MECHANIKA
TEORETYCZNA
I STOSOWANA
2, 26 (1988)

PROBLEMS IN CONNECTION WITH DESIGN, CALIBRATION AND USE OF PRESSURE CELLS*

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1. Introduction

The pessure cell problem makes a very good example from the Experimental Mechanics field of the complementarity of theory and experiment.

Experimental information is necessary in the process of developing better theories when dealing with particulate media whose properties are not very well known, and the pressure cell is one means of obtaining such information. On the other hand, theoretical considerations are necessary when designing the pressure cell so that its measuring accuracy when used in these not very well known materials can be predicted. These theoretical considerations must be based on simple material models; usually, linear elasticity is assumed.

We therefore see a development where pressure cells designed using very restrictive material characterizations are embedded in materials with more or less unknown material properties in order to obtain a better understanding of these materials' behaviour and thus improved theories.

The pressure cell example also shows how important it is to remember that the assumptions made are approximations to reality, so that both theoretical and experimental results have a certain inaccuracy, making calibration experiments under well known conditions essential.

Pressure cells are used to measure stresses in the fields of soil mechanics, road research, silo research, and structural engineering. The design bases for pressure cells are the same in the different fields, whereas the practical problems of using them may differ, leading to variations in geometry and stiffness.

The first application of pressure cells seems to have been in the silo field a hundred years ago [1], [2]. The development of design bases is of more recent date and has been slow in moving from assumptions of the pressure cell being embedded in a medium in an uniaxial state of stress to the general stress state and general stress-free strain state (shrinkage for instance). The development up to about 1970 is described in [3].

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* Praca wygłoszona na XII Sympozjum Doświadczalnych Badań w Mechanice Ciała Stałego, Warszawa-Jadwisin, 1986

In the following, only work carried out at the Department of Structural Engineering will be described. This work has consisted in developing design bases for wall cells and embedded cells under general stress states, i.e. work of a general nature. The applications described refer to silo research, but the conclusions drawn from it are also valuable in the fields of road research and soil mechanics.

The following 3 headings are used in the presentation:

Interface problem with no relative displacement.

Interface problem with relative displacement.

Embedded cell problem.

2. Interface problem with no relative displacement

2.1. Normal stress. The first design expressions for pressure cells measuring normal stress seem to have been based on practical experience and not on theoretical arguments, and not until 1956 [4] did a suggestion come based on a theoretical solution, and even then, the measuring error was underestimated. The analytical solution for a fluid-filled, membrane-type cell, assuming linear elasticity, was published at the Department in 1959 [5]. At the same time, the numerical solution for the piston type cell was given. These results, together with the expressions for the plate-type cell with or without fluid behind it, se fig. 1, are discussed in [6]. The error expressions have the form (1) and can be used when errors are small.

$$\frac{\Delta p}{p} = K \cdot \frac{E}{(1-\nu^2)\cdot a} \frac{\Delta w}{p} \tag{1}$$

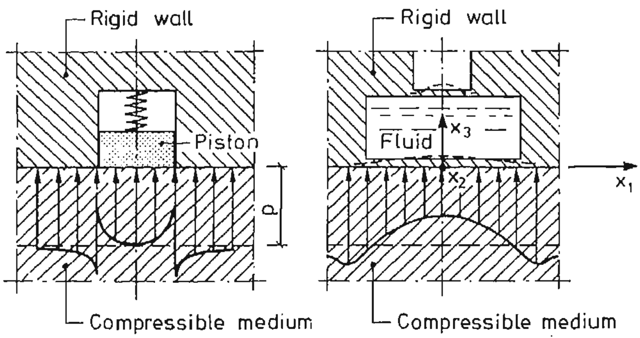

Fig. 1. Measuring principles for stress cells at an interface

Here, p is the uniform pressure on the undisturbed wall, and $p-\Delta p$ is the mean pressure transmitted to the pressure cell. $p-\Delta p$ depends on the displacement Δw of the cell's front surface under the load p. It also depends on the modulus E, Poisson's ratio ν for the compressible medium, and the radius a of the pressure cell. K is a constant depending on the cell type (about 1.9 for the stiff piston and 0.7 for a thin plate with no fluid behind it).

Expression (1) can also be used to evaluate error signals caused by temperature changes when materials with different coefficients of expansion are used in the cell-wall arrangement.

As shown in the following example the requirements to stiffness of the pressure cell are rigorous if small measuring errors are going to be obtained. Underestimation of these requirements still seems to be the major cause of errors in pressure cell work.

Example. In the case of a pressure level $p = 50$ KPa, if $a = 50$ mm, $E = 100$ MPa (sand), and $\nu = 0.3$, then even such a small displacement of the piston as 10^{-3} mm (for $p = 50$ KPa) will give a measuring error of about 10% when the pressure cell is of the rigid-piston type.

By making a suitable choice of cell stiffness $\frac{\Delta w}{p}$, a small error, say less than 2%, can be obtained for a range of values of E, indicating that the assumed value of E need not be very precise. The material need not even be linear elastic, as assumed, if even a reasonably good guess of the stiffness can be made and small errors are aimed at. It is thus justifiable to talk about the measurement of stress.

A typical pressure cell installation in a reinforced concrete wall is shown in fig. 2.

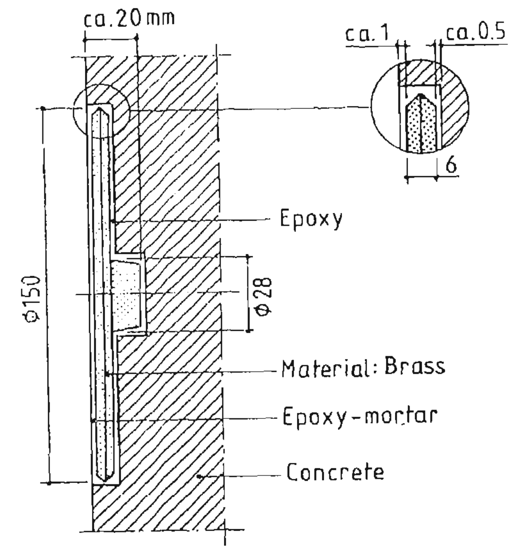

Fig. 2. Installation of normal stress cell in wall

The layers of epoxy and epoxy mortar are kept as thin as possible, and the free surface is given a roughness comparable with that of the surrounding concrete, even though the measuring signal from the cell is almost insensitive to deviations in roughness.

The design basis has been tested with the central bottom cell (no viscous paste layer) in a calibration chamber similar to the one shown in fig. 7. This cell has a front plate thickness of 0.3 mm. A mean deviation of 1% was found between the calculated and the measured sensitivity. The coefficient of variation was 0.02.

Tests with a pressure cell installed in a concrete specimen subjected to a strain state of the order of magnitude that can be expected in full-scale silos has shown only small changes in sensitivity and zero shift when thin front plates (0.3 mm) are used. In the case of thicker front plates, as the one in fig. 2, which are necessary in the case of coarse grains in the particulate medium, non-negligible changes must be expected. Such changes may also occur when curved front plates are used — in model, small-diameter silos, for instance.

2.2. Shear stress. The design basis for a shear cell has been developed by making a limit analysis of the case where an ellipsoidal inclusion is placed in a matrix under external load, as illustrated in fig. 3, [7].

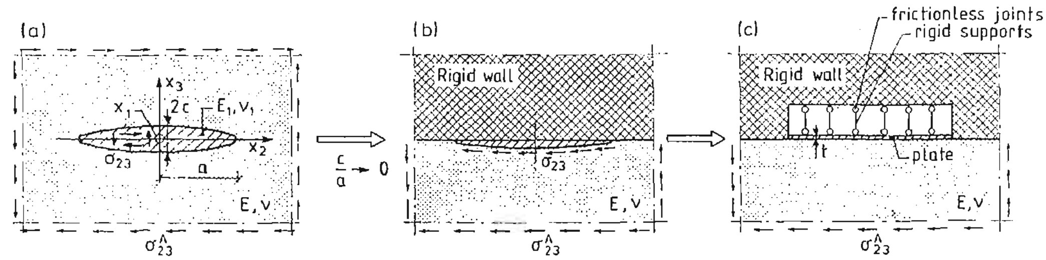

Fig. 3. Transference of ellipsoid in infinite medium into a plate fixed along the edge in an interface

The following expression was obtained for a cell type having a front plate of constant thickness and fixed along the edge:

$$\frac{\sigma_{23}}{\sigma_{23}^{A}} = \frac{8(1-\nu^2)}{\pi(2-\nu)U+8(1-\nu^2)}, \tag{2}$$

where:

$$U = \frac{Eu_2}{\sigma_{23}a}. \tag{3}$$

σ_{23} is the shear stress on the cell surface, σ_{23}^{A} is the shear stress in the matrix, which is to be measured, E and ν are the modulus of elasticity and Poisson's ratio, respectively, a is the cell radius, and u_2 is the displacement in the x_2 direction of the front plate centre caused by σ_{23}.

Example. Using the values $E \sim 100$ MPa (sand), $\nu \sim 0{,}3$, $a = 75$ mm, and $\sigma_{23} \sim 0{,}1$ MPa, it can be seen that to obtain a measuring error of less than 5%, the displacement u_2 for the stress $\sigma_{23} \sim 0{,}1$ MPa should be less than $5 \cdot 10^{-3}$ mm.

Calibration tests have been carried out with a cell of this type in the calibration set up shown in fig. 4. The shear cell was placed in the interface between a rigid wall and a sand layer of thickness of 150 mm and diameter of 1200 mm. The sand was encapsulated in an 0.3 mm thick rubber membrane and evacuated. The shear load was obtained by 90° rotation from the horizontal, as shown in the figure.

Had the diameter of the sand layer been very large, the shear stress in the interface around the stress cell would have been uniformly distributed and given by the weight of the volume of the column of sand over a unit of surface area.

For practical reasons, the diameter must be of limited size. In the chosen configuration, the shear stress can still be considered uniformly distributed, but a correction of 2% found by finite-element technique must be introduced. Good correspondence between the calculated and measured error of the shear cell was found being 7% and 9%, respectively.

A shear cell type, where the front plate is not fixed along the edge but moves in the plane as a whole has also been calibrated. The expression (2) seems to give a good idea of expected error in this case, too.

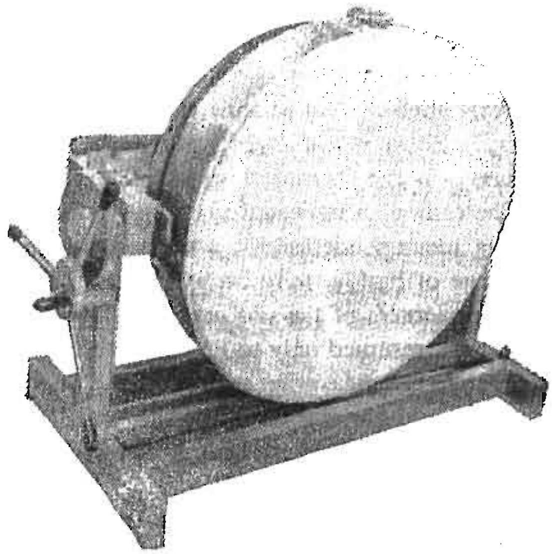

Fig. 4. Calibration equipment with shear cell in an interface

In fig. 5, a cell of that type is shown mounted in a mediumscale model silo [8]. By measuring the bottom lead and the shear stress distribution on the silo wall and knowing the weight of the silo medium (barley), an equilibrium check could be carried out to evaluate the shear cell behaviour. The correspondence seemed to be good, giving 3% difference between expected and measured shear load during filling at nearly full silo.

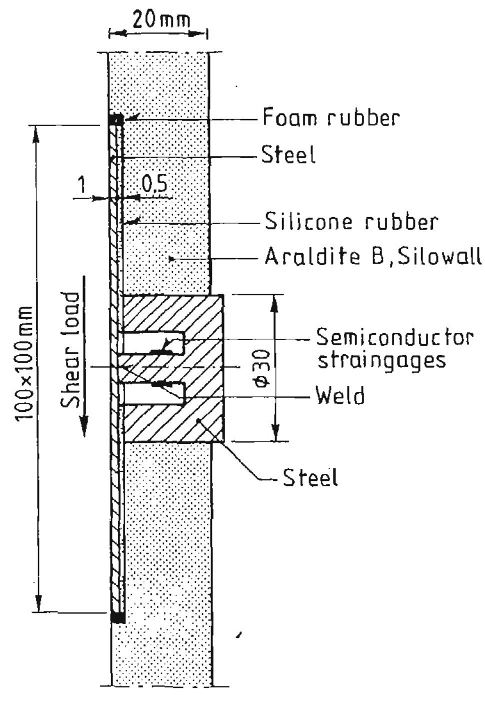

Fig. 5. Operating principle of shear cell

3. Interface problem with relative displacements

Large relative displacements between wall and medium occur during emptying in mass flow silos. The requirements to normal pressure cells used under these conditions have been investigated in [9].

Tests showed that serious errors could occur if the cell was not mounted flush with the wall. An angle of 1° between the cell's front surface and the wall surface thus gave a change in the measured stress of about 50%.

The tests also indicated that the relatively rapid variations in pressure about a slowly varying mean value, which are characteristic of pressure cell measurements during the discharge of barley, were caused by local irregularities in the geometry of the wall or in the compactness of the silo medium. These rapid variations in contrast to the more slow were thus presumed only to represent the condition in an area of the wall that is not much larger than the measuring surface of the pressure cell itself and should not normally cause serious bending moments in the silo wall. Measurements carried out in a fly-ash silo [10] have shown that pressure cells, when in contact with powder materials, represent a wall area much bigger than their own.

When measuring shear stresses and when the relative displacements occur in the interface itself, it becomes important to ensure that the coefficient of friction of the pressure cell is the same as that of the surrounding wall. If the relative displacements take place in the medium a few particle diameters from the wall, the only requirement to the coefficient of friction of the cell's surface is that this must be bigger than the internal friction in the silo medium. The equilibrium check carried out with friction cells in a model silo with barley mentioned previously was also done in the discharge situation, where semi-mass flow occurs, leading to large relative displacements near the wall. The deviations in two situations were measured as 6% and 10%.

4. Embedded cell problem

If a stress cell is made flat and thin enough, the measuring error can be made arbitrarily small whatever the material properties of the matrix and the cell.

For small errors, therefore, an almost free choice of the material properties is acceptable when establishing a design basis.

A design basis has therefore been formulated [11] [12] based on linear elasticity and on cells with axially symmetrical, ellipsoidal geometry.

$$\sigma_{33} = A\sigma_{33}^A + B(\sigma_{11}^A + \sigma_{22}^A) \tag{4}$$

is found for a normal stress cell and

$$\sigma_{23} = C\sigma_{23}^A \tag{5}$$

for a shear stress cell.

Here σ_{11}^A, σ_{22}^A, σ_{33}^A and σ_{23}^A are stress components in the surrounding medium with the shear σ_{23}^A lying in the plane of the flat stress cell and with the normal stress σ_{33}^A perpendicular to this plane. σ_{23} and σ_{33} are the corresponding stress components in the homogeneous ellipsoid. A, B and C are factors containing Poisson's ratio, the thickness-diameter ratio, and the ratio between the moduli of elasticity of the homogeneous ellipsoid and the surrounding medium.

By careful design and under certain restrictions, the almost ideal situation, where $A \sim 1{,}0$ and $B \sim 0$, can be obtained, up to even large deformations.

A load-history-dependent sensitivity will be encountered when stress cells are designed in such a way that A differs substantially from 1.0.

The approximations made can also be used to get an idea of the behaviour of strain cells. In this case, the ellipsoid is made long and thin, and an expression similar to (4) but with normal strains instead of normal stresses can be developed [12].

For both flat and long ellipsoids, expressions describing the situation where stress-free strain (shrinkage and differences in thermal expansion) occur, are also given in [12].

It is, of course, possible to refine the calculations by using better approximations of geometry and material properties and by using FEM techniques. But very little seems to be achieved by this where small errors are aimed at, nor do these more rigorous approaches render calibration tests superfluous.

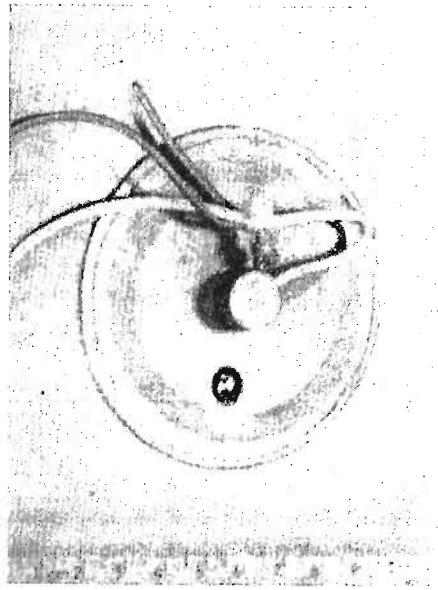

Fig. 6. Electro-hydraulic stress cell

The 75 mm dia. pressure cell shown in fig. 6 has been used in calibration tests described in [13] to get an idea of the overall accuracy that can be expected when using pressure cells.

These tests were performed in the calibration equipment shown in fig. 7 with the pressure cell embedded in wheat and in a similar but more rigid equipment when embedded in sand, which is a stiffer material. Compressibility of the calibration chamber in the vertical direction was obtained by making the cylinder of silicone rubber, while steel reinforcement in the hoop direction made the cylinder stiff in the radial direction.

When evaluating results from the calibration experiments, it is important to have obtained homogeneity in the stress state of the wheat mass. From fig. 8, this appears to be the case with very good accuracy.

In practice, pressure cells are used under load conditions which are only poorly known.

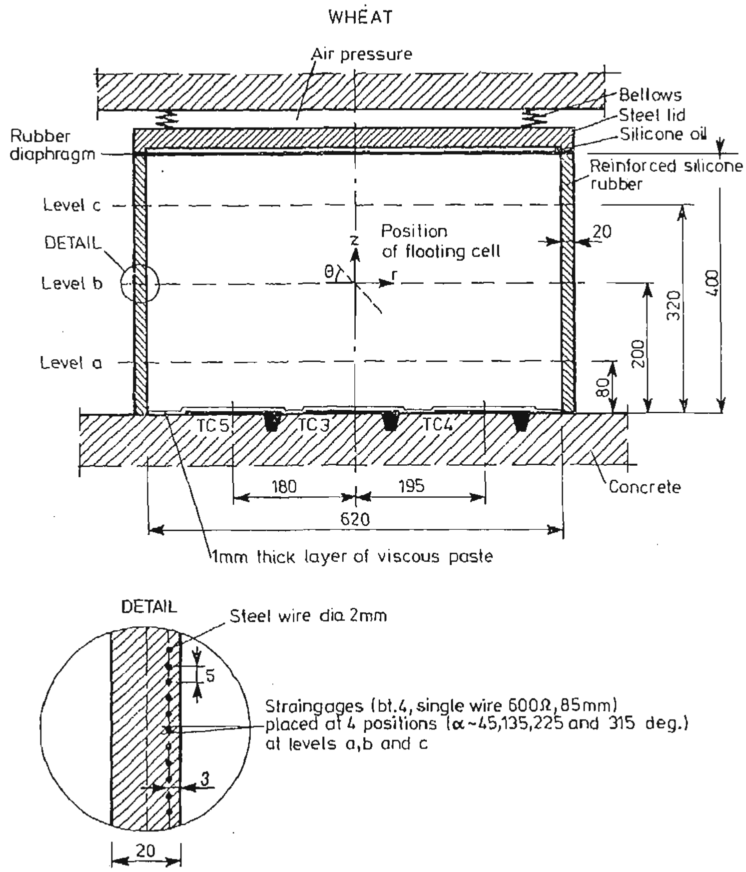

Fig. 7. Calibration chamber used for compressible materials

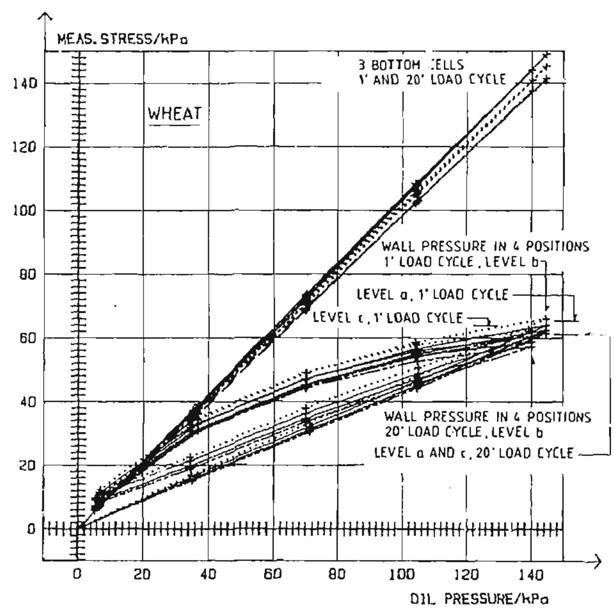

Fig. 8. Stress state in wheat filled calibration chamber

Therefore, to get an idea of the overall accuracy of the pressure cell under such conditions, calibration tests must be performed in such a way as to obtain greatly differing load situations.

In the present case, different load situations were produced by using different tilting angles Θ (0.45° and 90°, see fig. 7), whereby the pressure cell worked under different

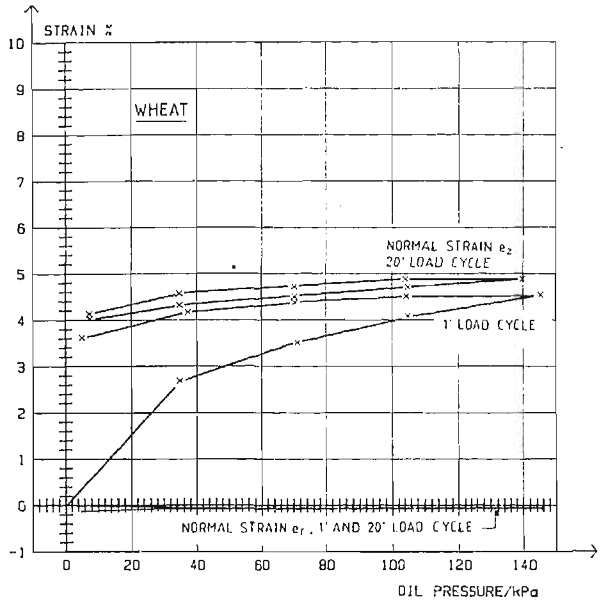

Fig. 9. Strain state in wheat filled calibration chamber

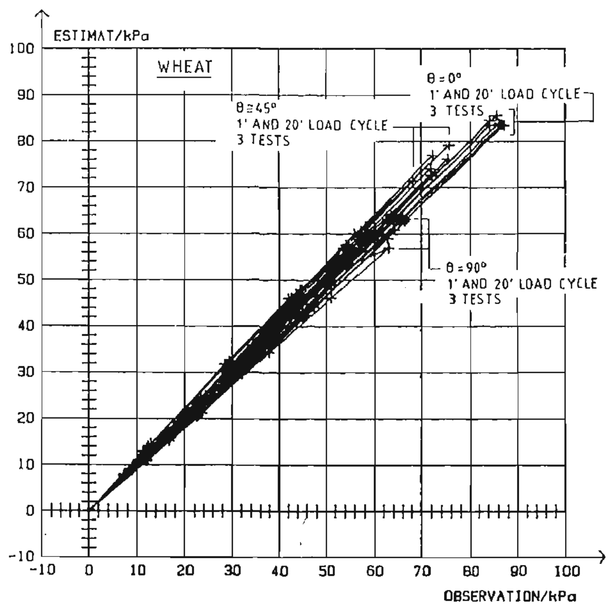

Fig. 10. Observed versus estimeated pressure q. 9 tests in wheat, Θ = 0.45 and 90 deg., 1 st and 20th load cycle

principal stress ratios. For $\Theta = 45°$, also shear stresses will act on the cell surface. The load history involved 20 load cycles, giving changes in material stiffness during tests as shown in fig. 9. All the tests were repeated twice giving a total of 9 tests.

The best estimate of the coefficients A and B in equation (4) from all 9 tests gave $A = 1.06$ and $B = \div 0.01$. Using these values on each test leads to fig. 10, indicating that an overall accuracy on the measurement of the normal stress using a pressure cell of this type in wheat corresponds to a coefficient of variation of 0.04.

Similar tests with sand in the calibration chamber led to $A = 0.93$ and $B = 0.07$ and a coefficient of variation of 0.05 on the measurement of the normal stress.

In both cases, the theoretically predicted A and B values were $A = 1.00$ and $B = 0.04$.

Even though calibration tests are necessary, the theoretical prediction of the coefficients A and B seems good, with deviations of 6 - 7% on A, which is the more important coefficient.

The materials used in the tests described here were loose materials, making it easy to install the pressure cell in the mass. In the case of more cohesive materials, installation may be more difficult, leading to bigger coefficients of variation on the results.

The behaviour of friction cells can be predicted by means of equation (5). To the author's knowledge, no description or use of such cells has been reported.

5. Conclusion

Theoretical considerations can lead to quite an accurate prediction of how a pressure cell will behave in practical use, whether it is embedded or placed in an interface. Such design expressions are given in the paper.

More precise information can, however, only be obtained by experiments, and such experiments are necessary because the assumptions on which the predictions are based may deviate considerably from the actual conditions during field tests.

Calibration tests and model tests with normal stress and shear stress cells in an interface show good agreement between calculated and measured values.

Good agreement is also obtained when embedded cells are tested in a calibration set up under greatly varying stress states.

The experimental results indicate that coefficients of variation of about 0.05 on normal stress measurements can be obtained in field tests with cells embedded in loose materials like sand and wheat, provided the installation conditions of the pressure cells are similar in full scale and calibration tests. Inhomogeneities in the loose materials may, of course, give a scatter in results even from closely spaced pressure cells, but this scatter cannot be attributed to the pressure cell itself.

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Резюме

ПРОЕКТИРОВАНИЕ, КАЛИБРОВКА И ПРИМЕНЕНЕНИЯ ДАТЧИКОВ ДАВЛЕНИЯ

Обсуждено работы над развитием датчиков для определения напряжений в сыпучих средах. Описано предположения для постройки датчиков помещеных в стенках силосных сооружений или во внутри сыпучой среды. Обсуждено также проблемы калибровки и точности измерений.

Streszczenie

PROBLEMY PROJEKTOWANIA, KALIBRACJI I UŻYTKOWANIA CZUJNIKÓW CIŚNIENIA

W artykule przedstawiono prace nad rozwojem czujników do pomiaru naprężeń w ośrodkach sypkich. Omówiono założenia dla budowy czujników umieszczonych w ścianach silosów lub zatopionych wewnątrz ośrodków. Omówiono problemy skalowania i dokładności pomiarów.

Praca wpłynęła do Redakcji dnia 1 czerwca 1987 roku.