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DYNAMIC STABILITY OF VISCOELASTIC CONTINUOUS SYSTEMS UNDER TIME-DEPENDENT LOADINGS

ANDRZEJ TYLIKOWSKI

Warsaw Technical University

1. Introduction

The problem of static buckling of viscoelastic columns under constant axial forces has been solved by DE LEEUW [1]. Applying the correspondence principle and analysing the properties of elasticity moduli the critical loadings for several viscoelastic models have been obtained. One of the first analyses of the dynamic stability of viscoelastic continuous systems has been made by GENIN and MAYBEE [2]. In this paper the stability of a beam made up of a linear Voigt-Kelvin material with viscoelastic boundary conditions has been investigated. In the next significant study PLAUT [3] has used the Liapunov method to determine the stability criteria of viscoelastic columns subjected to compresive axial loadings. Using the same method WALKER and DIXON [4] have examined the effect of a linear structural damping on the stability of plane membranes adjacent to a supersonic airstream.

The dynamic stability of continuous systems under time-dependent deterministic or stochastic loadings has also received much attention, e.g. (KOZIN [5], ARIARATNAM and TAMM [6], TYLIKOWSKI [7]). The problem was solved not only for a simple elastic column subjected to an axial time-dependent force but also for arches, panels, plates and shells. In most papers the dissipation of energy was described by an external viscous model of damping.

In the present article the applicability of the Liapunov method is extended to linear Voigt-Kelvin systems subjected to time-dependent deterministic or stochastic parametric excitations. Using appropriate functionals general sufficient conditions for the asymptotic stability, the almost sure asymptotic stability as well as the uniform stochastic stability are derived. The paper describes the two general approaches to the stability analysis and present some illustrative examples.

2. Problem Formulation

Consider a Hilbert space $\mathscr{H}$ of all summable functions having all generalized derivatives of order $\leqslant 2n$ on the open set Ω, summable to the power 2, independent of time, possessing a suitable inner product $\langle .,. \rangle$ and a dynamic system, which is assumed to be well defined by the equation

$$\ddot{u}+D\dot{u}+Ku+\sum_{i=1}^{l}(\xi_i+q_i)L_i u = 0, \quad x \in \Omega, \tag{1}$$

under the condition that at every fixed $t \in [0, \infty)$ the state of the system $(u, \dot{u})$ belongs to the product $Y \times Y$, where Y $(Y \subset \mathscr{H})$ is a subset of functions belonging to $\mathscr{H}$, which satisfy given linear time-independent boundary conditions on the boundary $\partial\Omega$ of Ω. Operators K, D, L_i are linear differential with respect to spatial variables. K is self-adjoint of order $2n$, D is of order $\leqslant 2n$, L_i are self-adjoint of order $\leqslant n$, q_i and ξ_i are constant and time-dependent loading components, respectively.

The question of interest is the stability of the eqilibrium $(u, \dot{u}) = (0, 0)$ for a general system of the form (1). To estimate deviations of solutions from the equilibrium state we introduce formal stability definitions using a scalar measure $||\cdot||$, which is the distance between a solution of equation (1) with nontrivial initial conditions and the trivial solution. The study of stability of equilibrium state splits into three branches. First, under the assumption that the time-dependent components of forces are deterministic functions of time, conditions of the asymptotic stability of the trivial solution, i.e. conditions that imply

$$\lim_{t\to\infty}||u|| = 0$$

are derived.

Our second purpose is to discuss the almost sure asymptotic stability of the trivial solution, i.e. that corresponding to the equality

$$P\{\lim_{t\to\infty}||u|| = 0\} = 1,$$

if the forces ξ_i are stochastic „nonwhite" processes.

In the third case, if the forces are the Gaussian white noises, we investigate the uniform stochastic stability, i.e. we formulate conditions implying the logic sentence

$$\bigwedge_{\varepsilon>0}\bigwedge_{\delta>0}\bigvee_{r>0} ||u(.,0)|| < r \Rightarrow P\left\{\sup_{t\geqslant 0}||u(.,t)|| > \delta\right\} < \varepsilon.$$

We are going to study the foregoing kinds of stability via the Liapunov functional approach. In order to employ the direct Liapunov method we construct the class of functionals as follows

$$V = \alpha\langle\dot{u}, \dot{u}\rangle+(1-\alpha)\langle\dot{u}+Du, \dot{u}+Du\rangle+\langle u, Ku\rangle+\left\langle u, \sum_{i=1}^{l} q_i L_i w\right\rangle, \tag{2}$$

where $0 < \alpha \leqslant \frac{1}{2}$.

For $\alpha = \frac{1}{2}$ we have the functional similar to „the best" functional applied by KOZIN [5]. The mentioned functionals are the same only if the dissipation operator corresponds to the viscous model of damping. For α arbitrarily small but positive we obtain the functional similar to that introduced by PLAUT and INFANTE [8].

The functional V satisfies the desired positive definite property if

$$\langle u, Ku\rangle + \left\langle u, \sum_{i=1}^{l} q_i L_i u\right\rangle \geqslant 0, \tag{3}$$

i.e. if the classical condition for the static stability is fulfilled.

3. Asymptotic Stability and Almost Sure Asymptotic Stability

We can give a unified treatment of stability analysis for both deterministic and stochastic „nonwhite" processes. Under this assumption a classicial stability analysis can be applied. We choose the Liapunov functional in the form (2) inserting $\alpha = \frac{1}{2}$.

$$V_1 = \frac{1}{2}\langle \dot u, \dot u\rangle + \frac{1}{2}\langle \dot u + Du, \dot u + Du\rangle + \langle u, Ku\rangle + \left\langle u, \sum_{i=1}^{l} q_i L_i u\right\rangle. \tag{4}$$

If condition (3) is satisfied functional (4) is positive definite and its time-derivative along equation (1) is

$$\frac{dV_1}{dt} = -\left\langle\left(K + \sum_{i=1}^{l} q_i L_i\right) u, Du\right\rangle - \left\langle 2\dot u + Du, \sum_{i=1}^{l} \xi_i(t) L_i u\right\rangle. \tag{5}$$

Our object is to obtain bounds on V_1 that will guarantee the asymptotic stability or the almost sure asymptotic stability. In order to do this we transform (5) into the form

$$\frac{dV_1}{dt} = -2\lambda V_1 + 2U_1, \tag{6}$$

where U_1 is the known functional and λ is a parameter describing the intensity of damping. We now attempt to construct a bound

$$U_1 \leqslant \chi V_1, \tag{7}$$

where the function χ is to be determined. Substituting (7) into (6) and solving the obtained differential inequality we have

$$V_1(t) \leqslant V_{10} \exp\left\{-2\lambda t + 2\int_0^t \chi(s)\,ds\right\}. \tag{8}$$

Thus, it immediately follows that the sufficient stability condition for the asymptotic stability with respect to the measure $\|\cdot\| = \sqrt{V_1}$ is

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t \chi(s)\,ds \leqslant \lambda, \tag{9}$$

or for the almost sure asymptotic stability, if the processes ξ_i are ergodic and stationary, is

$$E\chi \leqslant \lambda, \tag{10}$$

where E denotes the operator of the mathematical expectation.

4. Uniform Stochastic Stability

If the excitations are the Gaussian white noises equation (1) should be rewritten in the Itô differential form

$$du = vdt,$$
$$dv = -\Big(Ku+Dv+\sum_{i=1}^{l} q_i L_i u\Big)dt - \sum_{i=1}^{l} \xi_i L_i u dw_i, \tag{11}$$

where w_i are the standard uncorrelated Wiener processes with intensities σ_i. As realizations of the Wiener processes are not differentiable the Itô calculus has to be applied in the stability analysis (see e.g. CURTAIN and FALB [9]). Taking functional (2) we calculate its differential

$$dV = \Big\{2\alpha\Big\langle v, -Ku-Dv-\sum_{i=1}^{l} q_i L_i u\Big\rangle + 2(1-\alpha)\Big\langle v+Du, -Ku-\sum_{i=1}^{l} q_i L_i u\Big\rangle +$$
$$+2\langle v, Ku\rangle + 2\Big\langle v, \sum_{i=1}^{l} q_i L_i u\Big\rangle + \sum_{i=1}^{l} \sigma_i^2 \langle L_i u, L_i u\rangle\Big\}dt +$$
$$+2\Big\langle v+(1-\alpha)Du, -\sum_{i=1}^{l} q_i L_i u dw_i\Big\rangle. \tag{12}$$

On integrating with respect to t from s to $\tau_\delta(t)$, where

$$\tau_\delta(t) = \min\{\tau_\delta, t\},$$
$$\tau_\delta = \inf\{t: \|u\| > \delta > 0\}$$

and rearranging the integrand it follows that

$$V(\tau_\sigma(t)) = V(s) - 2\int_s^{\tau_\delta(t)} \Big\{\alpha\langle v, Dv\rangle + (1-\alpha)\langle Ku, Du\rangle + (1-\alpha)\Big\langle Dw, \sum_{i=1}^{l} q_i L_i u\Big\rangle -$$
$$-\frac{1}{2}\sum_{i=1}^{l} \sigma_i^2 \langle L_i u, L_i u\rangle\Big\}dt + 2\int_s^{\tau_\delta(t)} \Big\langle v+(1-\alpha)Du, \sum_{i=1}^{l} \sigma_i L_i u dw_i\Big\rangle. \tag{13}$$

We now take the conditional average of equation (13) remembering that the second integral is a stochastic one, so the conditional average of it is equal to zero

$$EV(\tau_\delta(t)) = V(s) - 2E\int_s^{\tau_\delta(t)} \Big\{\alpha\langle v, Dv\rangle + (1-\alpha)\langle Ku, Du\rangle +$$
$$+(1-\alpha)\Big\langle Du, \sum_{i=1}^{l} q_i L_i u\Big\rangle - \frac{1}{2}\sum_{i=1}^{l} \sigma_i^2 \langle L_i u, L_i u\rangle\Big\}dt. \tag{14}$$

We see that the functional $V(\tau_\delta(t))$ is a supermartingale, i.e. $EV(\tau_\delta(t)) \leqslant V(s)$, if the integrand of equation (14) is nonnegative. Neglecting the first positive term $\alpha\langle v, Dv\rangle$ and proceeding similarly to the proof of the Chebyshev inequality we have the following chain of inequalities

$$V(s) \geqslant EV(\tau_\delta(t)) = \int_\Gamma V(\tau_\delta(t))P(d\gamma) \geqslant \int_{\{\gamma: \sup_{t\geqslant s}\|u\| > \delta\}} V((\tau_\delta(t))P(d\gamma) \geqslant \delta^2 P\{\sup_{t\geqslant s}\|u\| > \delta\},$$

where $\gamma \in (\Gamma, \beta, P)$, i.e. γ belongs to the probability space Γ with σ — algebra β and probability measure P. Setting $s \to 0$ we conclude that the trivial solution of equation (11) is uniformly stochastically stable with respect to measure $\|\cdot\| = V^{1/2}$, if the following inequality is satisfied for every $u \in Y$

$$(1-\alpha)\left\langle Du, Ku+\sum_{i=1}^{l} q_i L_i u\right\rangle - \frac{1}{2}\sum_{i=1}^{l}\sigma_i^2\langle L_i u, L_i u\rangle \geqslant 0. \tag{15}$$

If α is arbitrarily small but positive, we shall obtain the largest stability region as a function of damping parameter and intensities σ_i, so the weak inequality (15) becomes

$$\left\langle Du, \left(K+\sum_{i=1}^{l} q_i L_i\right)u\right\rangle - \frac{1}{2}\sum_{i=1}^{l}\sigma_i^2\langle L_i u, L_i u\rangle > 0, \tag{16}$$

5. Asymptotic Stability and Almost Sure Asymptotic Stability of a Viscoelastic Beam Compressed by a Time-Dependent Force

Let us consider a straight simply supported beam of constant cross section. If the linear Voigt-Kelvin material is assumed the equation of transverse motion obtained by the correspondence principle has the form

$$\frac{\partial v}{\partial t} + 2\lambda v'''' + u'''' + (\xi + q)u'' = 0, \qquad x \in (0, 1) \tag{17}$$

where prime denotes the partial differentiation with respect to the spatial variable x, $v = \dfrac{\partial u}{\partial t}$, λ is the dimensionless retardation time, q and ξ are constant and time-dependent components of the axial force, respectively.

We choose the functional in the form (4)

$$V_1 = \int_0^1 \left[\frac{1}{2}v^2 + \frac{1}{2}(v+2\lambda u'''')^2 + (u'')^2 - q(u')^2\right]dx. \tag{18}$$

The functional (18) is positive definite if the Euler condition is fulfilled $q < \pi^2$. Upon differentiating V_1 along any solution of equation (17) we obtain the equation (6) where

$$U_1 = \int_0^1 [-\lambda(v'')^2 - \lambda(u'''')^2 - \lambda vu' + \lambda\xi(u''')^2 + \lambda q(u''')^2 +$$
$$+ \lambda(v^2 + 2\lambda vu'''' + 2\lambda^2(u'''')^2 + (u'')^2 - q(u')^2)]dx.$$

In order to determine the function χ that satisfies the inequality (7) we apply the variational calculus and solve the problem $\delta(U_1 - \chi V_1) = 0$ via the associated Euler equations. After extensive but straightforward computations we find the function χ to be

$$\chi = \lambda + \max_{n=1,2,\ldots} \left\{ n\pi \left| \lambda^2 (n\pi)^6 + \frac{\xi}{2} \right| \Big/ \sqrt{(n\pi)^2[1+\lambda^2(n\pi)^4] - q} - \lambda(n\pi)^4 \right\}.$$

The asymptotic stability regions as functions of σ^2, q, evaluated numerically in the case when the load is a deterministic periodic (sinusoidal) process are shown in Fig. 1,

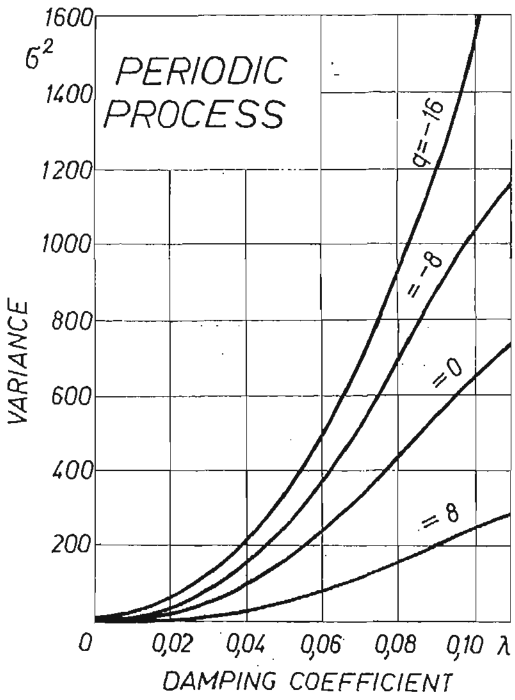

Fig. 1.

where the variance of the sinusoidal process is equal to the half of the amplitude squared. As the second numerical example we take the beam compressed by a Gaussian process. The dependence of the stability regions on the retardation time λ, variance σ and constant load q is shown in Fig. 2.

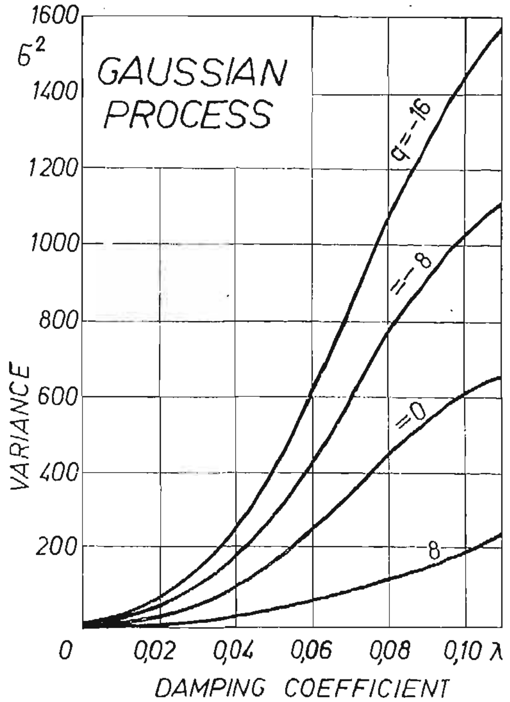

Fig. 2.

6. Uniform Stochastic Stability of a Viscoelastic Beam Compressed by the Gaussian white noise

If the load acting in the beam axis is a broad-band Gaussian process we model the excitation by means of a white noise of intensity σ and rewrite equation (17) in the Itô differential form

$$du = vdt,$$
$$dv = -(u'''' + 2\lambda v'''' + qu'')dt - \sigma u'' dw, \tag{19}$$

where w is the standard Wiener process. Using the functional V defined by (2) we obtain the simpler form of the general stability condition

$$\int_0^1 \left(\lambda(u'''')^2 - \lambda q(u''')^2 - \frac{\sigma^2}{4}(u'')^2\right) dx > 0.$$

Finally the condition for the uniform stochastic stability of the undeflected beam is given by

$$\lambda > \sigma^2/4\pi^2(\pi^2 - q).$$

7. Uniform Stochastic Stability of a Plane Bending Form of a Viscoelastic Thin-Walled Double-Tee Beam

Let us consider the flexural-torsional stability of a thin-walled double-tee simply supported beam subjected to broad-band Gaussian couples m acting on both ends in the plane of greater bending stiffness. Assuming the technical theory of thin-walled beams we neglect rotatory inertia terms and an influence of transverse forces on displacements of the beam and describe the displacement state by the axis displacements and the angle of torsion. As we are going to examine the stability of the plane form we can omit the equation of motion in the plane of the couples and describe the deviations from the plane state by the transverse displacement u of the beam axis and the angle of torsion φ. Using the correspondence principle we have the equations of motion in the form

$$\begin{aligned} &\frac{\partial v}{\partial t}+2\lambda e_1 v''''+e_1 u''''+m\varphi'' = 0, \\ &\frac{\partial \psi}{\partial t}+2\lambda e_2 \psi''''+e_2 \varphi''''-2\lambda e_3 \psi''-e_3\varphi''+mu'' = 0, \end{aligned} \tag{20}$$

where v and ψ are linear and angular velocities, respectively. Constants e_1, e_2, e_3 denote bending, warping and torsional stiffnesses, respectively. λ is the retardation time of a Voigt-Kelvin material. Modelling the broad-band couples as a sum of the constant component q and the white noise w with an intensity σ we rewrite equations (20) in the Itô differential form

$$\begin{aligned} du &= vdt, \\ dv &= -(e_1 u''''+2e_1 v''''+q'')dt-\sigma\varphi'' dw, \\ d\varphi &= \psi dt, \\ d\psi &= -(e_2\varphi''''+2\lambda e_2\psi''''-e_3\varphi''-2\lambda e_3\psi''+qu'')dt-\sigma u'' dw. \end{aligned} \tag{21}$$

We can now identify the operators

$$K = \begin{bmatrix} e_1(.)'''' & 0 \\ 0 & e_2(.)''''-e_3(.)'' \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & (.)'' \\ (.)'' & 0 \end{bmatrix}$$

$$D = 2\lambda K.$$

The functional is specified as follows

$$V = \int_0^1 \{\alpha(v^2+\psi^2)+(1-\alpha)\left((v+2\lambda e_1 u'''')^2+(\psi+2\lambda e_2\varphi''''-2\lambda e_3\varphi'')^2\right)+ \\ +e_1(u'''')^2+e_2(\varphi'''')^2+e_3(\varphi'')^2-2qu''\varphi\}dx. \tag{22}$$

The form (22) is similar to the functional used in a stochastic stability analysis of thin-walled beams with external viscous damping (TYLIKOWSKI [10]).

Assuming that the both ends are simply supported and are free to warp we have the following boundary conditions

$$\begin{aligned} \varphi(t,0) &= \varphi(t,1) = \varphi''(t,0) = \varphi''(t,1) = 0, \\ u(t,0) &= u(t,1) = u''(t,0) = u''(t,1) = 0. \end{aligned} \tag{23}$$

Functional (22) is positive definite if the well-known Timoshenko condition for lateral stability is satisfied by the constant component q

$$q < \pi\sqrt{e_1(e_3+e_2\pi^2)}.$$

Specifying general stability condition (16) we get

$$\int_0^1 \Big\{e_1^2(u'''')^2+e_2^2(\varphi'''')^2+e_3^2(\varphi'')^2+2e_2e_3(\varphi''')^2+qe_1u''''\varphi''+$$

$$+qe_2\varphi''''u''-qe_3\varphi''u''-\frac{1}{2\lambda}\sigma^2\left((u'')^2+(\varphi'')^2\right)\Big\}dx > 0. \tag{24}$$

Integrating by parts and using boundary conditions (23) one can show that

$$\int_0^1 u''''\varphi''dx = \int_0^1 u''\varphi''''dx,$$

Using this property and applying the elementary inequality

$$\pm ab \geqslant -\frac{1}{2\alpha^2}a^2-\frac{1}{2}\alpha^2b^2$$

to condition (24) we have

$$\int_0^1 \{[e_1^2-(e_1+e_2)^2/2\alpha^2](u'''')^2-1/2\alpha^2e_3^2(u')^2+e_2^2(\varphi'''')^2+$$

$$+e_3^2(\varphi'')^2+2e_2e_3(\varphi''')^2-q^2\alpha^2(\varphi'')^2-\sigma^2/2\lambda[(u'')^2+(\varphi'')^2]\}dx \geqslant 0,$$

where α^2 is to be determined. Taking into account the extremal property of minimal eigenvalue of boundary problem (23) we obtain the following inequality

$$\int_0^1 \{[(e_1^2-(e_1+e_2)^2/2\alpha^2)\pi^4-e_3^2/2\alpha^2-\sigma^2/2\lambda](u'')^2+$$

$$+[(e_2\pi^2+e_3)^2-q^2\alpha^2-\sigma^2/2\lambda](\varphi'')^2\}dx \geqslant 0. \tag{25}$$

Setting the first coefficient of integrand equal to zero and solving for the coefficient α^2 we find

$$\alpha^2 = [(e_1+e_2)^2\pi^4+e_3^2/[2(e_1^2\pi^4-\sigma^2/2\lambda)].$$

Substituting α^2 into inequality (25) we obtain the sufficient condition for the uniform stochastic stability with respect to measure $\|\cdot\| = V^{1/2}$

$$q < q_{cr}\sqrt{\left[1-\frac{\sigma^2}{2\lambda(e_2\pi^2+e_3)^2}\right]\left[1-\frac{\sigma^2}{2\lambda\pi^4e_1^2}\right]}, \tag{26}$$

where

$$q_{cr} = \sqrt{\frac{2\pi^4 e_1^2(e_2\pi^2+e_3)}{(e_1+e_2)^2\pi^4+e_3^2}}.$$

The condition (26) generates a stability region shown in Fig. 3.

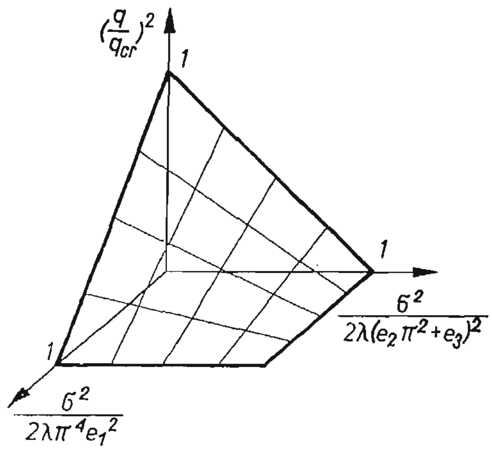

Fig. 3.

8. Conclusions

The applicability of the Liapunov method has been extended to linear Voigt-Kelvin continuous systems subjected to timedependent deterministic as well as stochastic parametric excitations. Two different dynamical models have been used, the first when the excitations are deterministic processes or stochastic nonwhite processes, the second one is applicable to describing the Gaussian white excitations. The class of Liapunov functionals useful for analysing both asymptotic stability and uniform stochastic stability has been proposed. Obtaining asymptotic stability and almost sure asymptotic stability criteria for the first model has been reduced to solving an auxiliary variational problem. The explicit stability criteria for stability of an Euler beam compressed by a periodic or stochastic force and a thin-walled double-tee beam bending by two broad -band Gaussian couples have been obtained as an application of the derived theory.

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Р е з ю м е

ДИНАМИЧЕСКАЯ УСТОЙЧИВОСТЬ ВЯЗКОУПРУГИХ СИСТЕМ ПОД ДЕЙСТВИЕМ НАГРУЗКИ ЗАВИСИМОЙ ОТ ВРЕМЕНИ

В работе представлена возможность применения прямого метода Ляпунова к исследованию устойчивости линейных непрерывных вязкоупругих систем при воздействии зависимого от времени детерминистического и стохастического параметрического возбуждения. Введен общий класс функционалов Ляпунова удобный в анализе устойчивости разнообразных непрерывных систем. Эффективно получены достаточные условия асимптотической устойчивости, асимптотической устойчивости с вероятностю 1 и равномерной стохастической устойчивости невыпученных форм систем из материала Фойгта-Кельвина. В виде примеров исследовано динамическую устойчивость стержня Эйлера сжатого периодической или стохастической силой и задачу динамической устойчивости плоской формы изгиба тонкостенного стержня при воздействии широкополосных нормальных моментов.

Streszczenie

S t r e s z c z e n i e

DYNAMICZNA STATECZNOŚĆ LEPKOSPRĘŻYSTYCH UKŁADÓW PODDANYCH DZIAŁANIU ZALEŻNEGO OD CZASU OBCIĄŻENIA

W pracy pokazano możliwość zastosowania bezpośredniej metody Lapunowa do badania stateczności liniowych lepkosprężystych układów ciągłych poddanych działaniu zależnego od czasu deterministycznego lub stochastycznego wymuszenia parametrycznego. Wprowadzono klasę funkcjonałów Lapunowa wygodnych w analizie stateczności różnych układów ciągłych. Efektywnie otrzymano dostateczne warunki asymptotycznej stateczności, prawie pewnej asymptotycznej stateczności i jednostajnej stateczności stochastycznej nieodkształconych postaci (rozwiązań trywialnych) układów Voigta-Kelvina. Jako przykłady zbadano dynamiczną stateczność pręta Eulera ściskanego okresową lub stochastyczną siłą oraz dynamiczną stateczność płaskiej postaci zginania cienkościennego pręta pod działaniem szerokopasmowych normalnych momentów.

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