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RECENT DEVELOPMENTS IN SHELL STABILITY ANALYSIS

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The present shell research activities at the Aerospace Engineering Department of the TH Delft are directed towards the development of an improved shell design criteria, which incorporates the latest theoretical findings and makes efficient use of the currently available computational facilities.

The establishment of an International Imperfection Data Bank is discussed. Characteristic initial imperfection distributions associated with different fabrication techniques are shown. It is demonstrated that the generation of reliability fuctions via the Monte Carlo Method, which displays the degrading effect of the expected initial imperfection distribution characteristic of a given fabrication process on the buckling load, offers the means of combining the Lower Bound Design Method with the notion of Goodness Classes. Thus shells manufactured by a process, which produces inherently a less damaging initial imperfection distribution, will not be penalized because of the low experimental results obtained with shells made by another process which produces a more damaging characteristic initial imperfection distribution.

1. Introduction

For buckling sensitive applications a typical shell design procedure, as recommended by any of the currently available shell design manuals [1], [2] consists of the following steps:

1. Lay-out the preliminary dimensions.
2. Select a wall construction and a stiffening concept.
3. Use one of the many shell-of-revolution codes to calculate the buckling load of the "perfect" structure taking into account the appropriate boundary conditions and the effect of prebuckling deformations.
4. Select a "knockdown" factor to account for the "imperfections" present in the finished product.
5. Apply the appropriate safety factor.

In the form of a formula one can write:

$$P_a \leqslant \frac{\gamma}{F.S.} P_c \tag{1}$$

where:

P_a = allowable load,

P_c = buckling load of the "perfect" structure,

γ = "knockdown" factor,

$F.S.$ = factor of safety.

The empirical "knockdown" factor γ is so chosen that when it is multiplied with P_cl the buckling load of the perfect structure, a lower bound to all available experimenta, data is obtained. For isotropic shells under axial compression this approach yields the lower-bound curve shown in Figure 1.

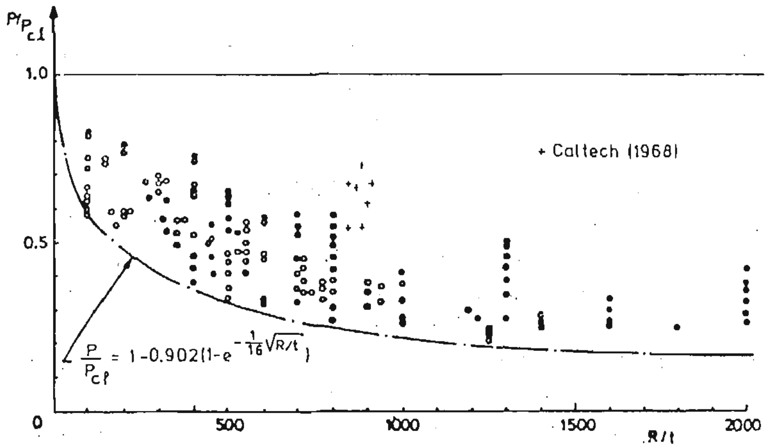

Fig. 1. Test data for isotropic cylinders under axial compression [3].

In principle, the use of empirical "knockdown" factors to account for the damaging effect of as yet unknown causes is an accepted engineering solution to a pressing problem. However, the question that immediately comes to one's mind is, where has scientific community failed? How comes that today, after so many years of concentrated research effort one cannot do any better and this despite the enormously increased computational facilities provided by todays high powered computers?

It is true that for many cases, especially in applications where the total weight of the structure is of no major concern, the Lower Bound Design Method provides safe and reliable buckling load prediction. However, it penalizes innovative shell design because of the poor experimental results obtained with shells produced and tested under completely different circumstances.

Thus the purpose of the Shell Stability Research currently being carried out at the Aerospace Enginnering Department of the Delft University of Technology is to derive an Improved Shell Design Procedure, which will incorporate the latest theoretical findings, especially that of the Imperfection Sensitivity Theory [4], and makes full use of the available computational facilities.

2. The imperfection sensitivity theory

Much effort has been spent in the past 30 years in trying to find the cause (or the causes) for the wide experimental scatter and for the poor correlation between the predictions based on a linearized small deflection theory with SS-3 ($N_x = v = w = M_x = 0$) boundary conditions and the available experimental results for axially compressed cylindrical shells. The concesus reached is that the experimental buckling loads are mainly affected by 3 factors, namely:

1. Initial geometric imperfections,
2. Boundary conditions,
3. Inelastic effects.

It has been shown [5] that for thin shells $\left(\frac{R}{t} > 300, \text{ say}\right)$ the inelastic effects may be neglected. Moreover, though different combinations of in-plane boundary conditions may affect the buckling loads considerably, for thin shells that buckle elastically initial imperfections have been accepted as the main cause of the wide scatter of experimental results (see also Fig. 1).

Thus a designer that wants to do better than the Lower Bound Design Method is faced by the following 3 questions:

1. Is the projected structure imperfection sensitive?
2. What are the shape and the amplitudes of the expected imperfections?
3. How does one calculate the buckling load of the imperfect structure?

Since it is well known that the largest portion of the knockdown factor used during the buckling load calculations of stiffened or unstiffened cylinders is due to initial geometric imperfections, therefore it makes sense to try to find shell configurations with stable post-buckling behaviour.

The imperfection sensitivity that a given structure will exhibit under certain loading conditions can be investigated very conveniently by so-called b-Factor Method, which is based on Koiter's general theory of elastic stability [4]. For those cases where the lowest buckling load is single valued and the bifurcation point is symmetric with respect to the buckling deflection, the initial post-buckling behaviour is governed by the following equation:

$$\frac{\lambda}{\lambda_c} = 1 + b\xi^2 + \dots \tag{2}$$

where: λ = applied load,
λ_c = perfect shell buckling load,
ξ = buckling displacement,
all normalized in a suitable fashion, and
b = second post-buckling coefficient.

Notice from Fig. 2 that if the post-buckling coefficient b is negative, then the equilibrium load λ falls following buckling and the buckling load λ_s of the real structure is expected to be imperfection sensitive. In this case the asymptotic relationship between the buckling load of the imperfect structure λ_s and the imperfection amplitude $\bar{\xi}$ is:

$$(1-\varrho_s)^{3/2} = \frac{3}{2}\sqrt{-3b}|\bar{\xi}|\varrho_s \tag{3}$$

where:

$$\varrho_s = \frac{\lambda_s}{\lambda_c}.$$

Here it is assumed that the shape of the initial imperfection is affine to the shape of the lowest buckling mode.

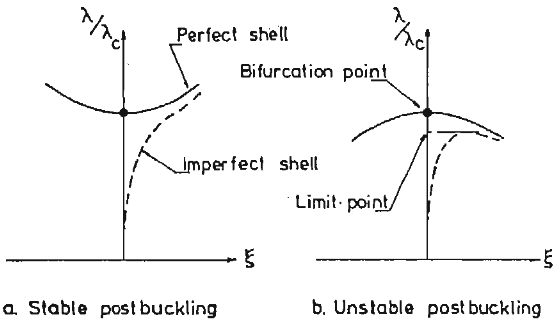

Fig. 2. Equilibrium path for perfect and imperfact shells.

That the idea of imperfection sensitivity is widely known and accepted is mainly due to the pioneering contributions of Koiter [4], [6] and the tireless efforts of the Harvard group under Budiansky and Hutchinson in the 60's [7], [8]. However, as the work of different investigators have shown, when computing the value of the b-factor for a particular configuration one must take into account the:

1. Effect of prebuckling analysis,
2. Effect of boundary conditions,
3. Apparent singularities due to the occurence of (nearly) simultanous buckling modes.

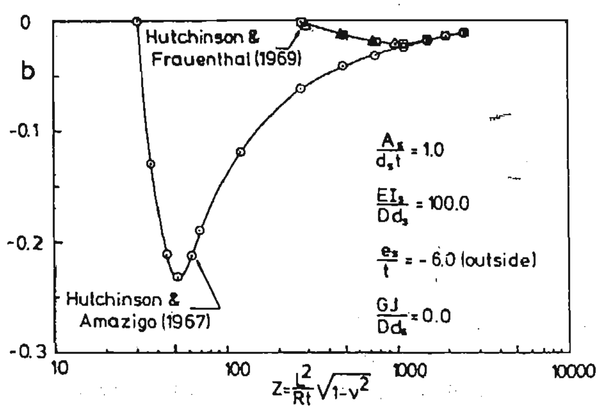

Fig. 3. Imperfection sensitivity of axially compressed stringer stiffened cylinders (HU-shells, $N_x = v = w = M_x = 0$).

Thus, for instance, considering Fig. 3 one sees that the large increase in imperfection sensitivity, obtained for Z values between 40 and 200 when one uses a membrane prebuckling analysis, disappears if the analysis is repeated using a nonlinear prebuckling analysis and when the SS-3 boundary conditions are enforced rigorously.

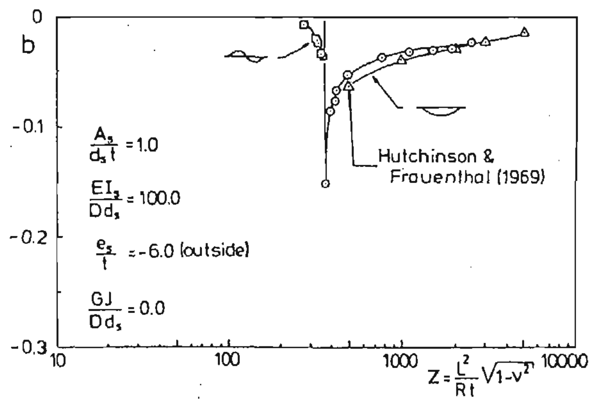

Fig. 4. Imperfection sensitivity of axially compressed stringer stiffened cylinders (HU-shells, $u = v = w = w_{,x} = 0$).

Furthermore, as can be seen from Fig. 4, the variation of the b-factor with Batdorf's Z-parameter exhibits a singular type behaviour at about $Z = 362$ if one uses nonlinear prebuckling analysis and C-4 ($u = v = w = w_{,x} = 0$) boundary conditions. The large negative peak of b, which corresponds to a large increase in imperfection sensitivity, is accompanied by a change in axaial dependence of the buckling mode from antisymmetric to symmetric. Obviously close to $Z = 362$ one has two nearly coincident buckling modes. As has been shown recently by several investigators [9], [10] in such cases the perturbation scheme used to compute the post-buckling coefficient b must be modified to account for the occurence of (nearly) simultaneous buckling modes and the resulting modal coupling effects.

The results shown in Fig. 4 have been computed by the computer code SRA [11] developed by Cohen, which has the capability of computing the first and second imperfection sensitivity factors "a" and "b" for general meridional shapes and wall constructions. The code provides for the use of different boundary conditions with either membrane, linear or nonlinear prebuckling analysis. However, the SRA program is based on the assumption that the lowest buckling load is isolated. Thus the results obtained near $Z = 362$ are uncertain.

Work is currently in progress at the TH-Delft to include the possibility of the occurence of (nearly) simultaneous buckling modes in an SRA-like code.

3. Characteristic imperfection distributions

Once the preliminary layout of a shell design has been completed and initial runs with an SRA-like code indicate that the buckling load of the proposed structure is sensitive to initial imperfections one has essentialy two options.

If the total weight of the structure and material costs are of no major concern one can employ the buckling formulas from the current design manuals, read the appropriate empirical "knockdown" factor γ from the charts to account for the effect of the unknown imperfections and use a factor of safety ($F.S. = 1.5 - 2.0$, say) to cover uncertainties in loading and other unforeseen damaging factors. This approach is the so-called Lower Bound Design Method.

If, however, the total cost and especially the total weight are of critical importance, then a more sophisticated design approach is called for. That is the designer must estimate how much the expected imperfections will decrease the buckling load of the chosen configuration. It is obvious that the main difficulty in using the Imperfection Sensitivity Theory in practical design problems dealing with weight sensitive applications is related to the fact, that it requires some advanced knowledge of the geometric imperfections that will be present once the structure under consideration has been built, an information that is rarely available.

For a prototype the imperfections can be measured experimentally and then they can be incorporated into the theoretical analysis to predict the buckling load accurately. This approach, however, is impractical for predicting the buckling loads of shells produced in normal production runs. The best one can hope to do for these shells is to establish the characteristic initial imperfection distribution, which a given fabrication process is likely to produce, and then to combine this information with some kind of statistical analysis of noth imperfections and the corresponding critical loads, a kind of Statistical Imperfection Sensitivity Analysis. The critical question thus is:

"Can we associate characteristic initial imperfection distributions with a specified manufacturing process?"

That the answer to this question is an uncoditional yes will be demonstrated by a few examples.

Figure 5 shows the measured initial imperfections of the integrally stringer stiffened aluminium shell AS-2, which has been tested at Caltech [12]. Figure 6 shows the measured initial imperfections of a similar shell KR-1 tested at Technion [13]. For further analysis the measured initial imperfections are decomposed in either a half-wave cosine

$$\overline{W}(x, y) = t\Sigma\Sigma\cos\frac{k\pi x}{L}\left(A_{kl}\cos l\frac{y}{R}+B_{kl}\sin l\frac{y}{R}\right), \tag{4}$$

or a half-wave sine Fourier series

$$\overline{W}(x, y) = t\Sigma\Sigma\sin\frac{k\pi x}{L}\left(C_{kl}\cos l\frac{y}{R}+D_{kl}\sin l\frac{y}{R}\right). \tag{5}$$

For the case of comparison Figures 7 and 8 display the variation of the measured half-wave sine Fourier coefficients as a function of the circumferential wave numbers (l) for selected axial half-wave numbers (k) for the shells AS-2 and KR-1. As one can see in both cases

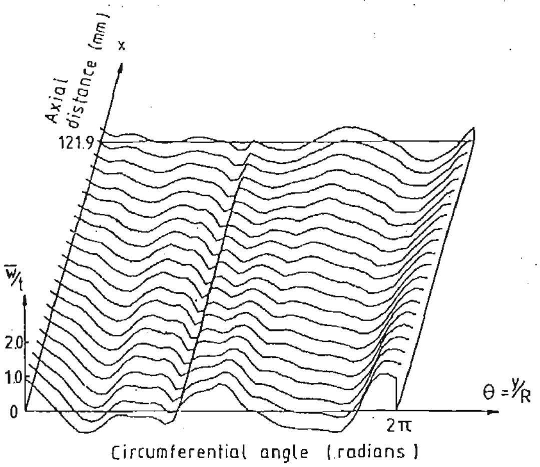

Fig. 5. Measured initial shape of the stringer-stiffened shell AS-2 [12].

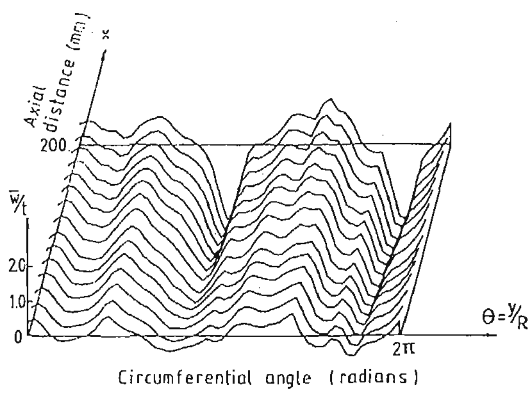

Fig. 6. Measured initial shape of the stringer-stiffened shell KR-1 [13].

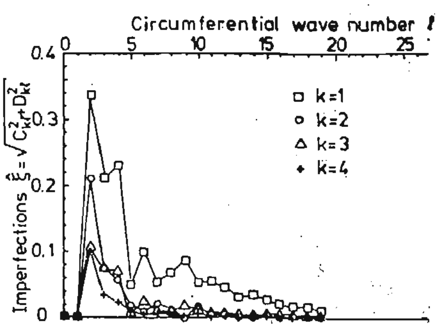

Fig. 7. Circumferential variation of the half-wave sine Fourier representation (Shell AS-2).

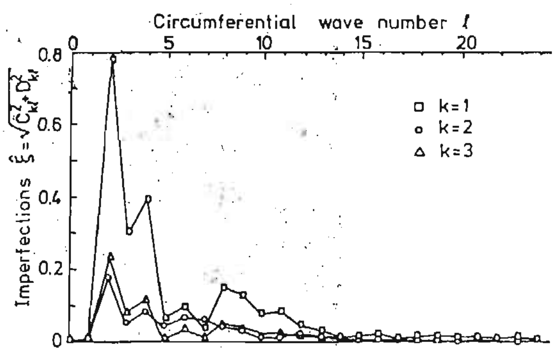

Fig. 8. Circumferential variation of the half-wave sine Fourier representation (Shell KR-1).

the amplitudes of the Fouries coefficients decay with increasing wave numbers both in the axial and in the circumferential directions. The Donnel-Imbert [14] analytical imperfection model

$$\bar{\xi}_{kl} = \sqrt{C_{kl}^2+D_{kl}^2} = \frac{\bar{X}}{k^r l^s}, \tag{6}$$

where the coefficients $\bar{X}$, r and s are determinated by least-square fitting the measured data displayed in Figures [7] and [8], represents the variation of the harmonic components with axial (k) and circumferential (l) wave numbers satisfactorily. Since both shells were machined out of seamless thick walled 7075-T6 aluminium alloy tubing, therefore the imperfection model given by Eq. (6) represents the characteristic imperfection distribution for this fabrication process.

Turning now to large scale of full scale shells, Figure 9 shows the 3-dimensional plot of measured initial imperfections of a large scale shell (945.8 mm radius, 0.635 mm wellthickness) tested at the Georgia Institute of Technology [15]. This shell was assembled

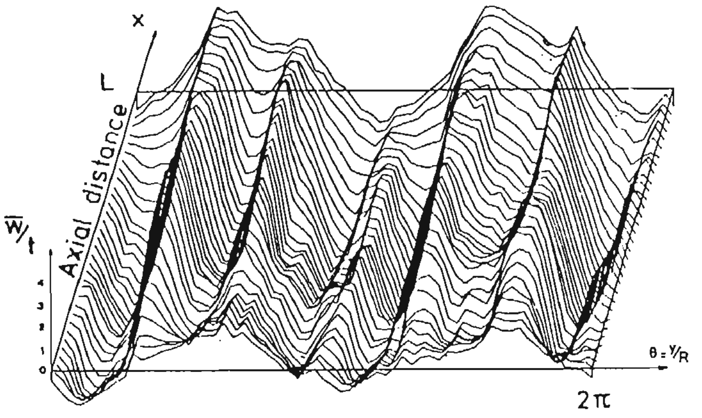

Fig. 9. Measured initial shape of Horton's shell HO-1 [15].

Fig. 10. Construction details of Horton's shell HO-1 [15].

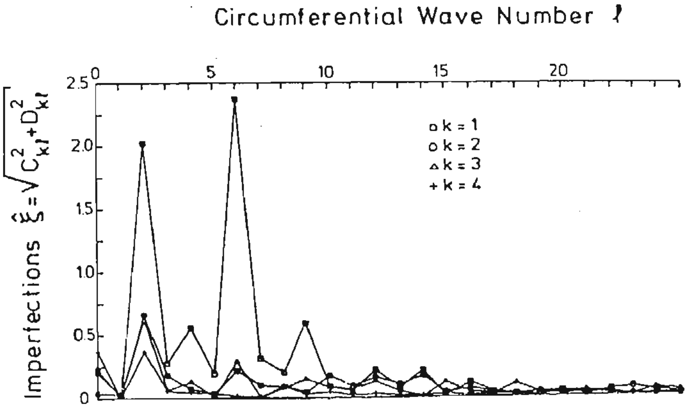

Fig. 11. Circumferential variation of the half-wave sine Fourier representation (Horton's shell HO-1).

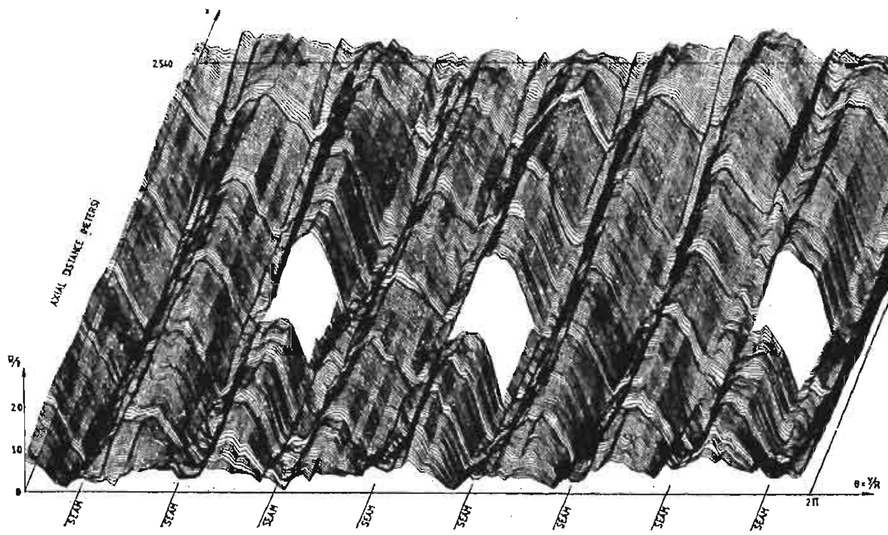

Fig. 12. Measured initial shape of the ARIANE shell AR23-1 [16].

from six identical longitudinal panels and reinforced by 312 closely spaced Z-shape stringers on the inside. One edge of each panel was joggled and two stringers were riveted along each joint line. As can be seen from Fig. 10 the shell was held circular by means of heavy rolled [-shaped external frames located 3.175 mm from each shell end. In addition 7 Z-shape equally spaced rings were riveted to the outer skin. As can be seen from Fig. 11 the amplitudes of the Fourier harmonics with a single half-wave in the axial direction have two distinct maxima, one at $l = 2$ (out of roundness) and another at $l = 6$ (number of panels the shell is assembled from). The Fourier coefficients with more than a single half-wave in the axial direction are in comparison much smaller.

In the last few years the Solid Mechanics Group of the Aerospace Engineering Department at the Delft University of Technology has carried out a number of imperfection surveys on the Ariane interstage I/II and II/III shells [16]. Figure 12 shows the 3-dimensional plot of the Ariane interstage II/III shell AR23-1 (1300.0 mm radius, 1.2 mm wall-thickness). These shells are assembled out of eight identical longitudinal panels. The joints between adjacent panels are joggled and one of the 120 equally spaced hat-shape stringers are riveted along the joint line on the outside. The shells are held by two precision-machined end-rings on the outside and five equally spaced [-shape rings on the inside. As can be seen from Fig. 13 the amplitudes of the Fourier harmonics with a single half-

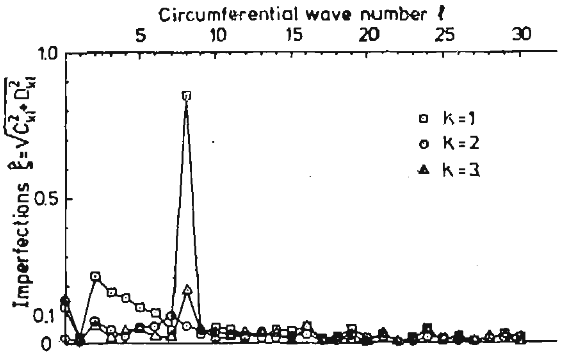

Fig. 13. Circumferential variation of the half-wave sine Fourier representation.

wave in the axial direction this time have a distinct maxima at $l = 8$ (which corresponds to the number of panels the shell is assembled from). There is also a sizeable $l = 2$ (out of roundness) component. All other Fourier coefficients are in comparison much smaller.

Thus, as has been pointed out by the same author in a recent survey lecture [17], it appears from the results presented in Figures 11 and 13 that for full-scale aerospace shells assembled out of a fixed number of curved panels the initial imperfections will be dominated by two components only, if the joints are riveted. Using the half-wave sine axial representation both components will have a single half-wave in the axial direction and, respectvely, two and N_p full-waves in the circumferential direction, where N_p is the number of full-length panels out of which the shell is assembled. By using accurately machined rigid endrings the $l = 2$ (out of roundness) component can be significantly reduced in size. The variation of the measured Fourier coefficients with axial half-wave (k) and circumferential full wave (l) numbers can be approximated by expressions of type:

$$\bar{\xi}_{kl} = \sqrt{C_{kl}^2 + D_{kl}^2} = \frac{1}{k^r}\left\{\frac{\bar{X}_1}{(l_1-l)^2+2\zeta_1 l^2} + \frac{\bar{X}_2}{(l_2-l)^2+2\zeta_2 l^2}\right\}, \tag{7}$$

where coefficients $\bar{X}_1, \bar{X}_2, r, l_1, l_2, \zeta_1$ and ζ_2 are determined by leastsquare fitting the measured data displayed in figures 11 and 13. Thus Equation (7) represents the characteristic imperfection distribution of full-scale aerospace shells assembled out of a fixed number of full-length panels by riveted joints.

The above examples demonstrate unequivocally that indeed characteristic initial imperfection distributions can be associated with the different fabrication processes. It must also be clear that further advances towards more accurate buckling load predictions of thin shells depend on the availability of extensive information about realistic imperfections and their correlation with manufacturing process. Hence the need for the establishment of an International Imperfection Data Bank.

The purpose of creating this International Imperfection Data Bank is twofold:

1. All the imperfection data obtained at different laboratories by different investigators are presented in identical format.

This makes the comparison and the critical evaluation possible resulting in characteristic imperfection distributions for the different manufacturing processes used.

2. For those who want to use the powerful nonlinear shell analysis codes on today's supercomputers the much needed realistic imperfection distributions are made available.

Besides contributions by Caltech, the TH Delft and Technion the International Imperfection Data Bank contains the results of initial imperfection surveys carried out at the University of Glasgow [18], at Det Norske Veritas [19] and others.

4. Stochastic stability analysis

Having demonstrated that indeed one can associate characteristic initial imperfection distributions with the different fabrication processes, one is faced with the next question, namely:

"Given a Characteristic Initial Imperfection Distribution, how does one proceed to incorporate this knowledge into a Systematic Design Procedure?"

Since initial imperfections are obviously random in nature some kind of Stochastic Stability Analysis is called for. The buckling of imperfection sensitive structures with small random initial imperfections has been studied by several investigators like Bolotin [20], Fraser and Budiansky [21], Amazigo [22], Roorda [23] and Hansen [24], just to name a few. In the absence of experimental evidence about the type of imperfections that occur in practice and in order to reduce the mathematical complexity of the problem all the above named investigators have worked with some form of idealized imperfection distribution.

It is not obvious to this author how these methods can be extended to the general imperfections observed in practice. Thus it was not until 1979 that a method has been proposed by Elishakoff [25] which makes it possible to introduce the results of the experimentally measured initial imperfections routinely into the analysis. The proposed approach is based on the notion of a reliability function $R(\lambda)$, where by definition:

$$R(\lambda) = Prob(\Lambda \geqslant \lambda). \tag{8}$$

Here λ is the normalized load parameter and Λ is the normalized random buckling load. As can be seen from Fig. 14 the knowledge of the reliability function permits the evaluation of the allowable load, defined as the load level λ_a for which the desired reliability is achieved, for the whole ensemble of similar shells produced by a given manufacturing process. Notice that the allowable load level λ_a is identical to the "knockdown" factor γ introduced in Eq. (1).

Basically Elishakoff has suggested to utilize the Monte Carlo Method to obtain the reliability function $R(\lambda)$ for a certain shell structure produced by a given fabrication process. The relative ease with which one can apply this procedure, once a sufficiently large sample of initial imperfection measurements is available, will be demonstrated for the case of axially compressed cylindrical shell with random axisymmetric imperfections.

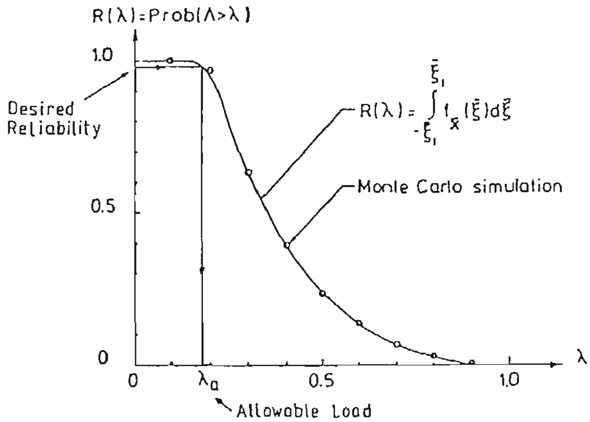

Fig. 14. Plot of the reliability function $R(\lambda)$.

Having a sample of N shells, the initial imperfections of which are given by:

$$\overline{W}(x)^{(m)} = t\Sigma A_i^{(m)} \cos i \frac{\pi x}{L}, \quad (m = 1, 2, \dots, N), \tag{9}$$

one proceeds by first calculating, by taking "ensemble averages", the estimated mean of the Fourier coefficients $A_i^{(m)}$

$$\overline{A}_i^{(e)} = \frac{1}{N} \sum_{m=1}^{N} A_i^{(m)}, \tag{10}$$

and then the estimated variance-covariance matrix

$$\sigma_{jk}^{(e)} = \frac{1}{N-1} \sum_{m=1}^{N} [A_j^{(m)} - \overline{A}_j^{(e)}] \cdot [A_k^{(m)} - \overline{A}_k^{(e)}]. \tag{11}$$

Since $\sigma_{jk}^{(e)}$ is a non-negative symmetric matrix, therefore it can be decomposed into a product of lower and upper triangular matrices (Cholesky decomposition)

$$\sigma_{jk}^{(e)} = \mathbf{C}\mathbf{C}^{\mathrm{T}}, \tag{12}$$

where $\mathbf{C}$ is a lower triangular matrix. Next the vector A of the simulated initial imperfections is obtained as:

$$A = \{A_i^{(s)}\} = \mathbf{C}\boldsymbol{r} + \overline{A}^{(e)}, \tag{13}$$

where: $\overline{A}^{(e)}$ = estimated "mean" vector

$\boldsymbol{r}$ = random vector.

The $\boldsymbol{r}$'s are normally distributed random numbers with zero mean and unit variance computed by standard random number generators available at every computing center. Taking, for example, 1000 different $\boldsymbol{r}$'s one gets 1000 diffrent $\boldsymbol{A}$'s, that is, 1000 different simulated shells with the $\boldsymbol{A}$'s as the Fourier coefficients of the initial imperfections. With these simulated shells (which are statistically equivalent to the initial experimentally measured sample) one can proceeds to carry out repeated buckling load analysis generating a so-called histogram of buckling loads. Since the reliability function $R(\lambda)$ has been defined as the probability that the random buckling load Λ will exceed the prescribed value, one then proceeds to calculated $R(\lambda)$ from the histogram of the buckling loads by the frequency interpretation (i.e. fraction of an ensemble).

As a test on the accuracy of the Monte Carlo Method one can use the results of Roorda and Hansen [26]. Assuming an axisymmetric initial imperfection of the form:

$$\overline{W}(x) = t\bar{\xi}\cos i_{cl}\frac{\pi x}{L}, \tag{14}$$

where: $i_{cl} = \frac{L}{\pi}\sqrt{\frac{2c}{Rt}}, \quad c = \sqrt{3(1-\nu^2)}$,

and $\bar{\xi}$ is a normally distributed random variable $\overline{X}$, they used Koiter's formula

$$(1-\lambda)^2 = \frac{3}{2}c|\bar{\xi}|\lambda, \tag{15}$$

as the nonlinear transfer function between the imperfection $\bar{\xi}$ and the buckling load λ. Since $\bar{\xi}$ is assumed to be a random variable therefore λ must also be a random variable yielding

$$(1-\Lambda)^2 = \frac{3}{2}c|\overline{X}|\Lambda, \tag{16}$$

or

$$\overline{X} = \frac{2(1-\Lambda)^2}{3c\Lambda}.$$

The reliability is then defined as the probability that the (random) buckling load Λ be greater or equal to some specified value λ. From the transfer function this is equivalent to the probability that the absolute value of the (random) imperfection $\overline{X}$ be less than or equal to the value given by Eq. (17.) Hence:

$$\begin{aligned} R(\lambda) &= Prob(\Lambda \geqslant \lambda) = Prob\left(|\overline{X}| \leqslant \frac{2(1-\lambda)^2}{3c\,\lambda}\right) = \\ &= Prob\left\{-\frac{2(1-\lambda)^2}{3c\,\lambda} < \overline{X} < \frac{2(1-\lambda)^2}{3c\,\lambda}\right\}. \end{aligned} \tag{18}$$

If one further assumes that the random variable $\overline{X}$ is normally distributed then its probability density is given by:

$$f_{\overline{X}}(\bar{\xi}) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{(\bar{\xi}-a)^2}{\sigma^2}\right\}, \tag{19}$$

where: a = mean value of $\overline{X}$,

σ = standard deviation of $\overline{X}$, and one can write

$$Prob(\Lambda \geqslant \lambda) = Prob(|\bar{X}| \leqslant \bar{\xi}) = \int_{-\bar{\xi}}^{\bar{\xi}} f_{\bar{X}}(\xi)\, d\xi . \tag{20}$$

For a $\equiv$ 0 one obtains Roorda and Hansen's result

$$R(\lambda) = Prob(\Lambda \geqslant \lambda) = \operatorname{erf}\left\{\frac{1}{\sqrt{2}}\,\frac{2(1-\lambda)^2}{3c\,\lambda\sigma}\right\}. \tag{21}$$

This expression has been ploted in Fig. 14 as the solid curve. The accuracy of the Monte Carlo Method can be seen from the close coincidence of the dots, representing the results obtained via the Monte Carlo Method, with the analytical solution. Further, it is evident from the initial imperfection surveys published sofar (see Figures 5, 6, 9 and

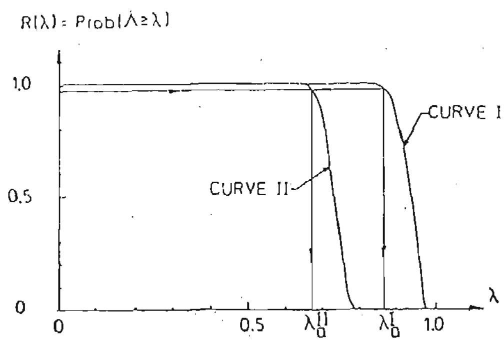

Fig. 15. Simulated reliability functions [29].

12) that in a realistic Stochastic Stability Analysis one must include both axisymmetric and asymmetric imperfections. Using the so-called Multi-Mode Analysis [27] Elishakoff and Arbocz [28] have demonstrated the feasibility of using the Monte Carlo Method to derive reliability functions for very general initial imperfections. As can be seen from Fig. 15 (here reproduced from Ref. [29]) the inclusion of asymmetric imperfection components results in a lower allowable load level λ_a for a given reliability (see curve II) than for the case of axisymmetric imperfections only (see curve I).

5. The improved shell design procedure

The improvements in the currently recommended shell design procedures are primarily sought in a more selective approach by the definition of the "knockdown" factor γ. Thus, for instance, if a company takes great care in producing its shells very accurately and if it can show experimentally that the boundary conditions are defined in such a way that no additional imperfections (especially at the shell edges) are introduced, then the use of an improved (higher) "knockdown" factor λ_a derived by a stochastic approach should be allowed. The proposed new Improved Shell Design Procedure can be represented by the following formula:

$$P_a \leqslant \frac{\lambda_a}{F.S.} P_c, \tag{22}$$

where: P_a = allowable buckling load,
P_c = buckling load of the "perfect" structure,
λ_a = reliability based improved (higher) "knockdown" factor,
$F.S.$ = factor of safety.

The steps involved in the definition of such a reliability based improved (higher) "knockdown" factor λ_a can be summarized for the group of 7 copper electroplated shells tested at Caltech [30] as follows:

1. Compute the Fourier coefficients of the initial imperfection surveys of a relatively small sample (say 7) nominally identical shells.
2. Calculate the mean vector and the variance-covariance matrix of the Fourier coefficients of the experimental sample.
3. Use Elishakoff's Method [31] to simulate a large sample of statistically equivalent imperfect shells.
4. Calculate the buckling loads of each of the simulated imperfect shells by one of the available deterministic methods [27], [32].
5. Determine the histogram of buckling loads from the results of step 4.
6. Compute the reliability function $R(\lambda)$ from the histogram of buckling loads via the frequency interpretation (i.e. fraction of an ensemble).
7. Determine the improved (higher) "knockdown" factor λ_a for a given reliability from the plot $R(\lambda)$ VS λ.

If the R/t values of the shells in the small experimental sample vary only slightly (see Caltech shells on Fig. 16) then it is sufficient to derive just a single reliability function $R(\lambda)$ for a group of shells produced by the same fabrication process. One uses then the

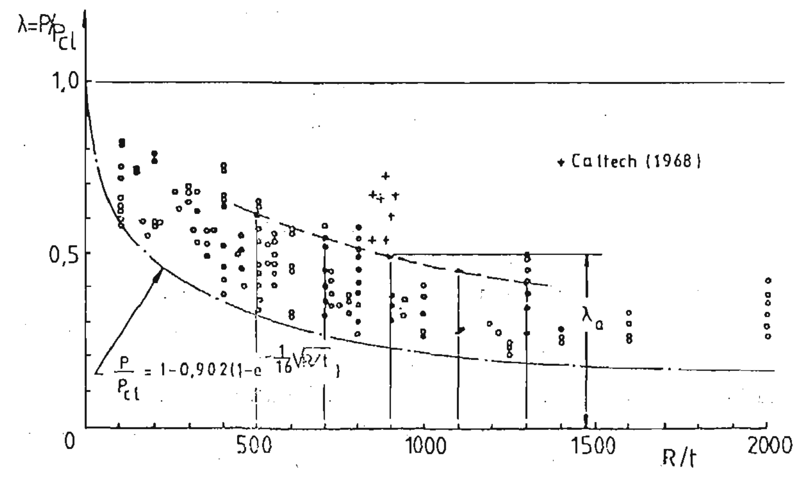

Fig. 16. Definition of the Improved Lower Bound Design Curve.

mean values for the geometric parameters involved like radius R, wall-thickness t, length L, Young's modulus E and Poisson's ratio ν. However, if the geometric parameters of the shells in question vary widely then it is necessary to calculate several reliability functions for a given fabrication process. Notice that one would have to derive at least (say) 4 reliability functions $R(\lambda)$ at different R/t values in order to get a reasonably well defined lower bound curve valid for (say) $300 < R/t < 1500$ (see Fig. 16).

To establish the accuracy of the new improved (higher) lower bound curve one must investigate the influence of the following factors

1. Confidence limits of the estimated statistical quantities;
2. Size of the experimental sample used;
3. Accuracy of the deterministic buckling analysis used;
4. Confidence limit of the Monte Carlo Method itself.

At the present time the effects of all these factors are being investigated at the Solid Mechanics Group of the Aerospace Engineering Department of the Delft University of Technology.

6. Conclusions

It has been shown that with the use of the reliability based "knockdown" factor λ_a it is possible to arrive at an Improved Shell Design Procedure, which for weight sensitive applications can result in large cost-savings.

Using the Monte Carlo Method to derive reliability functions one is combining the Lower Bound Design Philosophy with the notion of Googness Classes. Thus shells manufactured by a process, which produces inherently a less damaging initial imperfection distribution, will not be penalized because of low experimental results obtained with shells produced by another process, which generates a more damaging characteristic initial imperfection distribution.

For a successful implementation of the proposed Improved Shell Design Procedure the companies involved must be prepared to do the initial investment in carrying out complete imperfection surveys on a (small) sample of shells that are representative of their production-line. With the modern measuring and data systems one can carry out a complete surface map of very large shells at a negligible small fraction of their production cost. What is more expensive is the data reduction and the analysis that must be carried out in order to get the reliability functions.

The Solid Mechanics Group of the Aerospace Engineering Department of the Delft University of Technology is prepared to set up cooperative programs with interested companies in order to advice them how they can carry out the necessary imperfection surveys in an optimal manner, and to perform the necessary data reduction and the analysis involved in getting the reliability functions at minimal costs.

It is the author's opinion that, as the amount of data on characteristic initial imperfection distributions classified according to fabrication processes increases, we shall succeed with the help of the increased computational speed of the next generation of computers to make the Improved Shell Design Procedure available to more and more shell designers.

This, hopefully, will result in the desired dissemination of the wast amount of theoretical knowledge accumulated over the past 75 years about shell buckling behaviour. Thus, finally, the academic world will be able to point to the successful solution of one of the most perplexing problems in Mechanics.

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Резюме

ПОСЛЕДНИЕ ДОСТИЖЕНИЯ В ТЕОРИИ УСТОЙЧИВОСТИ ОБОЛОЧЕК

В работе представлено направления современной теории оболочек развиваемые в Институте Авиации политехнического института в Дельфт (Голландия). Сформулированы новые, улучшённые методы проектирования, которые учитывают последние результаты теоретических работ и современную электронно вычислительную технику.

Streszczenie

WSPÓŁCZESNE KIERUNKI ROZWOJU BADAŃ STATECZNOŚCI POWŁOK

W pracy omówiono kierunki rozwoju teorii powłok w ramach badań prowadzonych obecnie w Instytucie Inżynierii Lotniczej w Delft. Dotyczą one formułowania nowych ulepszonych kryteriów projektowania. Kryteria te uwzględniają najnowsze wyniki prac teoretycznych przy wykorzystaniu współczesnej techniki obliczeniowej.

Praca wpłynęła do Redakcji dnia 14 stycznia 1987 roku.