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MODELING AND ANALYSIS OF A TWIN-TUBE HYDRAULIC SHOCK ABSORBER

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In this paper, a physical and mathematical model was created for a twin-tube hydraulic shock absorber, using oil as the working medium. To analyze the model, methods of numerical integration were incorporated. The effect of the amplitude and frequency of the excitation, as well as the parameters describing the flow rate of oil through the valves, were examined. The basic characteristics of the damping force were obtained.

Key words: hydraulic damper, shock absorber, damping force, modelling, vehicle suspension

1. Introduction

Due to irregularities in the surface of the road, a moving car is often subjected to vertical, longitudinal and transversal displacements, which in turn greatly reduce the comfort of passengers. Additionally, periodical detachment of the wheels of the vehicle from the road surface lowers the effectiveness of both transmission of power and braking systems. In such a case, the manoeuvrability of the car, as well as the safety of the ride, is also worsened. In order to lower the amplitude of vibration, all sorts of shock absorbers are used. The so-called "hard" shock absorbers, provide better traction and stability in curves during turning, while the "soft" ones allow for the comfort of the ride to be improved. The hydraulic dampers that are used nowadays have characteristics which are generally unsymmetrical and nonlinear (Alonso and Comas, 2006; Cui *et al.*, 2010; Dixon, 2001; Dzierżek *et al.*, 2008).

The choice of these characteristics depends generally on the type of the vehicle, as well as the requirements given by the manufacturers (which differ for standard cars, SUVs, sports cars, motorcycles etc.) (Audenino and Belingardi, 1995; Dixon, 2001). Optimization of the characteristics is the result of both experimental (Cui *et al.*, 2010; Dzierżek *et al.*, 2008; Gardulski, 2009) and theoretical analysis. Theoretical research is typically based on the analysis of the quarter- or half-car model (Lee and Singh, 2008; Prabakar *et al.*, 2009). In order to evaluate the performance of the shock absorber used in the suspension system, it is best to consider the parameter that corresponds to vibration of the upper car body, as well as the wheel-ground traction (e.g. EUSAMA index). However, the vast majority of study in this field is directed at active and semi-active dampers (Lee and Singh, 2008; Maciejewski, 2010; Prabakar *et al.*, 2009; Spencer *et al.*, 1996), for which the description of characteristics is given through the use of equivalent models, such as Spencer's model (Spencer *et al.*, 1996). This is not only due to the fact that in the case of a passive damper, one cannot directly modify its parameters, also because of the relatively complicated mathematical description of the hydraulic damper.

In this paper, an attempt was made to create a model for a hydraulic damper that would additionally consider both the inertia and compressibility of the fluid. It was assumed that the control of the oil flow depends nonlinearly on the pressure difference. The basic characteristics of the shock absorber were found based on the simulation data, in accordance with the parameters corresponding to the structural solutions used in the dampers produced by Delphi company. For shock absorbers of this type, the characteristics obtained from both indoor and outdoor experiments were presented in a paper by Dzierżek *et al.* (2008).

The model that has the most similarity to the one shown in this paper, was presented by Lee and Moon (2006). However, in their work, the control of the oil flow depends on the displacement. Ramos *et al.* (2005) studied the influence of the thermal effect on characteristics of the model, paying less attention to the problem of flow control. An interesting model is also given by Alonso and Comas (2006), in which they analyzed to which degree the fluid cavitation influenced the characteristics of the system. A simplified model, that includes the compressibility of the fluid and its influence, is presented in the study of Ferreira *et al.* (2009). There are other works (Cui *et al.*, 2010; Liu and Zhang, 2002) with equivalent models, in which shock absorber characteristics are described using nonlinear functions of velocity and displacement. For example, Liu and Zhang (2002) considered a piecewise linear damping force-velocity curve of the absorber, while Cui *et al.* (2010) approximated the strength with the polynominal functions of speed, differing for the compression and rebound process. The choice of coefficients of these polynominals is based on experience. However, by using the last ones of the presented approaches to the problem of modeling the behavior of the hydraulic damper, it is not possible for the influence of structural parameters on the characteristics of the damper to be studied.

2. Model of the hydraulic damper

In this study, the most basic characteristics of the hydraulic damper using oil as a working medium, were determined. For this purpose, a model of a twin-tube shock absorber was created, as shown in Fig. 1. Typically, the upper mounting of the shock absorber is attached to the sprung part of the vehicle mass, while the lower one – to the unsprung parts (such as wheels, axles, bearings, brakes and some of the elements in the drive transmission system).

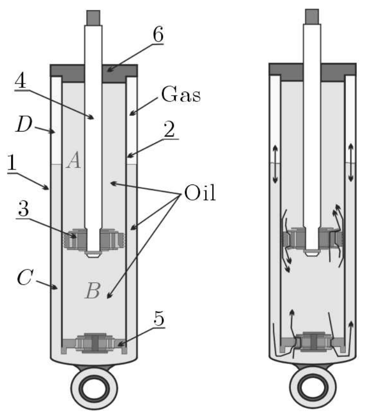

Fig. 1. Model of the hydraulic damper

The system, consists of two cylinders: external one (compensatory chamber) (1) and the internal (working) one (2). Inside, the working cylinder, a piston (3) is located, attached to the rod (4). The piston rod is equipped with the rod guide (6) that limits its movement in relation to the cylinders, to the longitudinal direction only. In the lower part of the internal cylinder, a basc valve (5) is located. Four main chambers can be distinguished: chamber above the piston "Extension Chamber" (A), chamber below the piston "Compression Chamber" (B) and the compensatory tank "Reservoir Chamber", which consists of two parts: chamber (C) located above the oil level and chamber (D) located below it. Chamber (D) is filled with nitrogen under low pressure (4-8 bar). The initial pressure of the nitrogen is defined by the parameter p_0, while its volume by the parameter V_0.

The principle behind the operation of the hydraulic shock absorber is quite simple. When the piston rod is pressed (and moves towards the bottom valve), the pressure in chamber B increases and the resistance force arises, which in turn causes the oil to flow through appropriate canals into the chamber A. The excess oil, passes through the bottom valve to the reservoir chamber, causing the pressure in chamber D to increase. In the rebound process, both piston and the rod move in the opposite direction, causing an increase of pressure in chamber A. The oil flows through the system of canals, back from the space above the piston, to the chamber below it. At the same time, oil from the compensatory tank flows through the bottom valve, in order to equalize the pressure in chamber B. The volume of the pumped fluid depends primarily on the piston displacement x, while the intensity of flow – on the piston velocity v in relation to the cylinder. The resistance force of the damper depends on the resultant force from pressure acting on the piston and from both dry and viscous friction. While trying to determine the characteristics of the hydraulic shock absorber, the analysis shall be limited to the variable of the pressure force F_p, given by the following equation

$$F_p = (p_A - p_0)A_1 - (p_B - p_0)A_2 \tag{2.1}$$

in which A_1 and A_2 define the size of the top and bottom area of the piston valve, while p_A and p_B are the values of pressure in chamber A and chamber B, respectively.

In order to determine the equations which could describe variation of oil pressure in chambers A, B and C, a relation needs to be considered: $\rho V = m$, which links the density ρ, volume V and mass m for each one of the shock absorber chambers. After its differentiation and inclusion of the relation

$$\frac{d\rho}{dp} = \frac{1}{K}\rho \tag{2.2}$$

the equations take the following form

$$\begin{aligned} &\frac{1}{K}\dot{p}_A V_A + \dot{V}_A = \frac{\dot{m}_A}{\rho_A} \qquad \frac{1}{K}\dot{p}_B V_B + \dot{V}_B = \frac{\dot{m}_B}{\rho_B} \\ &\frac{1}{K}\dot{p}_C V_C + \dot{V}_C = \frac{\dot{m}_C}{\rho_C} \end{aligned} \tag{2.3}$$

where V_A, V_B, V_C are the volumes of the chambers, ρ_A, ρ_B, ρ_C are the values of oil pressure inside them, while K stands for the compressibility modulus.

The changes in mass $\dot{m}_A$, $\dot{m}_B$, $\dot{m}_C$ are described by using of the mass flow equations

$$\begin{aligned} &\dot{m}_A = \dot{m}_{BA} - \dot{m}_{AB} \qquad \dot{m}_B = \dot{m}_{AB} - \dot{m}_{BA} + \dot{m}_{CB} - \dot{m}_{BC} \\ &\dot{m}_C = \dot{m}_{BC} - \dot{m}_{CB} \end{aligned} \tag{2.4}$$

The lower indices of the flow rates are associated with the chamber names between which the flow of oil occurs. For example, the mass flow rate $\dot{m}_{AB}$ determines the flow of oil from chamber A to B, has a non-zero value only if $p_A > p_B$ and differs from the mass flow rate $\dot{m}_{BA}$, mostly due to the different cross-sectional area size of the flow canal $(A_{BA} > A_{AB})$. These different area sizes are introduced in the construction of the valves in order to provide a higher resistance force during rebound (for $v > 0$), in comparison to the force that emerges during the compression process (for $v < 0$).

By introducing Heaviside's function, the discussed flow rates can be substituted by the following equations

$$\begin{aligned} \dot{m}_{AB} &= \rho_A \beta A_{AB} H(p_A - p_B) \sqrt{\frac{2(p_A - p_B)}{\rho_A}} \\ \dot{m}_{BA} &= \rho_B \beta A_{BA} H(p_B - p_A) \sqrt{\frac{2(p_B - p_A)}{\rho_B}} \end{aligned} \tag{2.5}$$

where β is the flow rate coefficient. The mass flow rates between chambers B and C can be described in a similar manner

$$\begin{aligned} \dot{m}_{BC} &= \rho_B \beta A_{BC} H(p_B - p_C) \sqrt{\frac{2(p_B - p_C)}{\rho_B}} \\ \dot{m}_{CB} &= \rho_C \beta A_{CB} H(p_C - p_B) \sqrt{\frac{2(p_C - p_B)}{\rho_C}} \end{aligned} \tag{2.6}$$

The integration of general equation (2.2), allows for the relations between the density and pressure of oil in each damper chamber, to be written

$$\begin{aligned} &\rho_A = \rho_0 \exp\Big(\frac{p_A - p_a}{K}\Big) \qquad \rho_B = \rho_0 \exp\Big(\frac{p_B - p_a}{K}\Big) \\ &\rho_C = \rho_0 \exp\Big(\frac{p_C - p_a}{K}\Big) \end{aligned} \tag{2.7}$$

where ρ_0 is the density of oil under the influence of atmospheric pressure p_a.

The characteristics of the damper depend to a significant degree on the parameters A_{BA}, A_{AB}, A_{BC}, A_{CB}, which influence the rate of mass flow (2.5), (2.6). The construction of valves shows that these parameters are actually functions, as they can be modified during the operation time. Since the inlets of the canals are covered by the set of discs, which are susceptible to deformation from respective pressure forces, the highest impact is from the difference of oil pressure between the adjacent chambers. Additionally, some of the inlets are uncovered only after the critical value of the pressure force is reached. In order for the functions A_{BA}, A_{AB}, A_{BC}, A_{CB}, to be properly described, the exact geometry of the valve, as well as physical relationships between the force acting on the disc and its deformations, are required (Czop *et al.*, 2009). The problem is simplified by assuming that the analyzed functions can be described using one constant and two variable components. The constant values represent permanently open canals. To describe the variable components, a new function $\vartheta(\Delta p, p_k)$ is introduced, that is defined by the pressure difference Δp and constant pressure p_k, referred to as "critical pressure". The functions A_{BA}, A_{AB}, A_{BC}, A_{CB} can be described by the equations of a similar structure

$$\begin{aligned} A_{AB} &= A_{AB}^{const} + A_{AB}^{max}[\delta_1\vartheta(p_A - p_B, 0) + \delta_2\vartheta(p_A - p_B, p_k)] \\ A_{BA} &= A_{BA}^{const} + A_{BA}^{max}[\delta_1\vartheta(p_B - p_A, 0) + \delta_2\vartheta(p_B - p_A, p_k)] \\ A_{BC} &= A_{BC}^{const} + A_{BC}^{max}[\delta_1\vartheta(p_B - p_C, 0) + \delta_2\vartheta(p_B - p_C, p_k)] \\ A_{CB} &= A_{CB}^{const} + A_{CB}^{max}[\delta_1\vartheta(p_C - p_B, 0) + \delta_2\vartheta(p_C - p_B, p_k)] \end{aligned} \tag{2.8}$$

in which the parameters labeled by the indices *const* and *max* are respectively the size of the cross-section area of the permanently open canals (including leakage) and the maximal size of the cross-section area of the canals with variable diameter.

The parameters δ_1 and δ_2 define the percentage of area size of the inlets, opened in a continuous manner from $\Delta p = 0$ and from $\Delta p = p_k$ (it is assumed that $\delta_1 + \delta_2 = 1$). The function $\vartheta(\Delta p, p_k)$ is approximated using arctan' function in a following way

$$\vartheta(\Delta p, p_k) = \frac{2}{\pi}\arctan[\mu(\Delta p - p_k)]H(\Delta p - p_k) \tag{2.9}$$

The above definition shows that for the pressure difference Δp exceeding the critical pressure p_k, function (2.9) rises in a way similar to the exponential from the value of zero to one. If the parameter μ, that defines the susceptibility of a disc covering a particular canal, is high enough, function (2.9) begins to resemble a unit step function.

In order to obtain the final form of the differential equations describing the pressure changes, values in equations (2.3) need to be substituted by the following relations

$$V_A = A_1(l - x) \qquad\qquad V_B = A_2(l + x) \tag{2.10}$$

An assumption is made here that complete length of the internal cylinder equals $L_w = 2l + h$, where h is the width of the piston valve, and that the displacement of the piston is given by the harmonic function $x = a\sin\omega t$. To determine the volume of oil in the external cylinder of the shock absorber, a constant volume relation of the reservoir chamber can be utilized $V_C + V_D = \mathrm{const}$, from which

$$\dot{V}_C = -\dot{V}_D \tag{2.11}$$

By assuming that the volume of gas V_D in the reservoir chamber satisfies the equation of polytropic processes

$$p_D V_D^n = p_0 V_0^n = \mathrm{const} \tag{2.12}$$

where n is the coefficient of the politrope and by stating that the pressure of both oil and gas in chambers C and D is equal $p_D = p_C$, by differentiating equation (2.12), it is possible to calculate the change in volume V_D

$$\dot{V}_D = -\frac{\dot{p}_D V_D}{n p_D} \tag{2.13}$$

After the additional use of equations (2.11)-(2.13)

$$\dot{V}_C = \frac{\dot{p}_D V_0 p_0^{\frac{1}{n}}}{n p_D^{1+\frac{1}{n}}} \tag{2.14}$$

Finally, the equations that describe the change of oil pressure in each chamber, take the following form

$$\begin{aligned}
&\dot{p}_A = \frac{K}{V_A}\Big(\frac{\dot{m}_A}{\rho_A} + A_1\dot{x}\Big) \qquad\qquad \dot{p}_B = \frac{K}{V_B}\Big(\frac{\dot{m}_B}{\rho_B} - A_2\dot{x}\Big) \\
&\dot{p}_C = \frac{\dot{m}_C}{\rho_C}\frac{K n p_C^{1+\frac{1}{n}}}{V_C n p_C^{1+\frac{1}{n}} + K V_0 p_0^{\frac{1}{n}}}
\end{aligned} \tag{2.15}$$

Equations (2.15) together with relations (2.4)-(2.10) and expression (2.1) provide the basis for determining the characteristics of the hydraulic damper resistance force.

3. Calculation results

The effect of the amplitude and frequency of the excitation, as well as selected parameters characterizing the flow of oil through valves, on the basic characteristics of the system is presented below. Based on the analysis of the specific solution for a damper manufactured by Delphi (Dixon, 2001)obtained using numerical computation, the following values of parameters were used: length $l = 0.195\,\mathrm{m}$, area size [cm^2]: $A_1 = 5.03$, $A_2 = 6.16$, $A_{BA}^{max} = 2A_{AB}^{max} = 0.204$, $A_{CB}^{max} = A_{BC}^{max} = 0.0679$, $A_{BA}^{const} = A_{AB}^{const} = \delta_0 A_{BA}^{max}$, $A_{BC}^{const} = A_{CB}^{const} = \delta_0 A_{CB}^{max}$, coefficients $\delta_0 = 0.1$, $\delta_1 = 0.4$, $\delta_2 = 0.6$, $\beta = 0.8$, volume $V_0 = 90\,\mathrm{cm}^3$, oil density $\rho_0 = 980\,\mathrm{kg/m}^3$, nominal pressure $p_0 = 6\,\mathrm{bar}$, compressibility modulus $K = 1.5\,\mathrm{GPa}$, coefficient $\mu = 1\,\mathrm{bar}^{-1}$, politrope index $n = 1.4$. The solution to equations (2.15) was found by making use of a variable-step Fehlberg integration algorithm, based on the Runge-Kutta methods of the 4th and 5th order.

Figure 2 illustrates the relation between the damping force and relative displacement for different values of amplitude (Fig. 2a for $f = 2\,\mathrm{Hz}$) and frequency (Fig. 2b for $a = 2\,\mathrm{cm}$) of the excitation. The extreme values of the force are achieved for the extreme velocities (close to the initial position of the piston) and depend nonlinearly on the value ωa. The ratio between the maximal and minimal force depends primarily on the ratio of the canal areas $A_{BA}^{(k)}/A_{AB}^{(k)}$ and $A_{BC}^{(k)}/A_{CB}^{(k)}$ $(k = 1, 2)$, responsible for the flow of oil during compression and rebound. Because in these calculations the ratio between the considered areas equals 2, the maximal forces (in the case of rebound) are around twice the minimal ones (in the case of compression).

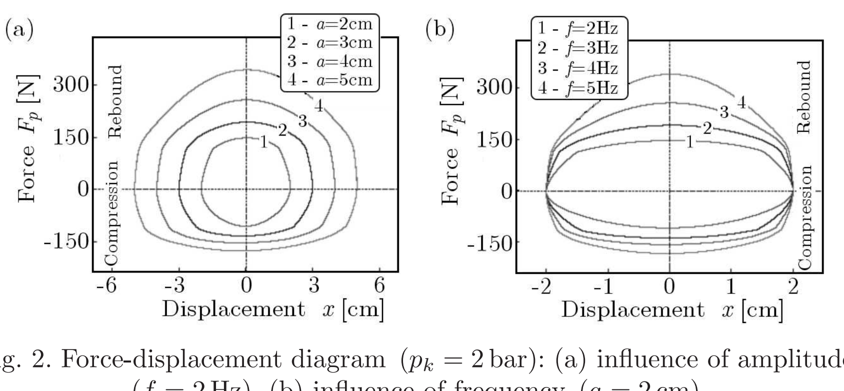

Fig. 2. Force-displacement diagram $(p_k = 2\,\mathrm{bar})$: (a) influence of amplitude $(f = 2\,\mathrm{Hz})$, (b) influence of frequency $(a = 2\,\mathrm{cm})$

With the increase in either amplitude or frequency, the characteristics of the force undergo certain changes – the effect of strong increase in oil pressure in chamber A, with a sudden drop of the pressure in chamber B (Fig. 3) causes the maximal forces to be increased.

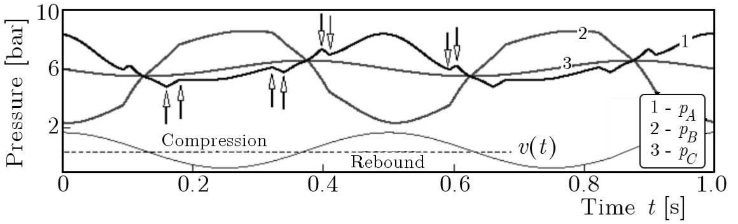

Fig. 3. Time histories of pressures p_A, p_B and p_C ($a = 5\,\mathrm{cm}$, $f = 2\,\mathrm{Hz}$, $p_k = 2\,\mathrm{bar}$)

Figure 3 shows time characteristics of the pressure in each shock absorber chamber, as well as velocities used to separate the process of compression and rebound. The graph of pressure p_C in the reservoir chamber is similar to a harmonic function, and is subjected to relatively small changes. More interesting are the graphs showing the pressure in chambers A and B. By studying these graphs, and especially the one showing pressure p_A, some characteristic points can be spotted (marked by arrows), in which the difference between the appropriate pressures equals the same as the critical pressure. These points correpond to the moments in which the additional holes in valves are opened and closed. During compression, for $v(t) < 0$, the canals are opened at first in the top valve, and then in the bottom one in order to prevent the oil from becoming too compressed in chamber B. The closing of the canals is performed in the inverse order. The similar situation takes place during the rebound process (for $v(t) > 0$). The order of opening and closing the canals is the same, but in this case the oil flows into chamber B.

Most useful information concerning the dynamical properties of the considered dampers can be acquired from Fig. 4 which shows the relation between the force and the relative velocity of the piston. A narrow hysteresis loop can be seen in the graphs, which proves the minor effect of the inertia of oil. These characteristics are asymmetrical since the damper puts more resistance during the rebound, which is actually desirable, especially in the case of the wheel falling into a deep hole or overriding a high obstacle.

In the case of absence of additional channels, opened only when the pressure difference exceeds the given value p_k (Fig. 4a for $p_k = 0$), the characteristic of the damper is progressive throughout the velocity range. For the value $p_k \neq 0$, the characteristics behave differently if the velocity range is high or low. In the presented graphs, several points of inflection can be seen, whoose locations play significant role in proper operation of the damper. However, the-

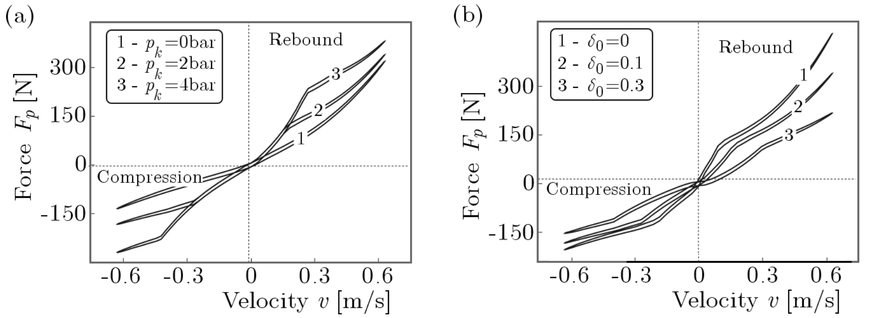

Fig. 4. Force-velocity diagram ($a = 5$cm, $f = 2\,$Hz): (a) influence of parameter p_k ($\delta_0 = 0.1$), (b) influence of parameter δ_0 ($p_k = 2\,$bar)

se locations depend mostly on the value of parameter p_k (Fig. 4a) and design parameters of the valve (especially the appropriate areas – Fig. 4b), and to a much lesser degree, on the parameters of excitation (amplitude, frequency).

The optimal characteristic of the damper depends also on the parameters of the suspension system and the inertial parameters which characterize the vehicle, in which the damper is to be mounted. Therefore, in order to determine this characteristic, it is necessary to analyze the appropriate vehicle model.

By analyzing the graph in Fig. 4b, an adverse effect of increasing the area size of the permanently open canals (including leakage) can be observed, mostly due to the decrease in difference between the maximal forces during compression and during rebound, and the fact that the characteristic is similar to degressive (e.g. for $\delta_0 = 0.3$) throughout the whole velocity range. The adverse effect of increased leakage is also confirmed by the experimental results, discussed in the paper Lee and Singh (2008).

4. Conclusions

The analysis of the modeled hydraulic shock absorber, allows for several conclusions to be drawn:

- The largest impact on the characteristics of the damping force is from the parameters that depend on the geometrical and physical properties of the design of top and bottom valves of the shock absorber.
- The most desirable (for the comfort of ride) asymmetry of the force characteristic can be obtained only through the application of a specific

design of the valve, which would provide a lower mass flow rate during rebound than during compression.

- A good convergence of the results in comparison with the results described in literature proves that the assumptions made during the modeling of the system where chosen correctly and appropriately.

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Modelowanie i analiza dwururowego tłumika hydraulicznego

Streszczenie

W pracy wprowadzono model fizyczny i matematyczny dwururowego amortyzatora hydraulicznego, w którym czynnikiem roboczym jest olej amortyzatorowy. Do jego analizy wykorzystano metody numerycznego całkowania. Zbadano wpływ amplitudy i częstości wymuszenia oraz parametrów opisujących przepływ oleju przez zawory. Wyznaczono podstawowe charakterystyki siły tłumienia.

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