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FATIGUE FAILURE CRITERION BASED ON STRAIN ENERGY DENSITY

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In this paper, the strain energy density approach is used to characterize the fatigue resistance of metals. The results suggest that in the high- and low-life regions the total strain energy density is a consistent damage parameter. Predictions of the proposed method are in agreement with the experimental data, and demonstrate the appropiateness to predict the mean stress effect on fatigue life. A number of relations describing the effect of mean stress on fatigue resistance, are shown to be particular cases of the one developed herein.

1. Introduction

One of the current research topics in fatigue failure is the determination of the suitable damage parameter which can predict the fatigue resistance of the material when subjected to various loads and/or deformation patterns. The failure process associated with nucleation and subsequent propagation of the fatigue cracks is generally controlled by the local stress/strain fields. Earlier research efforts were concerned with correlating the fatigue life with either the stress or strain amplitudes. The well known Coffin-Manson relationship was used to correlate high- and low-cycle fatigue in which the total strain amplitude divided into elastic and plastic components [1]. In this application the interrelation between the cyclic stress and plastic and/or elastic strains, and the fatigue damage process was generally overlooked [2].

In recent years however, a number of relationships have been proposed relating the low- and high-cycle fatigue life to the plastic strain energy stored in the material during a load cycle as a means of the area of the hysteresis loop or its part [2 - 12]. The plastic strain energy per cycle, ΔW, has the advantage of being nearly constant during the life, under strain-controlled conditions. However, as the strain range, $\Delta \varepsilon$, decreases, the plastic strain range component $\Delta \varepsilon_p \to 0$, and the corresponding plastic strain energy, $\Delta W \to 0$. It is obvious that in this case, the elastic strain energy controls the fatigue damage.

To unify both low- and high-cycle fatigue, it follows that the total cyclic strain energy density, ΔW_t, (plastic and elastic) appears to be a promising damage parameter [13].

The significance of the strain energy density approach is in its ability to unify microscopic and macroscopic testing data and to formulate multiaxial life prediction model [4, 8].

In this paper, the strain energy density approach is used to characterize the fatigue life of metals for fully-reversed constant strain or stress and mean-tensile-stress effect. It is shown that in the high- and low-life regions, the strain energy density is a consistent damage parameter. The strain energy density used as a damage parameter is also consistent with non-linear fracture mechanics approach to fatigue crack propagation through the integral. The approach described herein is applicable for both Masing and non-Masing type of materials. It is also shown that a number of relations describing the mean stress effect proposed earlier, can be derived as a particular case of the present one.

The predictions of the proposed method are compared with the experimental data and the agreement is found to be fairly good.

2. Cyclic strain energy

When a metal is subjected to a cyclic strain controlled test its stable cyclic stress-strain curve could be described by well known Ramberg-Osgood type of relation:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}, \tag{1}$$

where Δ represents „range" when used as a prefix for a variable, K' is a coefficient with dimensions of stress, and n' is the cyclic strain hardening exponent. The parameters K' and n' are determined through the best fit to the experimental data.

Figure 1 shows a stable hysteresis loop as a cyclic response of a material when subjected to a constant strain range of $\Delta\varepsilon$. The absorbed plastic strain energy per cycle, is the area of the hysteresis loop (area OABCO in Fig. 1) and for a Masing material is given by [3]:

$$\Delta W = \frac{1-n'}{1+n'}\Delta\sigma\Delta\varepsilon_p. \tag{2}$$

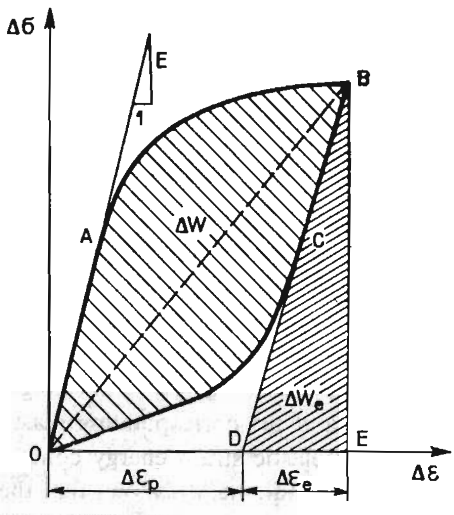

Fig. 1. Description of stable hysteresis loop

A material is said to exhibit a Masing description when the branches of the hysteresis loops can be described by the cyclic stress-strain curve, Eq. (1), magnified by a factor of two.

The expression for calculating the cyclic plastic strain energy for a non-Masing material response was developed in Ref. [10, 11]. A „master curve" different from that of cyclic curve is defined. This curve is obtained from matching upper (or lower) branches of the hysteresis loops through translating each loop along its linear response portion. The equation of the master curve where the origin corresponds to the tip of hysteresis loop with the minimum proportional range is given by [11]:

$$\Delta\varepsilon^* = \frac{\Delta\sigma^*}{E} + 2\left(\frac{\Delta\sigma^*}{2K^*}\right)^{1/n^*}. \tag{3}$$

The cyclic plastic strain energy (area of the hysteresis loop) is then calculated from:

$$\Delta W = \frac{1-n^*}{1+n^*}(\Delta\sigma - \delta\sigma_0)\Delta\varepsilon_p + \delta\sigma_0\Delta\varepsilon_p, \tag{4}$$

where:

$$\delta\sigma_0 = \Delta\sigma - \Delta\sigma^* = \Delta\sigma - 2K^*(\Delta\varepsilon_p/2)^{n^*} \tag{5}$$

is the increase in the proportional stress due to non-Masing behaviour of the material [11]. Note that for an ideal Masing description the master curve and that of obtained from cyclic curve will coincide, and in this case $\delta\sigma_0 = 0$.
The linear elastic strain energy, area BCDEB of Fig. 1, is given by,

$$\Delta W_e = \frac{1}{2}\Delta\sigma\Delta\varepsilon_e. \tag{6}$$

The total cyclic strain energy, can then be calculated as the sum of the area OABO and OBEDO, i.e.:

$$\Delta W_t = \frac{1}{2}\Delta W + \frac{1}{2}\Delta\sigma\Delta\varepsilon, \tag{7}$$

where ΔW can be determined from Eq. (2) or (4) depending on Masing or non-Masing material respectiviely.

Thus for a given strain or stress range ($\Delta\varepsilon$ or $\Delta\sigma$), the total strain energy density per cycle ΔW_t, can be calculated, if cyclic and the master curves are available. Note that for non-Masing material a minimum of two tests is required to specify the master curve Eq. (3), one with the smallest, the other with the largest hysteresis loop.

3. An energy-based fatigue failure criterion

A fatigue failure criterion based on the total strain energy density at half life was recently developed by Ellyin and Kujawski [13] for ASTM A-516 Gr. 70 carbon low alloy steel. The power law relationship of the form:

$$\Delta W_t = W_f(2N_f)^d + \Delta W_{end} \tag{8}$$

was proposed to predict fatigue lives in low- and high-cycle fatigue. In the above equation the parameters W_f and d are determined through the best fit to the experimental data, and ΔW_{end} is the elastic strain energy density associated with the material endurance level.

The relationship (8) was used to correlate experimental results of solid circular cylindrical speciments of this steel. The testing program included strain controlled tests, fully-reversed and with mean-tensile-strain, stress controlled tests, fully-reversed and with tensile or compressive prestrain. Further information regarding speciment preparation, material properties and testing techniques can be found in Ref. [10, 11, 13]. Figure 2 shows the total strain energy density per cycle at half life, ΔW_t, plotted versus the number of reversals to failure, $2N_f$.

The relationship (8) appears to describe the results fairly well. The unifying nature of the total strain energy density is evident from examining Fig. 2.

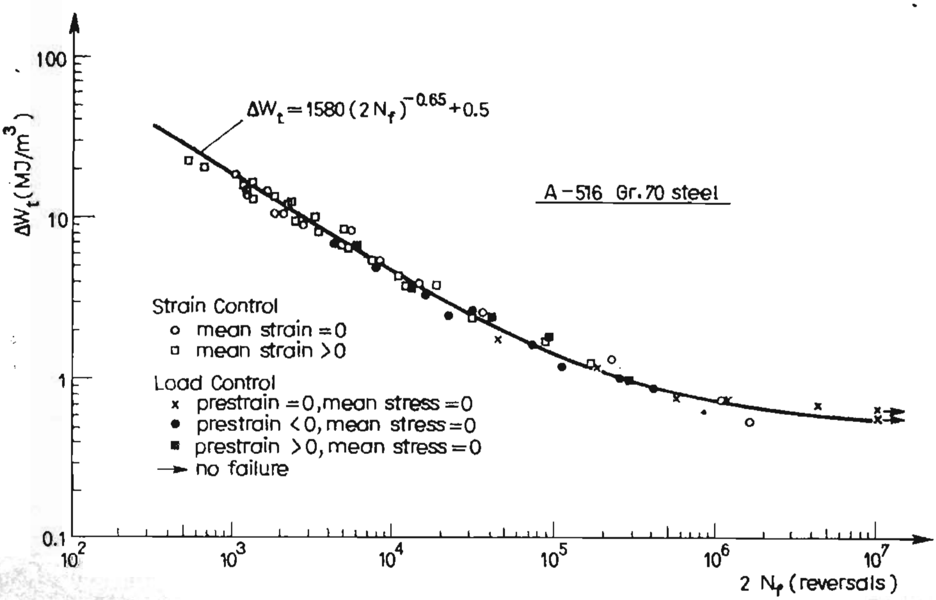

Fig. 2. Total strain energy density per cycle at half-life versus reversals to failure.

We can therefore conclude that the total strain energy density and the power law relationship (8) are fair representation of the fatigue resistance for stress-strain controlled tests without mean stress.

In order to take into account the mean stress effect on fatigue life the following relationship is proposed:

$$\left(\frac{\Delta W_{tm}}{\Delta W_t}\right)^{\frac{\bar{n}}{1+\bar{n}}} + \frac{\sigma_m}{k\sigma_f'} = 1\,, \tag{9}$$

Where: ΔW_t and ΔW_{tm} are total strain energy density per cycle for stress controlled tests fully-reversed and with mean stress $\sigma_m \neq 0$ respectively; σ_f' is the fatigue strengh coefficient; k is a parameter equal or less then one, and:

$$\bar{n} = \frac{1+n'(\Delta W_p/\Delta W_e)}{1+(\Delta W_p/\Delta W_e)}, \tag{10}$$

where ΔW_p and ΔW_e are the plastic and elastic components of the strain energy density for fully-reversed test:

$$\Delta W(\varepsilon) = \int_0^{\Delta\varepsilon} \Delta\sigma d(\Delta\varepsilon). \tag{11}$$

Note that for low-cycle fatigue $\Delta\varepsilon_p \gg \Delta\varepsilon_e$; $\Delta W_p \gg \Delta W_e$, $\bar{n} \approx n'$ and for high-cycle fatigue $\Delta\varepsilon_p \ll \Delta\varepsilon_e$; $\Delta W_p \ll \Delta W_e$, $\bar{n} \approx 1$.
In the case of high-cycle fatigue when $\Delta\varepsilon_p \ll \Delta\varepsilon_e (\bar{n} \approx 1)$ and neglecting plastic part of strain energy density the relationship (9) can be reduce to:

$$\frac{\Delta\sigma_m}{\Delta\sigma} + \frac{\sigma_m}{k\sigma_f'} = 1, \tag{12}$$

or using $\Delta\sigma = 2\sigma_a$ and $\Delta\sigma_m = 2\sigma_{am}$ we can write:

$$\frac{\sigma_{am}}{\sigma_a} + \frac{\sigma_m}{k\sigma_f'} = 1, \tag{13}$$

where σ_a and σ_{am} are the stress amplitudes at half life for fully-reversed and with mean stress respectively. The relation in terms of stress, similar to that of (13), can be obtained from (9) in the case of low-cycle fatigue where $\Delta\varepsilon_p \gg \Delta\varepsilon_e$, $\bar{n} \approx n'$ and neglecting the elastic part of strain energy density which is small compare to its plastic part.

4. Discussion

Let us now discusse the relation (13) which is a special case of relationship (9). Depending on the parameter k a number of previously proposed relationships can be obtained. For example, if $k = 1$ relation (13) reduce to well known relationship proposed by Morrow [2]:

$$\frac{\sigma_{am}}{\sigma_a} + \frac{\sigma_m}{\sigma_f'} = 1. \tag{14}$$

Alternatively, for $k = R_m/\sigma_f'$ and $k = R_e/\sigma_f'$ the relationships proposed by Goodman and Soderberg can be obtained respectively, where R_m is the ultimate strength and R_e is the yield stress of the material.

It have to be noted that in the case of smooth specimen subjected to stress-controlled test the relation (9) or (13) is applicable when the following condition is satisfied:

$$\sigma_{max} = \sigma_{am} + \sigma_m \leqslant R_m. \tag{15}$$

If this condition is not satisfied the maximum stress, σ_{max}, can induce unrestricted plastic deformation and failure in first reversal may occur. On the other hand, in the case of notches when the surrounding material will inhibit unrestricted plastic flow at the notch tip, and consequently relationship (9) (or 13) could be used even if condition (15) is not satisfied. For ductile metals the parameter $k = 1$ can be used.

Two different ductile materials were chosen to compare the correlation of the relationship (13), with $k = 1$, to the available experimental data for lives ranging from 10^2 to 10^5 cycles [14]. Smooth circular specimens were tested under constand cyclic stress range condition. Figures 3 and 4 show the corelation between experimental results and the relationship (13) and condition (15).

The correlation with experimental data is fairly good over the entire fatigue lives.

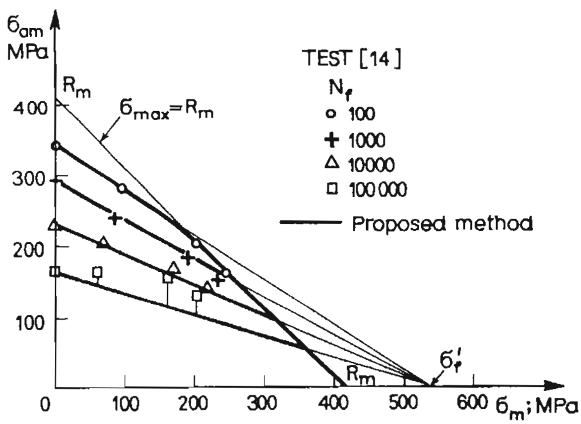

Fig. 3. Effect of mean stress for stress-controlled rests (A-201 steel).

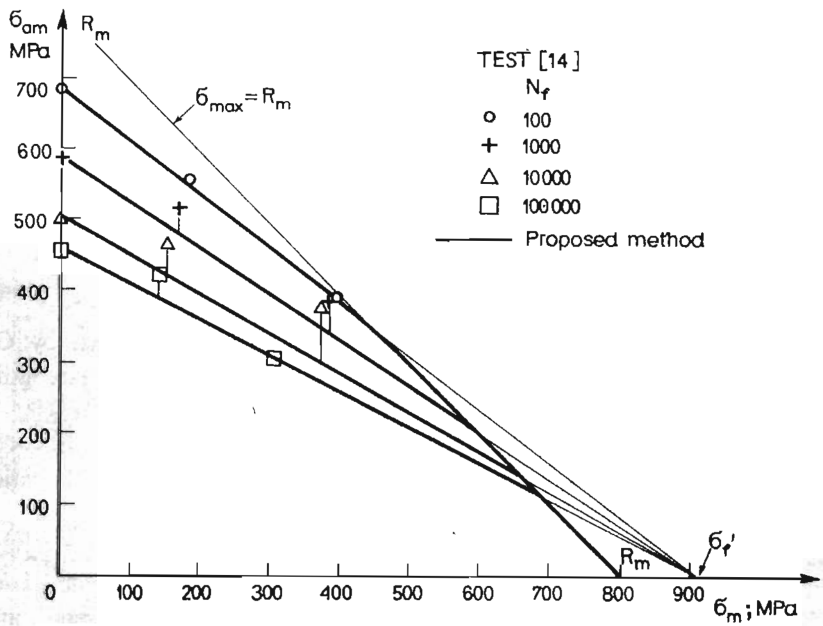

Fig. 4. Effect of mean stress for stress-controlled tests (A-517 steel).

5. Conclusion

From the results presented herein, it is seen that the total strain energy density is a consistent damage parameter in high- and low-life regions.

The proposed method predicts the mean stress effect on fatigue lives. A number of relations describing the effect of mean stress on fatigue resistance, are shown to be particular cases of the one developed herein.

The correlation with experimental data is found to be fairly good.

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Резюме

КРИТЕРИЙ УСТАЛОСТНОГО РАЗРУШЕНИЯ ОСНОВАННОЕ НА ПЛОТНОСТИ ЭНЕРГИИ ДЕФОРМАЦИИ

В работе представлен критерий усталостного разрушения, основанный на плотности энергии деформации.

Представленные исследования показывают, что плотность полной энергии деформации может

быть использована как основной параметр усталостного повреждения для малоцикловой усталости.

Показано, что предложенный критерий достаточно хорошо соответствует экспериментальным данным.

Известные из литературы критерии, определяющие влияние среднего напряжения на усталостное разрушение, можно полйяить как частные случаи представляемого метода.

Streszczenie

KRYTERIUM ZMĘCZENIOWEGO ZNISZCZENIA OPARTE NA GĘSTOŚCI ENERGII ODKSZTAŁCENIA

W pracy przedstawiono kryterium zmęczeniowego zniszczenia oparte na gęstości energii odkształcenia. Przedstawione wyniki badań sugerują, że gęstość całkowitej energii odkształcenia jest istotnym parametrem uszkodzeń zmęczeniowych w zakresie małej i dużej liczby cykli obciążeń.

Proponowane zależności umożliwiają określenie wpływu średnich naprężeń na trwałość zmęczeniową i są w dużej zgodności z danymi doświadczalnymi.

W pracy pokazano również, że znane z literatury zależności opisujące wpływ naprężenia średniego na trwałość zmęczeniową można otrzymać jako szczególne przypadki proponowanej metody.

Praca wpłynęła do Redakcji dnia 11 maja 1987 roku