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TRANSIENT RESONANCE OF MACHINES AND DEVICES IN GENERAL MOTION

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The new, more accurate and general possibility of assessments of maximum amplitudes of vibratory machines and vibroinsulated systems during their passage through the resonance was considered in the paper. The energy balance of the system was applied and the analytical solution for typical systems of several degrees of freedom during free coasting was given. The formulated method can be applied to systems of multiple degrees of freedom including continuous systems.

Key words: transient resonance, vibratory machines, vibration insulation, limited power systems

1. Problem formulation

Designing of technological lines in heavy industry branches requires an estimation of casing conditions of machines and devices in consideration of maximum amplitudes of vibrations in a steady state and during transient processes.

Dynamic loads, transferred to floors and ceilings and to supporting structures as well as conditions of cooperation in machine lines, depend on these amplitudes. This problem concerns a broad class of machines and devices having elastic suspensions such as: vibroinsulating systems, fans, compressors and vibratory machines as: vibrating screens, feeders or vibratory conveyers.

A transient resonance, occurring in vibrating systems when frequencies of excitation forces are passing via the natural frequencies range of the system, belongs to the most dangerous dynamic states of the devices. The resonance during the machine free coasting is specially hazardous since due to a longer duration of the process it leads to maximum amplitudes being 1 to 2 orders higher than the amplitudes in the steady state.

The first estimation of maximum amplitudes (Lewis, 1932) was obtained for an oscillator of one degree of freedom, at the excitation of a constant amplitude and linearly variable frequency. The solution for systems of the excitation amplitude proportional to the square of a force frequency was given in Kac (1947). These works were applied for the preparation of nomograms used nowadays in practice (Harris, 1957). An expansion of analysis into systems of some degrees of freedom by means of uncoupling the equations of motion was given in works Goliński (1979), Yanabe and Tamura (1980). Numerous further works were focused on mathematic problems related to solutions of equations of motion. One of the most important works Markert and Seidler (2001) is the study in which motion of the oscillator is forced by a linear combination of the assumed time function and its time derivative. Such description of exciting forces allows for the analysis of the transient resonance with taking into account not only the normal but also the tangent component (depending on the angular acceleration) of the force of inertia originated from an unbalanced rotor and the analysis of kinematic forcing by the base movement.

The common feature of the above cited works is the *a priori* assumption of the form of the time excitation function, without taking into consideration the influence of vibrations of the unbalanced rotor axis on its angular motion. As it was proved by the author (Michalczyk, 1995) such an approach leads to significant over-estimation of amplitudes during machine free coastings.

The reason for these errors result from the omission of the additional moment originating from the force of transportation inertia, occurring in non-inertial systems related to the vibrating rotor axis (Kononienko, 1964). The analysis of systems with taking into account the limited driving power were performed only in relation to the steady state resonance (due to difficulties in solving equations of motion) – see works of Sommerfeld, Kononienko (1964) and later e.g. Warmiński (2001).

Works of Agranowska and Blechman (1969) and Michalczyk (1993) based on the energy balance of the effect are without this fault. However, the first of these works, based on the kinetic and potential energy balance requires the knowledge of the frequency at which the maximum amplitude occurs (variable, depending on the angular acceleration in the circum-resonance zone and not known *a priori*), while the second one was formulated for vibrating systems of one degree of freedom only.

The transient resonance problem for the system of one degree of freedom, with taking into account couplings between the rotor and body motion, was undertaken by Cieplok (2009) who obtained (by means of digital analysis) nomograms for the determination of maximum amplitudes in the transient resonance. The fault of this work is the assumption of cophasal synchronous running of both drives. As it was indicated by Michalczyk and Czubak (2010) the state of the cophasal drives running is lost in the circum-resonance zone, which changes the forcing conditions assumed by Cieplok and causes the pseudo-resonance activations in other directions.

2. Energy method of the resonance amplitude estimation at the machine coasting

The basis of the proposed – in the hereby paper – method of determination of the maximum amplitudes in the transient resonance constitutes the observation (Michalczyk, 1995) that in the stage of increasing circum-resonance vibrations during coasting, the unbalanced rotor returns usually 3/4 to 8/9 of the collected kinetic energy.

Thus, in order to estimate the amplitudes of resonance vibrations it is possible – not committing any significant error – to perform the energy balance between the kinetic energy of the vibrator angular motion and the kinetic energy of the body vibrations. It is assumed that the entire energy, which the vibrator or a set of n synchronous vibrators possess at the moment of entering into the i-th resonance zone is transferred into increasing amplitudes of the machine body vibrations, which vibrates in accordance with the i-th form of its natural frequencies.

Thus, it occurs

$$n\frac{1}{2}J_{zr}\omega_{0i}^2 = \frac{1}{2}\dot{\boldsymbol{q}}_{max\,i}^{\top}\mathbf{M}\dot{\boldsymbol{q}}_{max\,i} \tag{2.1}$$

where: n is the number of identical synchronously running driving systems, J_{zr} – moment of inertia of the driving system reduced on the rotor shaft of the unbalanced vibrator, ω_{0i} – angular velocity at which the energy exchange occurs (generally different (Lewis, 1932) in a certain range from the i-th frequency of natural machine body vibrations on an elastic suspension system), $\boldsymbol{q} = \operatorname{col}\{x_s, y_s, z_s, \varphi_x, \varphi_y, \varphi_z\}$ – vector of coordinates, describing the system vibrations, $\dot{\boldsymbol{q}}_{max\,i}$ – velocity vector determined for the moment of the maximum amplitude of the i-th form in the i-th resonance

$$
\mathbf{M} = \begin{bmatrix}
m & 0 & 0 & 0 & 0 & 0 \\
 & m & 0 & 0 & 0 & 0 \\
 & & m & 0 & 0 & 0 \\
 & & & J_{xx} & -J_{xy} & -J_{xz} \\
 & \text{sym.} & & & J_{yy} & -J_{yz} \\
 & & & & & J_{zz}
\end{bmatrix} \tag{2.2}
$$

m — body mass, J_{ij} – corresponding elements of the tensor of inertia in the central system $Sxyz$, x_s, y_s, z_s – coordinates of the machine mass centre, $\varphi_x, \varphi_y, \varphi_z$ – angles of small rotation with respect to the axis x, y, z.

For harmonic vibrations of a frequency ω_{0i}, the maximum values of generalised velocities $\dot{\boldsymbol{q}}_{max\,i}$ are related to the maximum amplitudes of the displacement vector by the following dependence: $\dot{\boldsymbol{q}}_{max\,i} = \omega_{0i}\boldsymbol{q}_{max\,i}$, which leads to

$$
n\frac{1}{2}J_{zr}\omega_{0i}^2 = \frac{1}{2}\omega_{0i}^2\boldsymbol{q}_{max\,i}^\top\mathbf{M}\boldsymbol{q}_{max\,i} \tag{2.3}
$$

or after reduction

$$
nJ_{zr} = \boldsymbol{q}_{max\,i}^\top\mathbf{M}\boldsymbol{q}_{max\,i} \tag{2.4}
$$

Let vibratory motion of the machine body placed on an elastic suspension system be described by the equation of small, free and undamped vibrations

$$
\mathbf{M}\ddot{\boldsymbol{q}} + \mathbf{K}\boldsymbol{q} = \mathbf{0} \tag{2.5}
$$

where $\mathbf{K}$ is the symmetrical elasticity matrix.

For the basic – in applications - case of the machine placed on a system of $j = 1, \dots, \nu$ parallel, identical elastic elements of constants:

– in a vertical direction, k_z

– in horizontal directions, $k_x, k_y = k_{xy}$

and coordinates x_j, y_j, z_j of points where the elastic elements were mounted to the body in the static equilibrium of the machine, it is easy to prove the following

$$
\mathbf{K} = \begin{bmatrix}
nk_{xy} & 0 & 0 & 0 & k_{xy}\sum z_j & -k_{xy}\sum y_j \\
 & nk_{xy} & 0 & -k_{xy}\sum z_j & 0 & k_{xy}\sum x_j \\
 & & nk_z & k_z\sum y_j & -k_z\sum x_j & 0 \\
 & & & k_z\sum y_j^2 + k_{xy}\sum z_j^2 & -k_z\sum x_j y_j & -k_{xy}\sum x_j z_j \\
 & \text{sym.} & & & k_z\sum x_j^2 + k_{xy}\sum z_j^2 & -k_{xy}\sum y_j z_j \\
 & & & & & k_{xy}(\sum x_j^2 + \sum y_j^2)
\end{bmatrix} \tag{2.6}
$$

For the harmonic form of solutions $\boldsymbol{q} = \boldsymbol{q}_{max}[\sin(\omega t + \gamma)]$, the condition of existing of the non-zero solution to equation (2.5) leads to the dependence

$$\det[\mathbf{K} - \omega^2\mathbf{M}] = \mathbf{0} \tag{2.7}$$

This dependence allows one to determine the set of natural frequencies: ω_{0i}, $i = 1, \dots, 6$ and later on the modal vectors

$$\boldsymbol{\Psi}_i(\omega_i) = \mathrm{col}\,\{\psi_{1i}, \psi_{2i}, \psi_{3i}, \psi_{4i}, \psi_{5i}, \psi_{6i}\} \tag{2.8}$$

Let us assume for a moment that these frequencies are different and sufficiently distant (in a sense of circum-resonance vibration amplification). This allows one to write the amplitude vector for vibrations of the i-th frequency and form

$$\begin{aligned} \boldsymbol{q}_{max\,i} &= \mathrm{col}\Big\{\frac{\psi_{1i}}{\psi_{ki}}q_{max\,ki}, \frac{\psi_{2i}}{\psi_{ki}}q_{max\,ki}, q_{max\,ki}, \frac{\psi_{6i}}{\psi_{ki}}q_{max\,ki}\Big\} \\ &= q_{max\,ki}\,\mathrm{col}\Big\{\frac{\psi_{1i}}{\psi_{ki}}, \frac{\psi_{2i}}{\psi_{ki}}, 1, \frac{\psi_{6i}}{\psi_{ki}}\Big\} \end{aligned} \tag{2.9}$$

where $q_{max\,ki}$ means the maximum amplitude of arbitrarily selected for the representation of the i-th form of vibrations (on the assumption: $\psi_{ki} \neq 0$) coordinate q_k.

Denoting

$$\mathrm{col}\Big\{\frac{\psi_{1i}}{\psi_{ki}}, \frac{\psi_{2i}}{\psi_{ki}}, 1, \frac{\psi_{6i}}{\psi_{ki}}\Big\} = \boldsymbol{a}_{ki} \tag{2.10}$$

and substituting the above to (2.9) and (2.4), we finally obtain a dependence for the maximum amplitude of the k-th coordinate during the system passing via the resonance with the i-th natural frequency

$$q_{max\,ki} = \sqrt{\frac{nJ_{zr}}{\boldsymbol{a}_{ki}^\top \mathbf{M}\boldsymbol{a}_{ki}}} \tag{2.11}$$

This dependence constitutes an over-estimated assessment (in typical cases, quite accurate) of the maximum amplitude of the selected k-th coordinate in the i-th resonance, ou the condition that the vibrator exciting force operates at vibrations of the investigated form. This is equivalent to the demand that the relevant modal force is not zero. Not meeting this condition means passing through the resonance zone without exciting significant machine vibrations.

When equal multiple frequencies occur in the spectrum of natural frequencies of the system, the above given proceedings are usually not possible in relation to these frequencies due to ambiguity of the vibration form. The symmetrical systems for which we know the vibration forms, e.g. from the physical analysis, constitute an exception.

3. System with a vertical main inertial axis, $h \neq 0$

For better clarity of considerations, let us assume the case of a system with a vertical main axis of inertial (often occurring in practice), e.g. a two-drive vertical vibratory conveyer shown in Fig. 1.

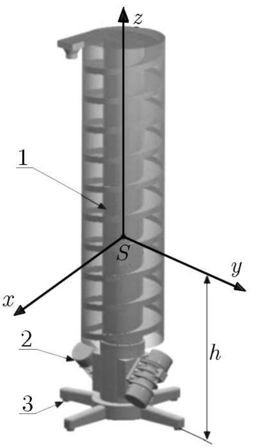

Fig. 1. Vertical vibratory conveyer (OFAMA vibratory conveyer PWS); 1 – machine body, 2 – vibrator, 3 – elastic support

The machine body motion will be described in the system $Sxyz$ of the vertical axis z, coinciding with the main axes of inertia of the machine in the static equilibrium under dead weight.

Additionally, we will assume that in the elastic supporting system (in accordance with the requirements concerning vibroinsulation systems and placements of vibratory machines) the conditions for equal static load of elastic elements hold: $\sum x_j = 0$, $\sum y_j = 0$, and that these elastic elements are fastened to the machine in the horizontal plane, being by a distance h below the machine mass centre: $z_j = -h$, and that at least one symmetry plane of distribution of elastic elements, zx or zy, exists. Then, as it is easily proved

$$\mathbf{K} = \begin{bmatrix} K_{xy} & 0 & 0 & 0 & -K_{xy}h & 0 \\ & K_{xy} & 0 & K_{xy}h & 0 & 0 \\ & & K_z & 0 & 0 & 0 \\ & & & K_{\varphi x}^S & 0 & 0 \\ & \text{sym.} & & & K_{\varphi y}^S & 0 \\ & & & & & K_{\varphi z} \end{bmatrix} \tag{3.1}$$

where

$$
\begin{aligned}
&K_z = \nu k_z && K_{xy} = \nu k_{xy} && K_{\varphi z} = k_{xy} \sum (x_j^2 + y_j^2) \\
&K_{\varphi x} = k_z \sum y_j^2 && K_{\varphi y} = k_z \sum x_j^2 && K_{\varphi x}^S = K_{\varphi x} + K_{xy} h^2 \\
&K_{\varphi y}^S = K_{\varphi y} + K_{xy} h^2
\end{aligned}
\tag{3.2}
$$

In addition, in the assumed main coordinate system S_{xyz}, mass matrix (2.2) contains elements on the main diagonal only.

Then

$$
[\mathbf{K} - \omega^2 \mathbf{M}] = \begin{bmatrix}
a_{11} & 0 & 0 & 0 & -K_{xy}h & 0 \\
 & a_{22} & 0 & K_{xy}h & 0 & 0 \\
 & & a_{33} & 0 & 0 & 0 \\
 & & & a_{44} & 0 & 0 \\
 & \text{sym.} & & & a_{55} & 0 \\
 & & & & & a_{66}
\end{bmatrix}
\tag{3.3}
$$

where $a_{11} = a_{22} = K_{xy} - \omega^2 m$, $a_{33} = K_z - \omega^2 m$, $a_{44} = K_{\varphi x}^S - \omega^2 J_{xx}$, $a_{55} = K_{\varphi y}^S - \omega^2 J_{yy}$, $a_{66} = K_{\varphi z} - \omega^2 J_{zz}$.

From condition of non-trivial solution (2.7), it is possible to obtain a set of system natural frequencies

$$
\begin{aligned}
\omega_1 &= \sqrt{\frac{K_z}{m}} \qquad \omega_2 = \sqrt{\frac{K_{\varphi z}}{J_{zz}}} \\
\omega_{3,4} &= \sqrt{\Big(\frac{K_{xy}}{2m} + \frac{K_{\varphi y}^S}{2J_{yy}}\Big) \pm \sqrt{\Big(\frac{K_{xy}}{2m} + \frac{K_{\varphi y}^S}{2J_{yy}}\Big)^2 - \frac{K_{xy}K_{\varphi y}}{mJ_{yy}}}} \\
\omega_{5,6} &= \sqrt{\Big(\frac{K_{xy}}{2m} + \frac{K_{\varphi x}^S}{2J_{xx}}\Big) \pm \sqrt{\Big(\frac{K_{xy}}{2m} + \frac{K_{\varphi x}^S}{2J_{xx}}\Big)^2 - \frac{K_{xy}K_{\varphi x}}{mJ_{xx}}}}
\end{aligned}
\tag{3.4}
$$

Substituting in equation

$$
[\mathbf{K} - \omega^2 \mathbf{M}]\boldsymbol{q}_{max} = \mathbf{0}
\tag{3.5}
$$

successive natural frequency values (3.4), it is possible to obtain modal vectors (2.8) which, after normalising due to the selected coordinates, determine vectors $\boldsymbol{a}_{ki}$ (2.10)

$$
\begin{aligned}
&\text{for } \omega_1: && \boldsymbol{a}_{z1} = \text{col}\{0,0,1,0,0,0\}\\
&\text{for } \omega_2: && \boldsymbol{a}_{\varphi z2} = \text{col}\{0,0,0,0,0,1\}\\
&\text{for } \omega_3: && \boldsymbol{a}_{\varphi y3} = \text{col}\{c_3,0,0,0,1,0\}\\
&\text{for } \omega_4: && \boldsymbol{a}_{x4} = \text{col}\{1,0,0,0,c_4,0\}\\
&\text{for } \omega_5: && \boldsymbol{a}_{\varphi x5} = \text{col}\{0,c_5,0,1,0,0\}\\
&\text{for } \omega_6: && \boldsymbol{a}_{y6} = \text{col}\{0,1,0,c_6,0,0\}
\end{aligned}
\tag{3.6}
$$

where

$$
\begin{aligned}
c_3 &= 2h\left[1 - \frac{K^S_{\varphi y}}{K_{xy}}\frac{m}{J_{yy}} - \sqrt{\Big(1 + \frac{K^S_{\varphi y}}{K_{xy}}\frac{m}{J_{yy}}\Big)^2 - 4\frac{K_{\varphi y}}{K_{xy}}\frac{m}{J_{yy}}}\right]^{-1}\\
c_4 &= \frac{1}{2h}\left[1 - \frac{K^S_{\varphi y}}{K_{xy}}\frac{m}{J_{yy}} + \sqrt{\Big(1 + \frac{K^S_{\varphi y}}{K_{xy}}\frac{m}{J_{yy}}\Big)^2 - 4\frac{K_{\varphi y}}{K_{xy}}\frac{m}{J_{yy}}}\right]\\
c_5 &= 2h\left[-1 + \frac{K^S_{\varphi x}}{K_{xy}}\frac{m}{J_{xx}} + \sqrt{\Big(1 + \frac{K^S_{\varphi x}}{K_{xy}}\frac{m}{J_{xx}}\Big)^2 - 4\frac{K_{\varphi x}}{K_{xy}}\frac{m}{J_{xx}}}\right]^{-1}\\
c_6 &= \frac{1}{2h}\left[-1 + \frac{K^S_{\varphi x}}{K_{xy}}\frac{m}{J_{xx}} - \sqrt{\Big(1 + \frac{K^S_{\varphi x}}{K_{xy}}\frac{m}{J_{xx}}\Big)^2 - 4\frac{K_{\varphi x}}{K_{xy}}\frac{m}{J_{xx}}}\right]
\end{aligned}
\tag{3.7}
$$

Substituting the above vectors into relationship dependence (2.11), it is possible to determine maximum amplitudes of individual coordinates during passing through the successive natural frequencies

$$
\begin{aligned}
&\text{for } \omega_1: && z_{max} = \sqrt{\frac{nJ_{zr}}{m}}\\
&\text{for } \omega_2: && \varphi_{z\,max} = \sqrt{\frac{nJ_{zr}}{J_{zz}}}\\
&\text{for } \omega_3: && \varphi_{y\,max} = \sqrt{\frac{nJ_{zr}}{mc_3^2 + J_{yy}}} && x_{max} = \varphi_{y\,max}c_3\\
&\text{for } \omega_4: && x_{max} = \sqrt{\frac{nJ_{zr}}{m + J_{yy}c_4^2}} && \varphi_{y\,max} = x_{max}c_4\\
&\text{for } \omega_5: && \varphi_{x\,max} = \sqrt{\frac{nJ_{zr}}{mc_5^2 + J_{xx}}} && y_{max} = \varphi_{x\,max}c_5\\
&\text{for } \omega_6: && y_{max} = \sqrt{\frac{nJ_{zr}}{m + J_{xx}c_6^2}} && \varphi_{x\,max} = y_{max}c_6
\end{aligned}
\tag{3.8}
$$

Resonances of the highest frequencies are usually the most dangerous, which is obvious on the grounds of the performed energy considerations.

Coordinates which do not occur in expressions (3.8) for the given frequency, do not participate in the circum-resonance growing of vibrations. In a similar fashion, there is none circum-resonance amplitude increase in relation to forms at which the exciting force is not performing work. Certain doubts can be raised in relation to two-drive vibratory machines of with linear translatory motion, in which the exciting force does not induce e.g. rotational vibrations.

However, if the vibrators experience a loss of the cophasal running stability in the circum-resonance zone (see Michalczyk and Czubak, 2010), the resonance for this form of vibrations will also occur.

4. The case of multiple frequencies, $h = 0$

In the case when $h = 0$, vibrations of the system become uncoupled and two of natural frequencies become equal. Carrying out the analogous analysis, we obtain

$$\begin{array}{lll} \omega_1 = \sqrt{\dfrac{K_z}{m}} & \omega_2 = \sqrt{\dfrac{K_{\varphi z}}{J_{zz}}} & \omega_3 = \sqrt{\dfrac{K_{xy}}{m}} \\ \omega_4 = \sqrt{\dfrac{K_{\varphi y}}{J_{yy}}} & \omega_5 = \sqrt{\dfrac{K_{xy}}{m}} & \omega_6 = \sqrt{\dfrac{K_{\varphi x}}{J_{xx}}} \end{array} \tag{4.1}$$

and $\omega_3 = \omega_5$

$$\begin{array}{llll} \text{for } \omega_1: & z_{max} = \sqrt{\dfrac{nJ_{zr}}{m}} & \text{for } \omega_2: & \varphi_{z\,max} = \sqrt{\dfrac{nJ_{zr}}{J_{zz}}} \\ \text{for } \omega_3: & x_{max} \leqslant \sqrt{\dfrac{nJ_{zr}}{m}} & \text{for } \omega_4: & \varphi_{y\,max} = \sqrt{\dfrac{nJ_{zr}}{J_{yy}}} \\ \text{for } \omega_5: & y_{max} \leqslant \sqrt{\dfrac{nJ_{zr}}{m}} & \text{for } \omega_6: & \varphi_{x\,max} = \sqrt{\dfrac{nJ_{zr}}{J_{xx}}} \end{array} \tag{4.2}$$

Symbols $\leqslant$ mean that depending on the exciting forces character the driving system energy can distribute itself into vibrations along the axes x and y in a different way. Sometimes there are physical grounds to consider that this distribution is equal (e.g. for a single drive machine with the a vertical rotor axis). In such a case

$$\text{for } \omega_{3,5}: \qquad x_{max}, y_{max} = \sqrt{\frac{nJ_{zr}}{2m}} \tag{4.3}$$

5. Conclusions

- The method of the assessment of the maximum amplitudes of systems with several degrees of freedom – during the transient resonance – was formulated in the paper. Especially, vibrations of bodies elastically supported in a way enabling small, arbitrary vibrations during the free coasting of the rotating unbalanced driving system, were analysed.
- The applied approach requires – in the engineering practice – only the knowledge of the basic and easily determined system parameters, and does not present computational problems. It provides the assessment of maximum amplitudes ('a top estimation'), which for typical systems is close to reality.
- The proposed approach can be successfully applied to the determination of maximum amplitudes in the transient resonance of vibrating systems with continuous distribution of mass e.g. beams, shafts, frames and plates (Michalczyk, 2012).

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Rezonans przejściowy w maszynach i urządzeniach w ruchu ogólnym

Streszczenie

W pracy wskazano na nową , dokładniejszą, możliwość oszacowania amplitud maksymalnych maszyn o ruchu drgającym i układów wibroizolowanych, podczas przejścia przez rezonans. Wykorzystano w tym celu bilans energetyczny układu i podano rozwiązanie analityczne dla typowych układów o wielu stopniach swobody podczas wybiegu swobodnego. Sformułowana metoda może mieć zastosowanie dla dowolnych układów wielomasowych, w tym, układów ciągłych.

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