_v \varrho \frac{\partial v}{\partial t}\right\} \tag{6b}$$

which yields the expression (6a) whenever

$$A_\sigma = A_x A_t^{-1} A_\varrho A_v. \tag{6c}$$

Thus, (6a) is invariant under the group of transformations G_8^B and its enlargement consisting of equations (5) when the group parameters satisfy equation (6c). Similarly the invariance of the differential form of (1e) implies

$$A_u = A_v A_t, \tag{6d}$$

and the invariance of differential form of (1c) yields

$$A_\sigma = A_x^{-\frac{1}{q}} A_t^{\frac{1}{q}} A_v^{\frac{1}{q}} A_\mu. \tag{6e}$$

Combination of (6c) with (6e) leads to

$$A_v = A_x^{-\frac{1+q}{q-1}} A_t^{\frac{1+q}{q-1}} A_\varrho^{-\frac{q}{q-1}} A_\mu^{\frac{q}{q-1}}. \tag{6f}$$

Substitution of (6f) into (6d) and (6c) gives

$$A_u = A_x^{-\frac{1+q}{q-1}} A_t^{\frac{2q}{q-1}} A_\varrho^{-\frac{q}{q-1}} A_\mu^{\frac{q}{q-1}}, \tag{6g}$$

$$A_\sigma = A_x^{\frac{-2}{q-1}} A_t^{\frac{2}{q-1}} A_\varrho^{\frac{-1}{q-1}} A_\mu^{\frac{q}{q-1}}. \tag{6h}$$

Finally, the boundary condition (3a) is invariant under G_8^B, whenever

$$A_{V_c} = A_x^{-\frac{1+q}{q-1}} A_t^{\frac{1+q}{q-1}-\delta} A_\varrho^{-\frac{q}{q-1}} A_\mu^{\frac{q}{q-1}}. \tag{6i}$$

Similar treatment of the differential expressions of equations (1b, d) and (2b) yields no additional independent relationships. Consequently, (6f, g, h, i) represent a system of four equations among the eight parameters; therefore at the most four of them can be considered to be independent. So, the 4-parameter group of transformations G_4^B assumes the form:

$$G_4^B: \begin{cases} S_4^B: \begin{cases} \bar{x} = A_x x, \quad \bar{t} = A_t t, & \text{(7a)} \\ \bar{\mu} = A_\mu \mu, \quad \bar{\varrho}\bar{q} = A_\varrho(\varrho q), & \text{(7b)} \\ \bar{V}_c = A_x^{-\frac{1+q}{q-1}} A_t^{\frac{1+q}{q-1}-q} A_\varrho^{-\frac{q}{q-1}} A_\mu^{\frac{q}{q-1}} V_c, & \text{(7c)} \end{cases} \\ \bar{u} = A_x^{-\frac{1+q}{q-1}} A_t^{\frac{2q}{q-1}} A_\varrho^{-\frac{q}{q+1}} A_\mu^{\frac{q}{q-1}} u, & \text{(7d)} \\ \bar{v} = A_x^{-\frac{1+q}{q-1}} A_t^{\frac{1+q}{q-1}} A_\varrho^{-\frac{q}{q-1}} A_\mu^{\frac{q}{q-1}} v, & \text{(7e)} \\ \bar{\sigma} = A_x^{-\frac{2}{q-1}} A_t^{\frac{2}{q-1}} A_\varrho^{-\frac{1}{q-1}} A_\mu^{\frac{q}{q-1}} \sigma. & \text{(7f)} \end{cases}$$

In the above relations S_4^B is a 4-parameter subgroup of G_4^B. Also, the equation of characteristic is conformally invariant under G_4^B (see theorem 3 in [7]). It turns out that the kinematical condition of compatibility across the wave front [8, 9, 10],

$$\left[\frac{du(x,t)}{dt}\right] = \left[\frac{\partial u(x,t)}{\partial t} + \frac{\partial u(x,t)}{\partial x}\frac{dx_w(t)}{dt}\right] \tag{8}$$

and the balance law of linear momentum

$$[\sigma(x,t)] = \varrho \frac{dx_w(t)}{dt}\left[\frac{\partial u(x,t)}{\partial t}\right], \tag{9}$$

across the singular surface, [8, 9, 10], are also invariant under the group G_4^B. The symbol of square bracket, $[\cdot]$, in (8) and (9) means the jump of the function across the wave front. The proof of invariance is similar to that given in [16], and is omitted here. This implies, that the conditions on the wave front do not give further restrictions among the group parameters and the parameters are essential.

The dimensional matrices associated with the dimensional group of transformations G_4^B assume the forms:

$$A:\begin{bmatrix} -\frac{1+q}{q-1}, & \frac{2q}{q-1}, & -\frac{q}{q-1}, & \frac{q}{q-1} \\ -\frac{1+q}{q-1}, & \frac{1+q}{q-1}, & -\frac{q}{q-1}, & \frac{q}{q-1} \\ -\frac{2}{q-1}, & \frac{2}{q-1}, & -\frac{1}{q-1}, & \frac{q}{q-1} \end{bmatrix}, \tag{10a}$$

$$B:\begin{bmatrix} 1, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0 \end{bmatrix}, \tag{10b}$$

$$C:\begin{bmatrix} 0, & 0, & 0, & 1 \\ 0, & 0, & 1, & 0 \\ -\frac{1+q}{q-1}, & \frac{1+q}{q-1}-\delta, & -\frac{q}{q-1}, & \frac{q}{q-1} \end{bmatrix}. \tag{10c}$$

The matrix BC, constructed by augmenting the matrix B with C, has the rank, $r = 4$, while the matrix C has the rank, $s = 3$ (since $q > 0$). These properties of the matrices BC and C indicate that since $r > s$ the similarity transformations for the problem formulated above, can be obtained.

Theorem 2 in [7] indicates that the group G_4^B has $[n+m+p-r] = [3+2+3-4] = 4$ functionally independent absolute invariants, where n is the number of dependent variables, m — independent variables, p — physical parameters. Making use of formulae (1.16) - (1.21) (1.21) of [7] we obtained respectively:

$$\eta = x[t]^{\Gamma_{12}}[\mu]^{\gamma_{11}}[\varrho q]^{\gamma_{12}}[v_c]^{\gamma_{13}}, \tag{11}$$

where Γ_{12} and γ_{1j}, $j = 1, 2, 3$, provide linearly independent solutions to

$$\Gamma_{12}\begin{bmatrix} b_{21} \\ b_{22} \\ b_{23} \\ b_{24} \end{bmatrix} + \sum_{\omega=1}^{3}\gamma_{1\omega}\begin{bmatrix} c_{\omega 1} \\ c_{\omega 2} \\ c_{\omega 3} \\ c_{\omega 4} \end{bmatrix} = -\begin{bmatrix} b_{11} \\ b_{12} \\ b_{13} \\ b_{14} \end{bmatrix}. \tag{12a}$$

Taking into account (10b, c), the above equation (12a), can be written as

$$\Gamma_{12}\begin{bmatrix}0\\1\\0\\0\end{bmatrix}+\gamma_{11}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}+\gamma_{12}\begin{bmatrix}0\\0\\1\\0\end{bmatrix}+\gamma_{13}\begin{bmatrix}-\dfrac{(1+q)}{q-1}\\ \dfrac{1+q}{q-1}-\delta\\ -\dfrac{q}{q-1}\\ \dfrac{q}{q-1}\end{bmatrix}=-\begin{bmatrix}1\\0\\0\\0\end{bmatrix}. \tag{12b}$$

The system of equations (12b) gives

$$\gamma_{11}=-\frac{q}{1+q},\quad \gamma_{12}=\frac{q}{1+q},\quad \gamma_{13}=\frac{q-1}{1+q}, \tag{13a}$$

$$\Gamma_{12}=-m,\quad \text{where } m=1+\delta\frac{1-q}{q+1}. \tag{13b}$$

Substituting (13a, b) into (11) we obtain

$$\eta=K\frac{x}{t^{m}}, \tag{14a}$$

where

$$K=\left\{\left(\frac{\mu}{\varrho q}\right)^{\frac{q}{1+q}}V_c^{\frac{1-q}{1+q}}\right\}^{-1}, \tag{14b}$$

$$m=1+\delta\frac{1-q}{1+q}>0. \tag{14c}$$

Next, the functionally independent absolute invariants are determined as new dependent variables F_j, $j=1,2,3$:

$$F_1=u[t]^{\Lambda_{12}}[\mu]^{\lambda_{11}}[\varrho q]^{\lambda_{12}}[v_c]^{\lambda_{13}}, \tag{15a}$$

$$F_2=v[t]^{\Lambda_{22}}[\mu]^{\lambda_{21}}[\varrho q]^{\lambda_{22}}[v_c]^{\lambda_{23}}, \tag{15b}$$

$$F_3=\sigma[t]^{\Lambda_{32}}[\mu]^{\lambda_{31}}[\varrho q]^{\lambda_{32}}[v_c]^{\lambda_{33}}, \tag{15c}$$

where Λ_{j2} and $\lambda_{j\omega}$ provide linearly independent solutions to

$$\Lambda_{j2}\begin{bmatrix}b_{21}\\b_{22}\\b_{23}\\b_{24}\end{bmatrix}+\sum_{\omega=1}^{3}\lambda_{j\omega}\begin{bmatrix}c_{\omega1}\\c_{\omega2}\\c_{\omega3}\\c_{\omega4}\end{bmatrix}=-\begin{bmatrix}a_{j1}\\a_{j2}\\a_{j3}\\a_{j4}\end{bmatrix},\quad j=1,2,3 \tag{16a}$$

Taking into account the elements of matrices B and C, from equations (10), equation (16a) becomes

$$A_{j2}\begin{bmatrix}0\\1\\0\\0\end{bmatrix}+\lambda_{j1}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}+\lambda_{j2}\begin{bmatrix}0\\0\\1\\0\end{bmatrix}+\lambda_{j3}\begin{bmatrix}-\dfrac{1+q}{q-1}\\ \dfrac{1+q}{q-1}-\delta\\ -\dfrac{q}{q-1}\\ \dfrac{q}{q-1}\end{bmatrix}=-\begin{bmatrix}a_{j1}\\a_{j2}\\a_{j3}\\a_{j4}\end{bmatrix},\quad j=1,2,3. \tag{16b}$$

In relations (16) $a_{j\alpha}$ are the elements of the matrix A, (10a). For $j = 1$ the solution of (16b) can be expressed in the form

$$\lambda_{11} = 0, \quad \lambda_{12} = 0, \quad \lambda_{13} = -1, \tag{17a}$$

$$\lambda_{12} = -(1+\delta), \tag{17b}$$

hence, the invariant F_1 assumes the form

$$F_1 = uV_c^{-1}t^{-(1+\delta)}. \tag{18}$$

If $j = 2$, the system of equations (16b) leads to

$$\lambda_{21} = 0, \quad \lambda_{22} = 0, \quad \lambda_{23} = -1, \tag{19a}$$

$$\lambda_{22} = -\delta. \tag{19b}$$

Substitution of (19) into (15b) results in

$$F_2 = vV_c^{-1}t^{-\delta}. \tag{20}$$

Finally, setting $j = 3$ we obtain form (16b)

$$\lambda_{31} = -\frac{q}{q+1}, \quad \lambda_{32} = -\frac{1}{q+1}, \quad \lambda_{33} = -\frac{2}{q+1}, \tag{21a}$$

$$\lambda_{32} = -\frac{2\delta}{q+1}. \tag{21b}$$

The last invariant of the group G_4^B assumes the form

$$F_3 = \sigma t^{-\frac{2\delta}{q+1}}\mu^{-\frac{q}{q+1}}(\varrho q)^{-\frac{1}{q+1}}V_c^{-\frac{2}{q+1}}. \tag{22}$$

The set $\{\eta, F_1, F_2, F_3\}$ of independent absolute invariants of G_4^B, given by (14a), (18), (20) and (22), gives the following similarity transformations:

$$\eta = K\frac{x}{t^m}, \tag{23a}$$

$$u(x,t) = V_c t^{(1+\delta)}F_1(\eta), \tag{23b}$$

$$v(x,t) = V_c t^{\delta}F_2(\eta), \tag{23c}$$

$$\sigma(x,t) = K_1 t^{m_1}F_3(\eta), \tag{23d}$$

where

$$K = \left(\frac{\mu}{\varrho q}\right)^{-\frac{q}{q+1}} V_c^{-\frac{1-q}{1+q}}, \tag{23e}$$

$$m = 1+\delta\frac{1-q}{1+q}, \tag{23f}$$

$$K_1 = (\mu^q \varrho q V_c^2)^{\frac{1}{1+q}}, \tag{23g}$$

$$m_1 = \frac{2\delta}{q+1}. \tag{23h}$$

4. Similarity Representation

Making use of the similarity transformations (23), the system of equations (1) and auxiliary conditions (2) can be reduced to an ordinary boundary value problem. The partial derivatives appearing in equations (1), can now be expressed in terms of similarity transformations as

$$\frac{\partial u}{\partial t} = V_c t^{\delta}[(1+\delta)F_1(\eta) - m\eta F_1'(\eta)], \tag{24a}$$

$$\frac{\partial v}{\partial t} = V_c t^{(\delta-1)}[\delta F_2(\eta) - m\eta F_2'(\eta)], \tag{24b}$$

$$\frac{\partial \sigma}{\partial x} = K_1 K t^{(m_1-m)} F_3'(\eta), \tag{24c}$$

$$-\frac{\partial v}{\partial x} = -K V_c t^{(\delta-m)} F_2'(\eta), \tag{24d}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{\sigma}{\mu}\right)^q\right] = q\left(\frac{K_1}{\mu}\right)^q t^{m_1 q-1} F_3^{(q-1)}(\eta)[m_1 F_3(\eta) - m\eta F_3'(\eta)]. \tag{24e}$$

Substituting (24a) and (23c) in (1e) results

$$F_2(\eta) = (1+\delta)F_1(\eta) - m\eta F_1'(\eta), \tag{25a}$$

the equations of motion, (1a), can be expressed in terms of similarity transformations as

$$qF_3'(\eta) = -\delta F_2(\eta) + m\eta F_2'(\eta), \tag{25b}$$

and the constitutive law (1c), taking into account (24d, e), assumes the form

$$F_2'(\eta) = -qm_1 F_3^q(\eta) + qm\eta F_3^{q-1}(\eta) F_3'(\eta). \tag{25c}$$

The boundary condition (2a) also can be transformed to the similarity space, as

$$F_2(0) = 1, \tag{26a}$$

and by the use of equations (25a, b) and (26a) we obtain

$$F_1(0) = \frac{1}{1+\delta}, \quad F_3'(0) = -\frac{\delta}{q}. \tag{26b, c}$$

It should be pointed out, that boundary conditions (26) are not linearly independent and only one of them can be taken into account for further consideration.

The boundary conditions on the wave front will be determined on the basis of the similarity characteristic — relationship [6, 7] and the relation between $F_3(\eta)$ and $F_1'(\eta)$. The system of coupled equations (1) leads to the partial differential equations of second order in terms of stress as

$$\frac{\partial^2\sigma}{\partial x^2} = \frac{\varrho q}{\mu q}\sigma^{q-1}\frac{\partial^2\sigma}{\partial t^2} + \frac{\varrho q(q-1)}{\mu^q}\sigma^{q-2}\left(\frac{\partial\sigma}{\partial t}\right)^2. \tag{27}$$

Making use of theorem 3, given in [7], we know that the equation of characteristics of quasi-linear partial differential equation (27) is invariant under the group G_4^B. This allows us to transform the location of the wave front into the similarity space. The characteristic equation of (27) has the form

$$\frac{dt}{dx} = \left(\frac{\varrho q}{\mu q}\right)^{1/2}\sigma^{\frac{q-1}{2}}. \tag{28a}$$

Making use of the equation (23d), (28a) becomes

$$\frac{dt}{dx} = \left(\frac{\varrho q}{\mu q}\right)^{1/2} K_1^{\frac{q-1}{2}} t^{\frac{m_1(q-1)}{2}} F_3^{\frac{q-1}{2}}(\eta) \tag{28b}$$

and hence

$$t^{\frac{m_1(1-q)}{2}} dt = \left(\frac{\varrho q}{\mu q}\right)^{1/2} K_1^{\frac{q-1}{2}} F_3^{\frac{q-1}{2}}(\eta)\,dx. \tag{28c}$$

Integration of both sides of (28c) gives

$$\frac{1}{m}t^m = \left(\frac{\varrho q}{\mu q}\right)^{1/2} K_1^{\frac{q-1}{2}} F_3^{\frac{q-1}{2}}(\eta)x + c. \tag{28d}$$

For the characteristic passing through the origin the constant c becomes zero, hence on the wave front the following relation holds

$$\frac{1}{m}t^m = \left(\frac{\varrho q}{\mu q}\right)^{1/2} K_1^{\frac{q-1}{2}} K^{-1} t^m F_3^{\frac{q-1}{2}}(\eta_w)\eta_w, \tag{28e}$$

where η_w is the location of the wave front in the transform space.
Finally, after substituting the values of K_1 and K in (28e), the similarity characteristic relation assumes the form

$$\eta_w = \frac{1}{m}[F_3(\eta_w)]^{\frac{1-q}{2}}. \tag{29}$$

It can be easily shown that whenever the characteristic relation, (29) holds, the kinematical condition of compatibility, (8), across the singular surface and the balance law of linear momentum, (9), are identically satisfied. The calculations are similar to that given in [16] and are omitted here. The relation (29) locates the wave front, however, this is given implicitly. In order to state full boundary value problems we need one more boundary condition. This will be obtained by the use of the relation between $F_3(\eta)$ and $F_1'(\eta)$. The constitutive law, (1c), is equivalent to

$$\sigma(x,t) = \mu\left(-\frac{\partial u(x,t)}{\partial x}\right)^{\frac{1}{q}}. \tag{30}$$

Making use of the similarity transformations (23), it is found that (30) becomes

$$F_3(\eta) = \left(-F_1'(\eta)\right)^{\frac{1}{q}}. \tag{31}$$

Now, we can eliminate $F_2(\eta)$ and $F_3(\eta)$ between equations (25a, b, c) to deduce a single differential equation in terms of $F_1(\eta)$:

$$\left\{[-F_1'(\eta)]^{\frac{1-q}{q}} - m^2\eta^2\right\} F_1''(\eta) - m(m-2\delta-1)\eta F_1'(\eta) - \delta(\delta+1)F_1(\eta) = 0, \tag{32a}$$

with boundary conditions

$$F_1(0) = \frac{1}{1+\delta}, \tag{32b}$$

$$F_1(\eta = \eta_w) = 0 \quad \text{and} \tag{32c}$$

$$\eta_w = \frac{1}{m}[-F_1'(\eta_w)]^{\frac{1-q}{2q}}. \tag{32d}$$

During the derivation of equation (32a) the relation (31) has been used and the characteristic relation (29) becomes (32d) after the substitution of (31). The boundary condition (32c) is obtained on the basis of physical consideration, since the displacement $u(x, t)$ is a continuous function and must equal zero at the wave front.

The boundary value problem (32) is in agreement with a special case of a more general problem given in [17], however the solution in [17] is obtained only for almost nonlinear case. The boundary value problems (32) can not be considered for arbitrary values of the parameters δ and q, suitable restriction given by equation (14c) must be taken into account. Also in order to include the physically interesting case of an applied velocity impact which is infinitely large at $t = 0$ followed by a decay in time, the parameter δ is permitted to take on negative values. However, it seems reasonable to consider only those cases for which both the impulse and displacement at the origin are finite. According to equations (23b, c) we must then require

$$\delta > -1. \tag{33}$$

The restriction on parameter δ and q given by (14c), which requires m to be positive and inequality (33) implies that for $q = 1$, linear case, δ can assume any real number greater than -1. It turns out that this assumption is valid not only for $q = 1$, but also for any $0 < q \leqslant 1$. However, when $q > 1$, δ has to satisfy the inequality

$$-1 < \delta < \frac{1+q}{q-1}. \tag{34}$$

5. Closed Form Solutions of Some Special Cases

a). Linear elastic rod, q = 1, subjected to time dependent velocity impact $\delta > -1$

For some special cases closed form solutions can be obtained for the system of equations (32). For instance, if we consider a linear elastic material ($q = 1$) then the function

$$F_1(\eta) = \frac{1}{1+\delta}(1-\eta)^{1+\delta}, \quad 0 \leqslant \eta \leqslant 1, \quad \delta > -1, \quad q = 1, \tag{35}$$

satisfies equation (32a) and boundary conditions (32b, c, d), see [6, 18]. The similarity function, $F_3(\eta)$, may then be obtained by substitution of equation (35) into equation (31)

$$F_3(\eta) = (1-\eta)^\delta, \quad 0 \leqslant \eta \leqslant 1, \quad \delta > -1, \quad q = 1. \tag{36}$$

The displacement and the stress distribution in the original (x, t) — space can now be easily expressed by making use of equations (35) and (36) in equations (23b) and (23d) to obtain

$$u(x,t) = V_c t^{1+\delta} \frac{1}{1+\delta}\left(1-K\frac{x}{t}\right)^{1+\delta}, \quad \delta > -1, \quad q = 1, \tag{37a}$$

$$\sigma(x,t) = K_1 t^\delta \left(1-K\frac{x}{t}\right)^\delta, \quad \delta > -1, \quad q = 1, \tag{37b}$$

or

$$u(x,t) = \frac{1}{1+\delta} V_c\left(t-\frac{x}{c}\right)^{1+\delta}, \quad \delta > -1, \quad q = 1. \tag{38a}$$

$$\sigma(x,t) = (\mu\varrho V_c^2)^{1/2}\left(t-\frac{x}{c}\right)^\delta, \quad \delta > -1, \quad q = 1, \tag{38b}$$

where c is the velocity of wave propagation in the linear elastic rod. We can express the above relations in nondimensional form for convenience in the evalutaion of numerical results. For this purpose we set

$$\bar{x} = \frac{x}{x_0} \quad \text{and} \quad \bar{t} = \frac{t}{t_0} \tag{39}$$

where $\bar{x}$ and $\bar{t}$ are dimensionless, x_0 and t_0 have the same dimension as x and t respectively, otberwise they have nonzero but arbitrary magnitudes. On this basis we obtain the following nondimensional expressions:

$$\bar{u} = \frac{u(\tilde{x},\bar{t})}{V_c t_0^{1+\delta}} = \frac{1}{1+\delta}(\bar{t}-\tilde{x})^{1+\delta}, \quad \delta > -1, \quad q = 1, \tag{40a}$$

$$\bar{\sigma} = \frac{\sigma(\tilde{x},\bar{t})}{(\mu\varrho)^{1/2} V_c t_0^\delta} = (\bar{t}-\tilde{x})^\delta, \quad \delta > -1, \quad q = 1. \tag{40b}$$

where

$$\tilde{x} = \bar{x}\frac{x_0}{t_0} = \eta\bar{t}. \tag{40c}$$

b). Nonlinear elastic rod $q > 0$ subjected to step velocity impact, $\delta = 0$

The second class of closed form solutions is for step velocity loading, $\delta = 0$. In this case equation (32a) reduces to

$$\left\{[-F_1'(\eta)]^{\frac{1-q}{q}} - m^2\eta^2\right\} F_1''(\eta) = 0 \tag{41}$$

which is identically satisfied if $F_1''(\eta) = 0$. Thus, the general solution is given by

$$F_1(\eta) = c_1 + c_2\eta. \tag{42}$$

The boundary conditions (32b, c, d) give

$$c_1 = 1 \quad \text{and} \quad c_2 = -1, \tag{43}$$

which implies, that for any $q > 0$

$$\eta_w = 1. \tag{44}$$

Hence,

$$F_1(\eta) = 1-\eta, \quad 0 \leqslant \eta \leqslant 1, \quad \delta = 0, \quad q > 0. \tag{45}$$

Thus, for a nonlinear elastic bar with step velocity impact, the function $F_3(\eta)$ related to stress by equation (23d) assumes the form

$$F_3(\eta) = \left(-F_1'(\eta)\right)^{\frac{1}{q}} = 1, \quad 0 \leqslant \eta \leqslant 1, \quad q > 0, \quad \delta = 0, \tag{46}$$

In the manner similar to case a) the displacement u(x, t) and the stress $\sigma(x, t)$ for the constant velocity impact can be expressed in nondimensional form. Taking into account (39) and the solution (45) and (46) and the similarity transformation (23) we obtain respectively:

$$u(x, t) = V_c t\left(1-K\frac{x}{t}\right) = V_c(t-Kx), \quad q > 0 \tag{47a}$$

$$\sigma(x, t) = K_1, \quad q > 0 \tag{47b}$$

or

$$\bar{u} = \frac{u(\bar{x}, \bar{t})}{V_c t_0} = (\bar{t}-\bar{x}), \quad q > 0, \quad \delta = 0, \tag{47c}$$

$$\bar{\sigma} = \frac{\sigma(\bar{x}, \bar{t})}{(\mu^q \varrho V_c^2)^{\frac{1}{1+q}}} = q^{\frac{1}{1+q}}, \quad q > 0, \quad \delta = 0, \tag{47d}$$

where

$$\bar{x} = \bar{x}\frac{x_0}{c_q t_0} = \eta\bar{t}, \tag{47e}$$

and $c_q = \frac{1}{K}$, where K given by (23e), is the velocity of propagation of the elastic wave in the non-linear material. When $q = 1$, $\bar{\sigma} = 1$ which is in agreement with that obtained in [18].

c). Almost non-linear material for q close to unity

A valid analytical approximation can be obtained for the parameter of the non-linearity q close to unity.

For an almost nonlinear rod we assume that the parameter q assumes the values close to unity such that

$$[-F_1'(\eta)]^{\frac{1-q}{q}} \cong 1. \tag{48a}$$

It is understood in equation (48a) that the slope of similarity function $F_1(\eta)$ is not zero and does not tend to infinity at any point $0 \leqslant \eta \leqslant \eta_w$. With the above approximation the similarity representation given by equations (32a, b, c, d) assumes the form

$$[1-m^2\eta^2]F_1''(\eta)-m(m-2\delta-1)\eta F_1'(\eta)-\delta(\delta+1)F_1(\eta) = 0, \quad 0 \leqslant \eta \leqslant \eta_w, \tag{48b}$$

$$F_1(\eta = 0) = \frac{1}{1+\delta}, \tag{48c}$$

$$F_1(\eta = \eta_w) = 0, \tag{48d}$$

where

$$\eta_w = \frac{1}{m}, \quad \text{and} \quad q \cong 1, \quad m > 0. \tag{48e}$$

It should be pointed out, that the parameter of nonlinearity of material, q, is still included in the similarity representation (48), since the parameter m depends on q. The approximation is made only in one term, namely $[-F_1'(\eta)]$. Following the method given in [17] and applying directly the theorem 5, page 369 of Kaplan, [19], two linearly independent solutions of equation (48b) are obtained as

$$F_1^{(1)}(\eta) = 1 + \sum_{s=1}^{\infty} \eta^{2s} \frac{(1+q)^{s/q}}{2^s s!} \frac{\delta(\delta+1)}{(1+q)} \frac{(\delta-2m+1)(\delta-2m)}{3(1+q)} \times \frac{(\delta-4m+1)(\delta-4m)}{5(1+q)} \cdots \frac{(\delta-2m(s-1)+1)(\delta-2m(s-1))}{(2s-1)(1+q)} \tag{49a}$$

$$F_1^{(2)}(\eta) = \eta\left\{1 + \sum_{s=1}^{\infty} \eta^{2s} \frac{(1+q)^{s/q}}{2^s s!} \frac{(\delta+1-m)(\delta-m)}{3(1+q)} \times \frac{(\delta-3m+1)(\delta-3m)}{5(1+q)} \cdots \frac{(\delta-2m(s-1)+1-m)(\delta-m(2s-1))}{(2s+1)(1+q)}\right\}. \tag{49b}$$

The general solution of equation (48b) in terms of two linearly independent functions $F_1^{(1)}(\eta)$ and $F_1^{(2)}(\eta)$ can be written as

$$F_1(\eta) = c_1 F_1^{(1)}(\eta) + c_2 F_1^{(2)}(\eta), \tag{49c}$$

where c_1 and c_2 are constants to be determined from the boundary conditions. Making use of the boundary condition (48c) the value of c_1 is obtained as

$$c_1 = \frac{1}{1+\delta}, \tag{50a}$$

and on the basis of the boundary condition (48d) we obtain

$$c_2 = -\frac{F_1^{(1)}(\eta)}{(1+\delta)F_1^{(2)}(\eta_w)}, \tag{50b}$$

where η_w is given by (48e).

Thus, on the basis of equations (50a, b) the solution (49c) can be written as

$$F_1(\eta) = \frac{1}{1+\delta}\left[F_1^{(1)}(\eta) - \frac{F_1^{(1)}(\eta_w)}{F_1^{(2)}(\eta_w)} F_1^{(2)}(\eta)\right], \tag{51a}$$

$$0 \leqslant \eta \leqslant \frac{1}{m}, \tag{51b}$$

and under the condition that the parameters q and δ must satisfy the inequalities (33) or (34). Furthermore, it may be remembered that the solution holds for the values of q close to unity. The numerical analysis shows that the solution given by (51) gives an acceptable approximation for $0.5 \leqslant q \leqslant 1.5$. This would approximate the behaviour of such engineering materials which are not idealy linearly elastic but are close to it.

The nondimensional expressions for displacement $\bar{u}(\hat{x}, \bar{t})$ and stress $\bar{\sigma}(\hat{x}, t)$ for the almost nonlinear case assume the forms

$$\bar{u} = \frac{u(\hat{x}, \bar{t})}{V_c t_0^{\delta+1}} = \bar{t}^{(\delta+1)} F_1(\eta), \tag{52a}$$

$$\bar{\sigma} = \frac{\sigma(\hat{x}, t)}{(\mu^q \varrho V_c^2)^{\frac{1}{1+q}}} = \left[\frac{1}{1+\delta}\bar{t}^{(\delta+1)} \frac{F_1^{(1)}(\eta_w)}{F_2^{(2)}(\eta_w)}\right]^{\frac{1}{q}}, \quad \eta = 0,$$

$$= [-t^{(\delta+1)} F_1'(\eta)]^{\frac{1}{q}}, \quad \eta \neq 0, \tag{52b}$$

where $F_1'(\eta)$ is the derivative of $F_1(\eta)$ evaluated on the basis of (51). The results are in agreement with those obtained in [17].

6. Numerical Solution

Numerical solution of nonlinear similarity representation, equations (32), is obtained by Gear method for precision and convenience [20, 21]. The Gear subroutine package is available, for instance, in MULTICS computer system. It solves the initial value problem for a system of ordinary differential equations given in the form

$$\dot{y} = f(y, t), \tag{53a}$$

with initial values

$$y(t_0) = y_0, \tag{53b}$$

where y, $\dot{y}$ and f are vectors of order $N \geqslant 1$. With a subroutine for the calculation of f, the GEAR package computes a numerical solution of equations (53) at values of the independent variable t in some interval $[t_0, T]$, as desired by the user. It must be remembered that the right-hand side f of the ODE's must be a well defined function of $y = y(t)$ and t. Thus, it cannot involve y at previous values of t as for example in delay or retarded ordinary differential equations or in integro-differential equations. The approach used in the GEAR package are linear multipoint methods of the form

$$y_n = \sum_{j=1}^{k_1} \alpha_j y_{n-j} + h \sum_{j=1}^{k_2} \beta_j \dot{y}_{n-j}, \tag{54}$$

where y_k is an approximation to $y(t_k)$, $\dot{y}_k = f(y_k, t_k)$ is an approximation to $\dot{y}(t_k)$, and h is a constant step size: $h = t_{k+1} - t_k$. In the case of the Adams method of order l we have $k_1 = 1$ and $k_2 = l-1$. In the case of the backward differentiation formula (BDF) of order l, also called Gear's stiff method, we have $k_1 = l$ and $k_2 = 0$. The *BDF*'s are so called because, on dividing throught by $h\beta_0$, they can be regarded as approximation formulas for $\dot{y}_n$ in terms of $y_n, y_{n-1}, \ldots, y_{n-l}$. In either case, α_j and β_j are constants associated with the method, and $\beta_0 > 0$. The latter means that equation (54) is an implicit equations for y_n and is in general a nonlinear algebraic system that must be solved on every step. The fact that the order of a given method is l means that, if equation (54) is solved for y_n

with all past values being exact, then y_n will differ from the correct solution of the ODE by a local truncation error that is 0 (h^{l+1}) for small h.

A prime feature of GEAR package is its ability to solve stiff ODE problems. Also, it contains, as an option, a method well suited for non-stiff problems as well, namely the implicit Adams method with functional (or fixpoint) corrector iteration, also called the Adams-Bashforth-Moulton method. In this analysis both the stiff and non-stiff methods are implemented in a manner which allows both the step size and the order to vary in a dynamic way throughout the problem.

For details concerning the Gear's stiff method we refer the reader to Hindmarsh [20], where a description of method, testing examples, and listings of subroutines can be found.

In application of Gear method to the solution of the system of equations (32), first of all it is reduced to a system of two first order equations

$$\dot{y}_1(\eta) = y_2(\eta), \tag{55a}$$

$$\dot{y}_2(\eta) = \frac{m(m-2\delta-1)\eta}{(-y_2(\eta))^{\frac{1-q}{q}} - m^2\eta^2} y_2(\eta) + \frac{\delta(\delta+1)}{(-y_2(\eta))^{\frac{1-q}{q}} - m^2\eta^2} y_1(\eta) \tag{55b}$$

with initial conditions

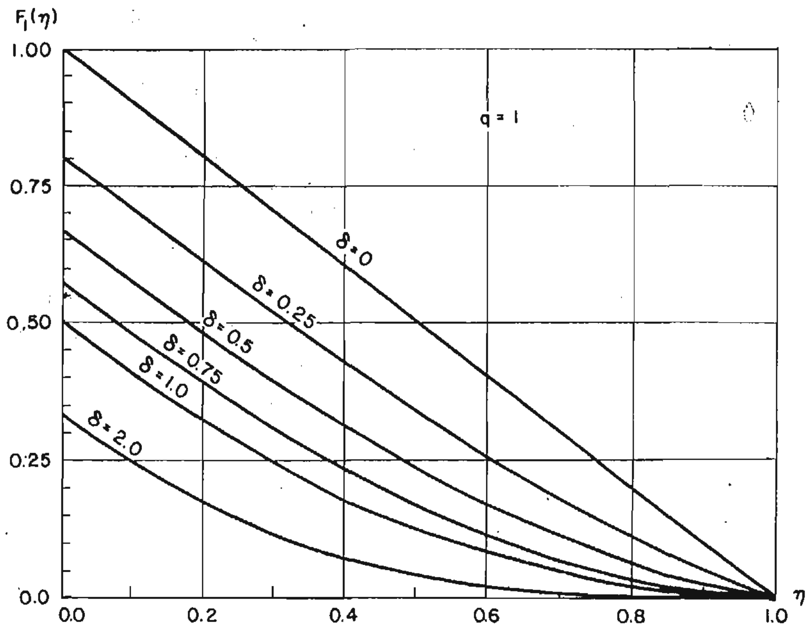
$$y_1(0) = \frac{1}{1+\delta}, \tag{56a}$$

$$y_2(0) = F_1'(0) = \alpha, \tag{56b}$$

Where, as first approximation, α is evaluated from the series solution for almost nonlinear case, equations (51). It turns out that the slope of the function $F_1(\eta)$ does not change sharply with the parameter of nonlinearity q in the neighbourhood of $\eta = 0$. Thus, the second initial condition (56b) can be determined from the analytical solution of almost nonlinear case. The correction for α is obtained by taking into account the boundary condition (32c, d) in such a way that the error in η_w, equation (32d) is kept less than 10^{-3}. Then, the boundary value problem is numerically solved by making use of Gear method. Computations were made for $\delta = 0$ and $q = 1.25$ and results were compared with the corresponding solution for almost nonlinear case. The numerical results obtained by Gear method were also compared with those obtained as close form solutions for $q = 3$ and $\delta = 0$. In both the above cases the numerical results were in good agreement with the corresponding exact solutions.

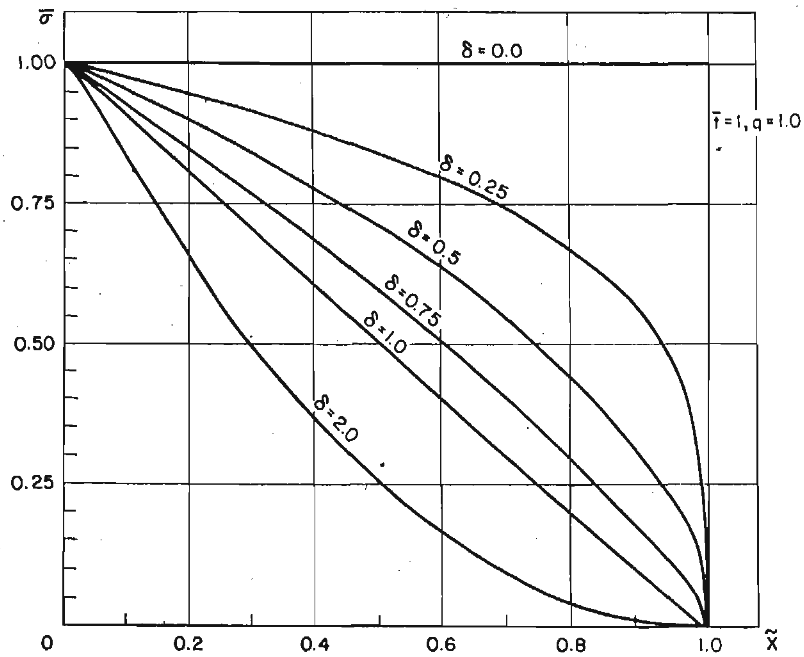
Effect of time dependence of impact, through the variation of parameter δ, is shown in Fig. 1 for a linear case, $q = 1$. It may be noted that whereas the value of $F_1(\eta)$, in general, decreases with increase of δ, the value of η_w is independent of δ and remains fixed as unity. Corresponding variation of $\bar{\sigma}$ as against $\bar{x}$ are shown in Fig. 2. Solution for almost nonlinear case with $\delta = 1$ is given for values of $0.5 \leqslant q \leqslant 1.5$ in Fig. 3. It is clear that in this case η_w varies with q.

Corresponding values of $\bar{\sigma}$ are given in Fig. 4, where it is seen that the values of $\bar{\sigma}$ and $\bar{x}_w$ decrease with decrease in the values of q. In Fig. 5 is shown the effect of variations in the parameter of nonlinearity, q for a constant velocity impact, $\delta = 0$ and a fixed $\bar{t}$, it is seen that value of $\bar{\sigma}$ approaches unity as $q \to \infty$.



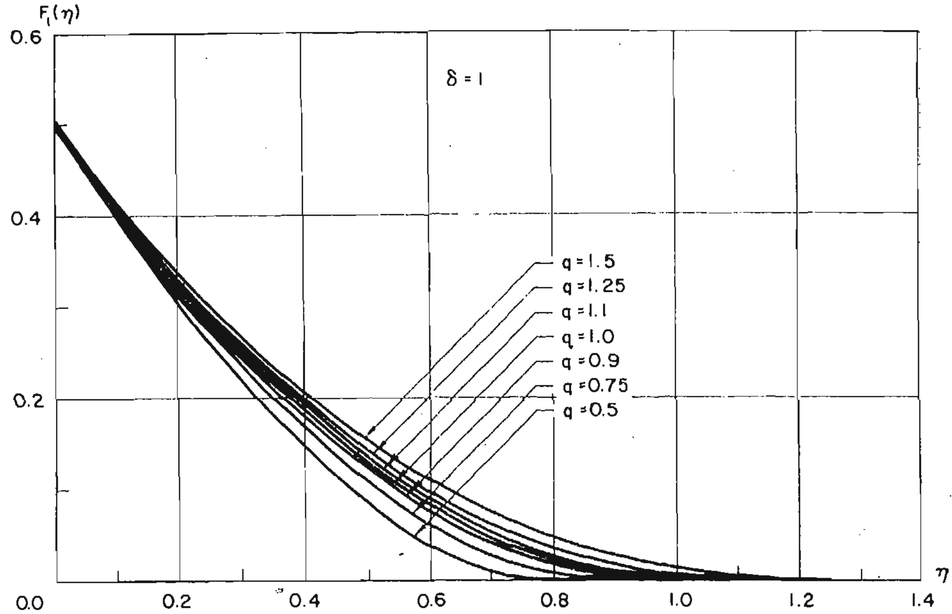
F₁(η)
1.00
0.75
0,50
0.25
0.0
0.0
0.2
0.4
0.6
0.8
1.0
η
q = 1
δ = 0
δ = 0.25
δ = 0.5
δ = 0.75
δ = 1.0
δ = 2.0

Rys. 1



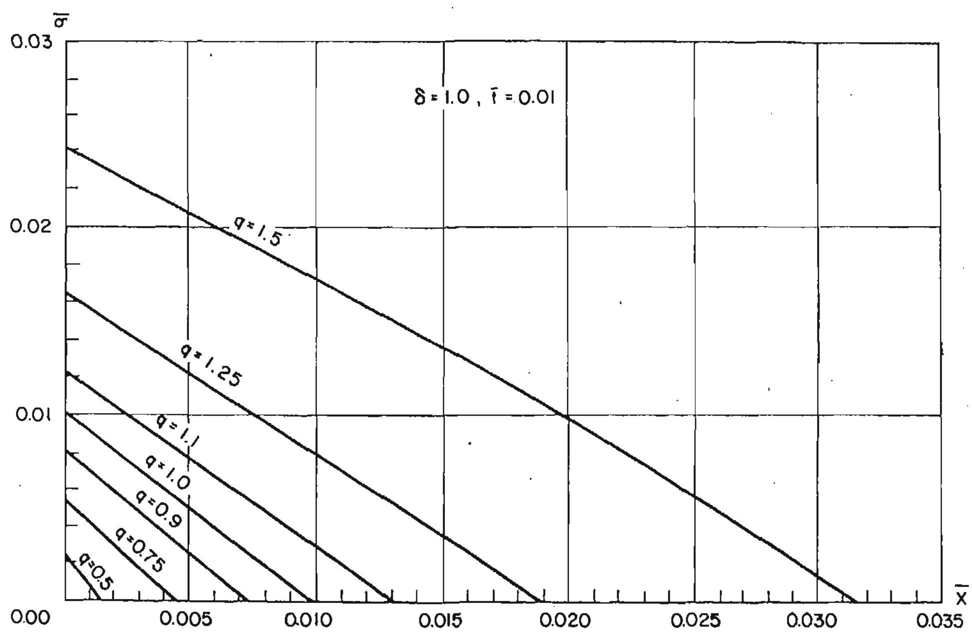
σ̄
1.00
0.75
0.50
0.25
0
0.2
0.4
0.6
0.8
1.0
x̃
δ = 0.0
t̄ = 1, q = 1.0
δ = 0.25
δ = 0.5
δ = 0.75
δ = 1.0
δ = 2.0

Rys. 2



$F_1(\eta)$
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
η
δ = 1
q = 1.5
q = 1.25
q = 1.1
q = 1.0
q = 0.9
q = 0.75
q = 0.5

Fig. 3



$\bar{\sigma}$
0.03
0.02
0.01
0.00
0.005
0.010
0.015
0.020
0.025
0.030
0.035
$\bar{x}$
δ = 1.0 , $\bar{t}$ = 0.01
q = 1.5
q = 1.25
q = 1.1
q = 1.0
q = 0.9
q = 0.75
q = 0.5

Fig. 4

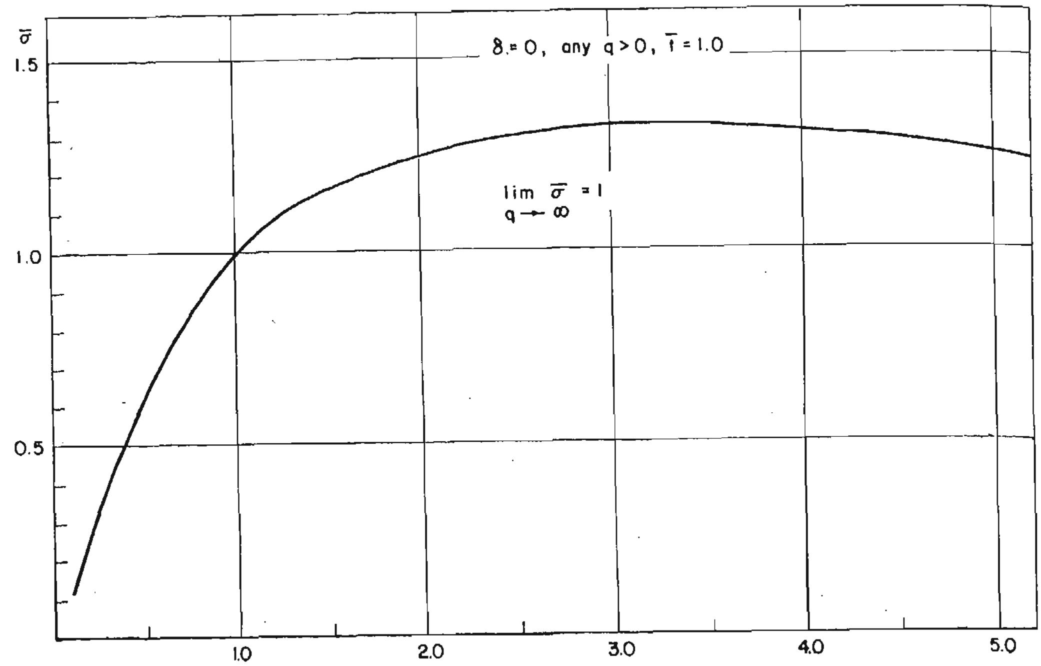


Fig. 5

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Резюме

РЕШЕНИЕ ПОДОБИЯ НЕЛИНЕЙНОГО УПРУГОГО СТЕРЖНЯ ПОД ВОЗДЕЙСТВИЕМ ИМПУЛЬСА СКОРОСТИ С ПОМОЩЮ МЕТОДА ГРУППОВЫХ ПРЕОБРАЗОВАНИЙ

Преобразования подобия для основных уравнений движения нелинейного упругого стержня под действием соударения со скоростью зависимой от времени получены путём применения групп преобразований. Подобие представлено в виде системы нелинейных обыкновенных дифференциальных уравнений с граничными усповиями в точке возникновения волны и на волновом фронте. Решения замкнутого типа были получены в нелинейном случае при соударении с постоянной скоростью и в линейном случае при соударении с переменной скоростью. Решение в виде ряда получено для почти нелинейного случая, так как для общего нелинейного случая получены численные решения.

Streszczenie

TRANSFORMACJE PODOBIEŃSTWA W PRZYPADKU ZAGADNIENIA NIELINIOWEGO PRĘTA SPRĘŻYSTEGO POD DZIAŁANIEM IMPULSU PRĘDKOŚCI PRZY POMOCY METODY PRZEKSZTAŁCEŃ GRUPOWYCH

Transformacje podobieństwa dla podstawowych równań ruchu nieliniowego pręta sprężystego pod działaniem impulsu prędkości otrzymano przy wykorzystaniu przekształceń przestrzennych.

Przedstawienie podobieństwa otrzymano jako system nieliniowych, zwyczajnych równań różniczkowych, w warunkach brzegowych w początku układu i na froncie fali.

Wyprowadzono równania w postaci zamkniętej dla przypadku liniowego zależnego od czasu impulsu prędkości oraz nieliniowego niezależnego od czasu impulsu prędkości.

Cały szereg rozwiązań otrzymano w przypadku prawie nieliniowym, natomiast w ogólnym przypadku nieliniowym przedstawiono rozwiązania numeryczne.

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