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CREEP-LIKE CRACK PROPAGATION IN FIBER REINFORCED COMPOSITES DUE TO DAMAGE ACCUMULATION PROCESS

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Introduction

The process of localization and spread of damage ahead of the dominant matrix crack is viewed as a sequence of nucleation and propagation phases both of which may be described by use of the internal damage parameter. This scalar quantity reflects the ratio of the current crack (or pore) density to its saturation, or critical level. It is shown that the Continuous Damage Mechanics (CDM) approach is useful in modeling a damage field consisting mainly of the fiber breaks generated ahead of the matrix crack and clustered around the plane of a prospective fracture, thus forming the so-called "damage band" embedded within the stress field of the dominant crack.

Description of this type of damage applies to the failure process which follows formulation of the "characteristic damage state" (CDS) observed in a number of multiphase materials.

Derivation of the Governing Equation

According to the continuous damage mechanics (CDM) view of fracture, the presence of the dominant crack is always accompanied by a region of microdefects in which damage is being continually built up, cf. Wnuk (1983) and Wnuk and Kriz (1985). Integration of the Kachanov law of damage accumulation

$$\frac{d\omega}{dt} = C\left(\frac{\sigma_\Sigma}{1-\omega}\right)^{\nu}, \qquad 0 \leq \omega \leq 1 \tag{1}$$

in which $\omega = \omega(t)$ is a scalar function describing the intensity of damage, σ_Σ denotes the stress ahead of the dominant crack while C and ν are material parameters, leads to the following criterion oa fracture

$$C\int^{t} \sigma_\Sigma^{\nu}[r(t'),t']dt' = \Omega_c \tag{2}$$

$$\sigma_\Sigma[r(t'),t'] = \sigma_\Sigma(a,a') = \frac{K_I(a')}{\sqrt{2\pi\rho_*}}\, Y(\rho_*+a-a')$$

This criterion may be verbalized as follows: "... for a collapse of a material element located at the distance r from a dominant crack tip of half-length 'a' it is necessary that the time integral of ν-th power of the stress at that point attains the critical value $\Omega_C = (1+\nu)^{-1}$."

Decomposing the integral in Eq. (2) into two parts:

$$C\int_0^{t_1} [\sigma_\Sigma^{\nu}]_{\dot{a}=0}\, dt' + C\int_{t_1}^{t} [\sigma_\Sigma^{\nu}]_{\dot{a}\neq 0} \tag{3}$$

and identifying the first part, say Ω_1, with the damage generated during the latent phase of the failure process associated with a buildup of microdefects while the dominant crack remains stationary, we obtain

$$C\int_{t_1}^{t} [\sigma_\Sigma^{\nu}]_{\dot{a}\neq 0}\, dt' = \Omega_c - \Omega_1 \tag{4}$$

The first critical time t_1 is defined as the time at which the material element adjacent to the tip of an initial crack collapses. This occurs when the integral

$$C\int_0^{t_1} \sigma_\Sigma^\nu(a_o)dt', \qquad \sigma_\Sigma(a_o) = \sigma_\Sigma(a_o,a_o) \tag{5}$$

attains the critical value, Ω_c. Since the stress $\sigma_\Sigma(a_o) = K_I(a_o)Y_*/\sqrt{2\pi\rho^*}$ does not depend on time, the integral (5) equals $C\,\sigma_\Sigma^\nu(a_o)t_1$, and therefore

$$t_1 = (\Omega_c/C)\,\frac{(K_I(a_o)Y_*)^\nu}{(2\pi\rho_*)^{\nu/2}}, \qquad Y_* = Y(\rho_*) \tag{6}$$

Applying the fracture criterion (2) to the second phase of the failure process, we can evaluate the amount of damage generated within the time interval $t_1 \le t' \le t$. Here, t coincides with the instant at which the material element located at point P collapses. Now, the stress distribution σ_Σ varies not only with the distance from the current crack tip but, also, it is time dependent, since at any fixed point in space, say P, the stress becomes elevated when the crack front propagates toward this point. The integral

$$\Omega_2(t) = C\int_{t_1}^{t} [\sigma_\Sigma^\nu(P,P')]_{\dot{a}\neq 0}\, dt' \tag{7}$$

may be replaced by an integral with respect to the current crack length

$$\Omega_2(a) = C\int_{a_o}^{a} \sigma_\Sigma^\nu(a,a')\,\frac{da'}{\dot{a}(a')} \tag{8}$$

When the nondimensional crack length $x = a/\rho_*$ and the nondimensional time $\theta = t/t_1$ are introduced, the expression (7) assumes the form

$$\Omega_2(x) = \Omega_c\int_{x_o}^{x} \Phi^\nu(x')[Y(x,x')/Y_*]^\nu\,\frac{dx'}{\dot{x}(x')}, \qquad \dot{x} = dx/d\theta \tag{9}$$

in which the geometry dependent function Φ is defined as the ratio of the stress intensity factor for a current crack and that for the initial crack, i.e.,

$$\Phi(x) = K_I(x)/K_I(x_o) \tag{10}$$

The function $Y(x,x')$ is the familiar nondimensional stress ahead of a moving crack, $Y(\xi)$, in which the distance ξ is expressed as $r/\rho_* = 1+x-x'$, see Fig. 1. We note that the rate of crack propagation $\dot{x}$ appearing in the expression (9) has to satisfy the governing equation (13), or (15).

Next, we eliminate time by replacing it with the time-like variable, a(t). Replacing dt' by $da'/\dot{a}(a')$, we arrive at the following equation of motion for a propagating crack

$$C\int_{a_o}^{a} \sigma_\Sigma^\nu(a,a')\,\frac{da'}{\dot{a}(a')} = \Omega_c - \Omega_1(a) \tag{11}$$

When this equation is differentiated with respect to time, remembering that $d[\ ]/dt = \dot{a}d[\ ]/da$, one arrives at

$$\dot{a}C\int_{a_o}^{a} \sigma_\Sigma^\nu(a,a')[da'/\dot{a}(a')] + C\sigma_\Sigma^\nu(a,a) = -\dot{a}d\Omega_1/da \tag{12}$$

Solving for the rate of crack growth gives

$$\dot{a} = \frac{-C\sigma_\Sigma^\nu(a)}{C\int_{a_o}^{a} \frac{\partial}{\partial a}\left[\sigma_\Sigma^\nu(a,a')\right]\frac{da'}{\dot{a}(a')} + d\Omega_1/da} \tag{13}$$

which is an integro-differential equation defining the unknown function a(t). We note that both terms appearing in the denominator of (7.13)involve not only the stresses $\sigma_\Sigma(a,a')$ and $\sigma_\Sigma(a,a_o)$ but also their gradients.

These gradients are always negative; therefore, the entire expression on the right-hand side of Eq. (13) is positive.

Let us introduce the nondimensional variables

$$x = a/\rho_* \ , \qquad \theta = t/t_1$$
$$x' = a'/\rho_* \ , \qquad x_o = a_o/\rho_* \tag{14}$$

in which ρ_* denotes the characteristic structural length (such as aggregate size), while the critical time t_1 is defined by Eq. (6). Substitution of (14) into Eq. (13) yields the nondimensional version of the governing equation, useful in further numerical studies, namely

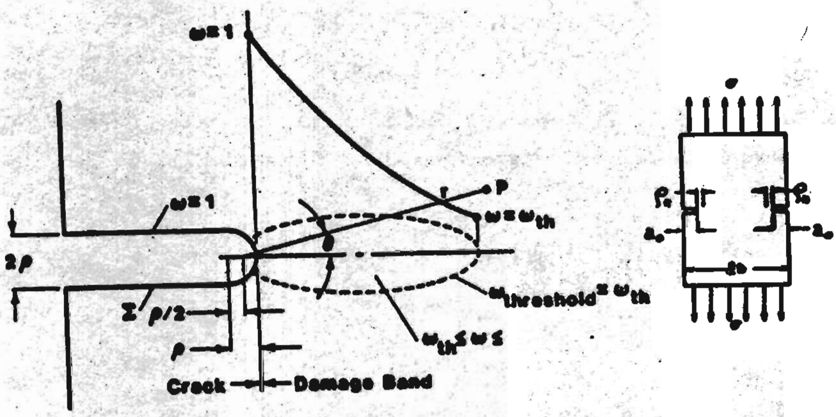

Fig. 1 Specimen geometry (DEN) and details of the damage zone ($\omega_{th} \le \omega \le 1$) adjacent to the crack front. It is assumed that when the crack propagates, the root radius ρ assumes a constant value ρ_o which is regarded as a microstructural characteristic.

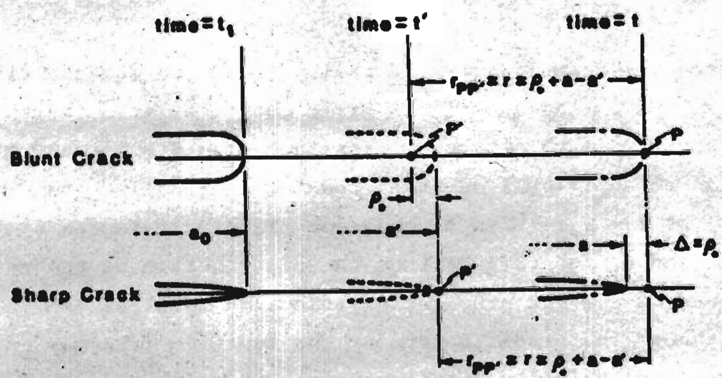

Fig. 2 Location of crack front at

t_1 - onset of crack propagation,

t' - intermediate time,

t - current instant.

Note that the damage accumulation process at the material point P and time t is influenced by the preceding stages of crack growth.

$$\frac{dx}{d\theta} = \frac{\nu^{-1}\Phi^{\nu}(x)}{\int_{x_o}^{x} \Phi^{\nu}(x')F_1(x,x')[dx'/\dot{x}(x')] + F_2(x)} \tag{15}$$

Here, the function $\Phi(x)$ is geometry dependent as indicated by Eq. (10), while the other two auxiliary functions F_1 and F_2 are defined as follows:

$$F_1(x,x') = -\left\{\left(\frac{Y(\xi)}{Y_*}\right)^{\nu-1} \frac{dY(\xi)}{Y_* d\xi}\right\}_{\xi(x,x') = 1+x-x'}$$

$$F_2(x) = F_1(x,x_o) \ , \quad \text{or} \quad \xi(x,x_o) = 1+x-x_o \tag{16}$$

These forms were used in generating the curves shown in Fig. 2.

The function $Y(r)$ is obtained from a continuum mechanics or a finite element solution to the problem of a crack interacting with its own damage zone. Let us suppose that this solution is of a form

$$\sigma_\Sigma(a,a') = \frac{K_I(a')}{\sqrt{2\pi\rho_*}} [Y(r')]_{r' = \rho_* + a - a'} \tag{17}$$

in which K_I denotes the Mode I stress intensity factor associated with crack of half-length a'. The quantity ρ_* is a characteristic length parameter, such as the length of the process zone, see Fig. 1, and it is treated here as an additional material constant. Symbol Y_* denotes the value of $Y(r)$ at $r = \rho_*$. When the current crack half-length a' varies between a_o and a, Eq. (17) defines the distribution of the near-tip stress σ_Σ ahead of a crack propagating gradually into its own damage zone. Although the specific form of the function $Y(r)$ may be difficult to obtain analytically, it is possible to solve the appropriate boundary value problem by any of the known numerical methods. For the simplest assumption of a valid LEFM field, the function $Y(r)$ is given by a familiar $\sqrt{\rho_*/r}$ expression. Curves shown in Fig. 2 were generated by assuming a double-edge notch crack configuration (see Fig. 3) and the $\sqrt{\rho_*/r}$ form for the Y-function. The K-factor for such crack configuration is given by the expression, cf.

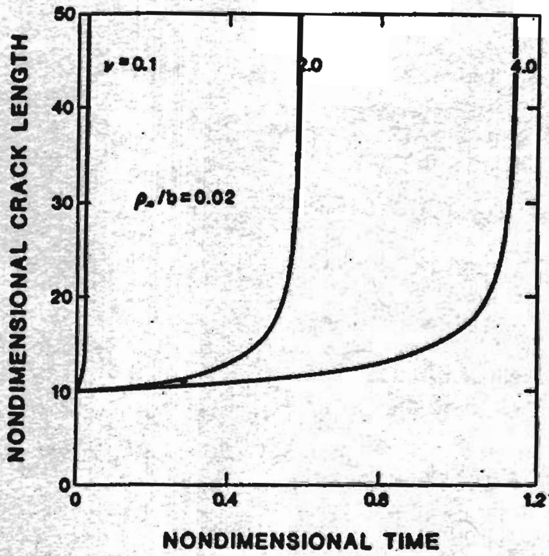

Fig. 3 Three crack growth histories obtained by numerical integration of Eq. (13) for the double-edge-notch specimen configuration when Kachanov's exponent ν equals 0.1, 2.0 and 4.0.

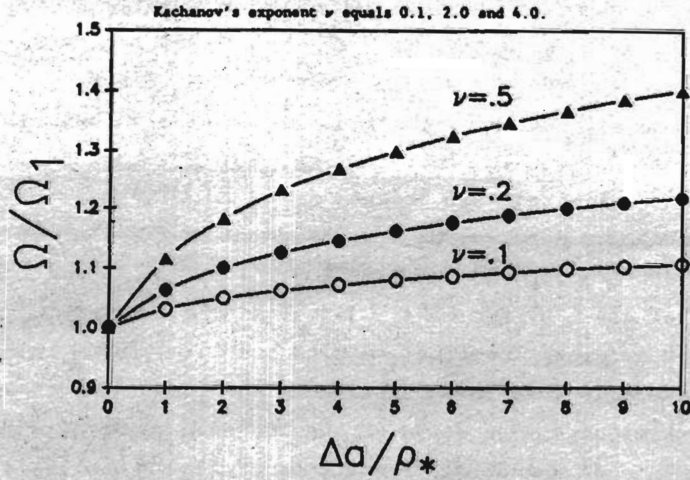

Fig. 4 R-curves in damaging material as predicted by Eqs. (27-28). Total damage Ω at the material point is represented as a sum of damage accumulated in the latent period of fracture development, Ω_1, and the damage Ω_2 generated while the crack front approaches the given control point P. Note a pronounced sensitivity of the R-curve shape to the magnitude of Kachanov's exponent ν.

the K-factors catalog by Tada, Paris and Irwin (1973):

$$K_I(a') = \sigma(t') \sqrt{\pi a'} \left\{(1 - \frac{a'}{b})^{-1/2} [1.122 - 0.561 (\frac{a'}{b}) - 0.205 (\frac{a'}{b})^2 + 0.471 (\frac{a'}{b})^3 - 0.190 (\frac{a'}{b})^4]\right\} \tag{18}$$

Symbol 2b denotes the width of a component, see Fig. 3, while $\sigma(t)$ is the stress applied remotely from the crack site.

<u>Resistance Curve in Damaging Materials</u>

If we denote the left-hand side of Eq. (11) by Ω_2, i.e.,

$$\Omega_2(a_o,a) = C\int_{a_o}^{a} \sigma_\Sigma^\nu(a,a') \frac{da'}{\dot{a}(a')} \tag{19}$$

in which the stress σ_Σ is defined by the second equation in (2), then the quantity so defined becomes a measure of damage associated with crack growth, and, therefore, a measure of an energy absorbed due to microdefects developed ahead of the dominant crack during the <u>second</u> phase of fracture involving stable growth of the macroscopic crack. Therefore, the quantity Ω_2 may be used as a suitable parameter describing material resistance to cracking during the early (stable) stages of fracture development in close analogy to plastic deformation associated with the ductile fracture process, cf. Wnuk (1981).

In order to obtain an R-curve, $\Omega_2 = \Omega_2(a)$, we must first substitute expression (13) for the function $\dot{a} = \dot{a}(a')$, and then evaluate the integral given by Eq. (19). This leads to a rather lengthy numerical procedure. The final results are illustrated in Fig. 4. Of special interest in this investigation is the lowest curve shown in Fig. 4, which corresponds to a value of the Kachanov exponent ν approaching zero. This is a case of fracture occurring in a brittle material. Despite the apparent complicated algebra, an equation describing the R-curve when $\nu \to 0$ can be derived in a closed form. Referring to this case as a "limit case," we proceed as follows.

First, we omit* the integral appearing in the denominator of Eq. (15), which, with $\xi'(x',x_o) = 1+x'-x_o$, gives

*The validity of such simplification was proven numerically.

$$\dot{x}(x') = \frac{\frac{1}{\nu}\Phi^{\nu}(x')}{y^{\nu-1}(1+x'-x_o)y'(1+x'-x_o)} \tag{20}$$

Here, a compact notation is introduced, namely

$$y(\xi) = Y(\xi)/Y_*, \quad \dot{x} = dx/d\theta$$
$$y' = dy/d\xi, \quad \xi = 1+x-x' \tag{21}$$

Next, using expression (9) we rewrite Eq. (19) in a nondimensional form

$$\Omega_2(x_1,x_o) = -\nu\Omega_c\int_{x_o}^{x}\Phi^{\nu}(x')y^{\nu}(1+x-x')y^{\nu-1}(1+x'-x_o)y'(1+x'-x_o)\frac{dx'}{\Phi^{\nu}(x')} \tag{22}$$

It is seen now that the geometry dependent function Φ cancels out. For ν approaching zero, expression (22) can be further simplified, namely

$$[\Omega_2]_{\nu\to 0} = -\nu\Omega_c\int_{x_o}^{x}\frac{y'(1+x'-x_o)}{y(1+x'-x_o)}dx' \tag{23}$$

We will attempt to integrate the latter expression in a closed form. Particularly attractive appears the case of a LEFM field for which

$$y(\xi) = 1/\sqrt{\xi}$$
$$y'(\xi) = -1/(2\xi^{3/2}) \tag{24}$$

Thus, the integral (23) reduces to

$$[\Omega_2]_{\nu\to 0} = \frac{\nu\Omega_c}{2}\int_{x_o}^{x}\frac{dx'}{1+x'-x_o} \tag{25}$$

which is elementary. Now, our R-curve is defined by a logarithmic function

$$[\Omega]_{\nu\to 0} = \Omega_1 + [\Omega_2]_{\nu\to 0} = \frac{\nu\Omega_c}{2}\ln[1-x_o+x] + \Omega_1 \tag{26}$$

This, indeed, is a "universal" geometry independent R-curve, as usually is the case for an R-curve obtained for quasi-brittle solids. Since $\rho_*(x-x_o)$ represents the increment in crack length, Δa, we may rewrite Eq. (26) in this way

$$[\Omega]_{\nu\to 0} = \Omega_1 + \frac{\nu\Omega_c}{2} \ln[1 + \frac{\Delta a}{\rho_*}] \tag{27}$$

The slope of this curve $d\Omega/da$, is given by

$$[\frac{d\Omega}{da}]_{\nu\to 0} = \frac{\nu\Omega_c}{2} \frac{1}{1 + \frac{\Delta a}{\rho_*}} \tag{28}$$

Equations (27) and (28) were used to construct the curves shown in Fig. 4. The R-curves experimentally obtained for various cementitious composites resemble very closely the R-curve shown in Fig. 4, cf. Wnuk et al. (1984). An obvious drawback of all the models developed so far, including the present one, is their deterministic nature. Further research aimed at incorporating stochastic features into the mathematical model of damage accumulation process is underway.

Conclusions

The Kachanov damage accumulation law is modified so that the effective stress which enters in the damage evolution equation reflects the elevation of stress due to the presence of a dominant macro-crack. This, in turn, triggers an interaction process between the damage zone, which precedes the crack front, and the crack itself. Coupling between micro- and macro-defects provides the time dependent mechanism responsible not only for an accelerated damage accumulation but also for the propagation of the macro-crack. Total damage Ω has been partitioned into the damage Ω_1 accumulated at a given material point when the macro-crack is not propagating (during the so-called latent stage of fracture), and Ω_2 which represents the damage build-up due to an increase in stresses and stress gradients observed at a stationary control point while the crack front is approaching. The quantity Ω has been evaluated in a closed form for the case $\Omega_1 >> \Omega_2$, which is the so-called "graceful fracture" case. The opposite situation, when $\Omega_1 << \Omega_2$, corresponding to the "sudden death" type of fracture, requires further numerical studies. The diagrams Ω vs. Δa, shown in Fig. 4, represent the final outcome of this investigation, namely the resistance curves in damaging materials.

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