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TRANSIENT AND STEADY VIBRATION OF HELICAL GEARS

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1. Introduction

In view of modern practical design, the article emphasizes the transient state of motion and considers the load distribution beetwen cooperating teeth. Attention is given to engineering aspects of vibration problem of the gear for solving transient and steady state relative displacement (u,u') of the wheels. Dynamic cooperation of teeth are dealt with and dynamic load distribution along the path and lines of contact changes stiffness and damping force of a system. A nonlinear diferential equation of motion is integrated using the analitical-numerical methods taking into account random pitch errors (relation with kinematic errors is also shown). This paper describes methods to find the density function of the maximum dynamic load along the path of contact during start-stop operations under variable external loads and speeds. If we stop the clock, then we will find a load distribution on tooth like in fig. 1.

Load p(x,y) will produce the stresses σ_H, σ_1, σ_2, σ_{12}, where x,y are variables along the path of contact and width of tooth. There are three important zones A,B,C shown in Fig. 1 (tip, pith and root)-when random densities of max. unit load have been found in these zones next we can apply appropriate theories to find factor of safety or the probability of damage (for example rain flow count method), Serensen et al. (1975) or Kocańda and Szala (1985).

The paper deals with a certain useful method solving dynamically indeterminate contact problems which are applied to find the density of loading of a helical gear, however many other important technical problems (bearings, cluthes, breakes, etc.) can be solved in similar way.

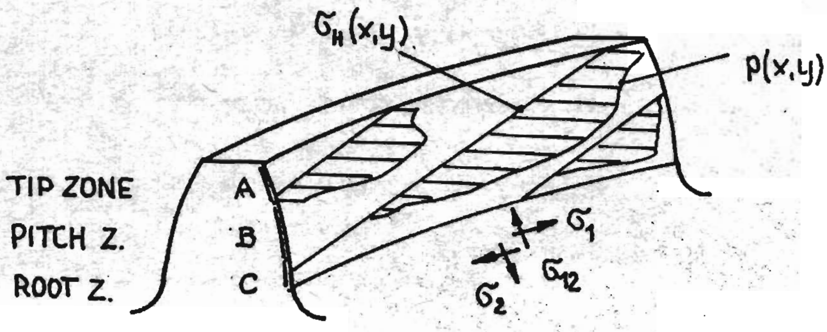

Fig 1. Load distribution on a tooth of helical gear.

2. Static unit load

A static load distribution is important because it makes possibility to compare dynamic solution with the static ones and then to find relation between density of probability of pith errors and kinematic errors of cooperating wheels. External moments acting on wheels (driver and follower) produces external force P. Rectangle of theoretical contact ABCD

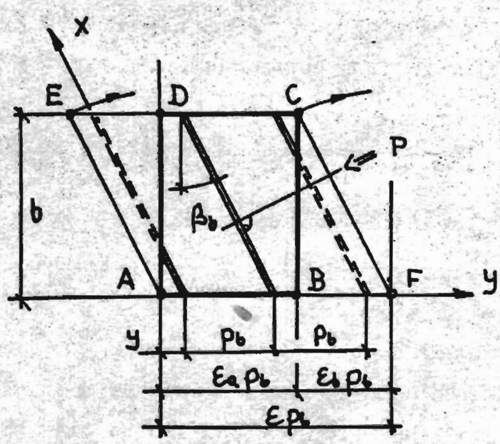

Fig 2. Rectangle of theoretical contact ABCD.

in a plane of pressure is shown fig. 2, where: x,y-variables along lines of contact and path of contact, p_b-base pith, β_b-base angle of h. gear, ε-total contact ratio.

As a consequence, it appears to be possible to replace a rectangle of contact by a parallelogram ABCD (fig. 3).

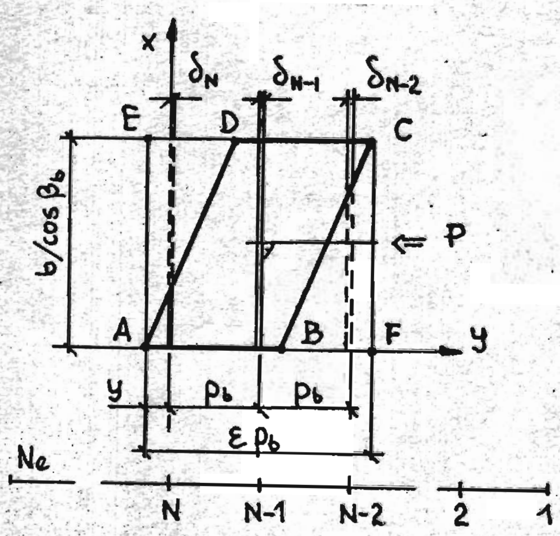

Fig 3. Computation rectangle of contact AECF.

The new rectangle of contact AFCE is more convenient whenwe assume that in triangles ADE and BCF stiffness value is equal 0. In such a way we don't have to control theoretical lines of contact when $0 \le y \le p_b$ and $N=1,2,...N_e$. Every number of pair of teeth N has got his own random pitch error δ_N which can be covered nearly by the Gaussian parameter σ in $<-0.5\ \delta_0, 0.5\ \delta_0>$ internal.

Consequently, a restoring force has a random character. An example of restoring-force-displacement diagram is plotted in fig. 4, either for 3 lines observed inside rectangle of contact (fig. 4a) or for 2 lines respectively (fig. 4b). When $2<\varepsilon<3$ the kinematic error (ortogonal to line contact diretion) of wheels is described by $x_{13}, x_{12}, \sigma_{13}, \sigma_{12}$, notation as in fig.4. In particular cases , it will be found that

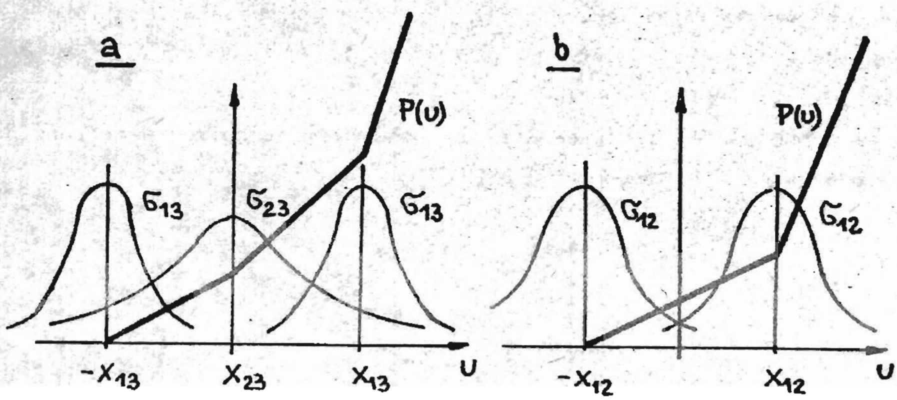

Fig. 4. Gaussian density describing restoring force of teeth when $2<\varepsilon<3$.

1' $1<\varepsilon<2$

$$x_1=A_1\ \sigma\ \Delta\varepsilon\ ;\ \sigma_1=\sigma\sqrt{1-\Delta\varepsilon(1-B_1^2)}\ , \tag{2.1}$$

2' $2<\varepsilon<3$

$$x_1=\sigma\ [A_1+\Delta\varepsilon\ (C_1-A_1)];\ \sigma_1=\sigma\sqrt{B_1^2-\Delta\varepsilon(B_1^2-D_1^2)}\ ,$$

3' $3<\varepsilon<4$

$$x_1=\sigma\ [C_1+\Delta\varepsilon\ (G_1-C_1)];\ \sigma_1=\sigma\sqrt{D_1^2-\Delta\varepsilon(D_1^2-K_1^2)}\ ,$$

where

$\Delta\varepsilon=\varepsilon-\mathrm{INT}\varepsilon$,

$\sigma_k=\sigma_1/\cos\beta_b$-Gaussian parameter of kinematic errors, $A_1=0.567$; $B_1=0.814$; $C_1=-0.862$; $D_1=0.679$; $G_1=-1.04$; $K_1=0.56$ (as an example $\sigma=0.58$; $\varepsilon=2.7$; $\sigma_1=0.41$).

According to equation (2.1) fig. 5 shows comparison of Gaussian parameter of kinematic errors σ_k with Gaussian parameter of pitch errors σ. In order to determine constant parameters $A_1,\ldots K_1$, some simple computer programs were set up, the Author does not know analytical solution of this mathematical problem.

The maximum kinematic error δ_k is determined by the following equation

$$\delta_k = Y\ \delta_o\ /\ \cos\beta_b\ . \tag{2.2}$$

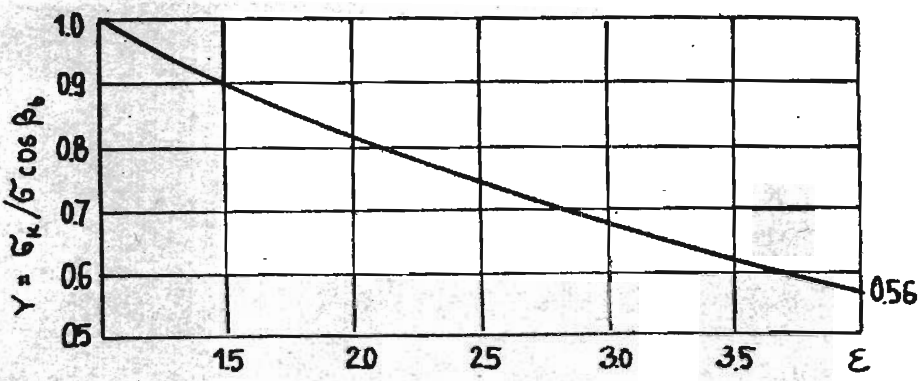

Fig. 5. Relation between kinematic errors σ_k with pitch errors σ .

It therefore seems to be possible to account for maximum pitch error δ_0 using kinematic tests (with external static load $P \cong 0$).

Let us consider the static load and deformation u if $N=1,\ldots N_e$ ($N_e \geq 10^3$) and for every N, $0 \leq y \leq p_b$.

When

$$0 \leq y < \Delta\varepsilon \quad p_b \qquad k = \mathrm{INT}\varepsilon + 1 \quad , \tag{2.3}$$

$$\Delta\varepsilon \quad p_b \leq y \leq p_b \qquad k = \mathrm{INT}\varepsilon \quad ,$$

where k-number of theoretical lines of contact (fig.3). The problem of evaluation of internal loadings is statically indeterminate.

External load for $i=1 \ldots n$ equals

$$p_i = \sum_{l=1} \sum_{j=1} (u - \delta_l)\, C_{j,i+(l-1)n} \quad , \tag{2.4}$$

$$C_{j,i+(l-1)n} = 0 \quad \text{when} \quad u < \delta_l \quad ,$$

where

$\delta_1 = \delta_N$, $\delta_2 = \delta_{N-1}$, $\delta_k = \delta_{N-k}$,

$\Delta x = B/m$, $\Delta y = p_b/n$, $\Delta\varepsilon = q\,\Delta y$,

$C_{i,j}$ -matrices of stiffness in a new rectangle of contact like in fig.3, which is devided into three zones:

$$
\begin{array}{lll}
A & \text{when} & 0 \le y \le \Delta\varepsilon\, p_b \,, \qquad \Delta\varepsilon = \varepsilon - \mathrm{INT}\varepsilon \\
B & \text{when} & \Delta\varepsilon\, p_b < y < (\varepsilon-\Delta\varepsilon)\, p_b \\
C & \text{when} & (\varepsilon-\Delta\varepsilon)\, p_b \le y \le \varepsilon\, p_b
\end{array}
\qquad (2.5)
$$

If $0 \le i \le q$ and $1 \le j \le m$, then unit loading equals

$$
\begin{aligned}
P_A &= \mathrm{MAX}(P_{1,j}) \\
P_C &= \mathrm{MAX}(P_{k,j}) \\
P_B &= \mathrm{MAX}(P_{2,j} \dots P_{k-1,j})
\end{aligned}
\qquad (2.6)
$$

and if $q < i \le n$ then $P_B = \mathrm{MAX}(P_{1,j} \dots P_{k-1,j}\ , P_B)$

$$
\begin{aligned}
P_{i,j} &= (u - \delta_i)\, C_{j,i+(i-1)n} \qquad \text{when} \quad u-\delta_i > 0 \\
P_{i,j} &= 0 \qquad\qquad\qquad\qquad\ \ \text{when} \quad u-\delta_i \le 0
\end{aligned}
$$

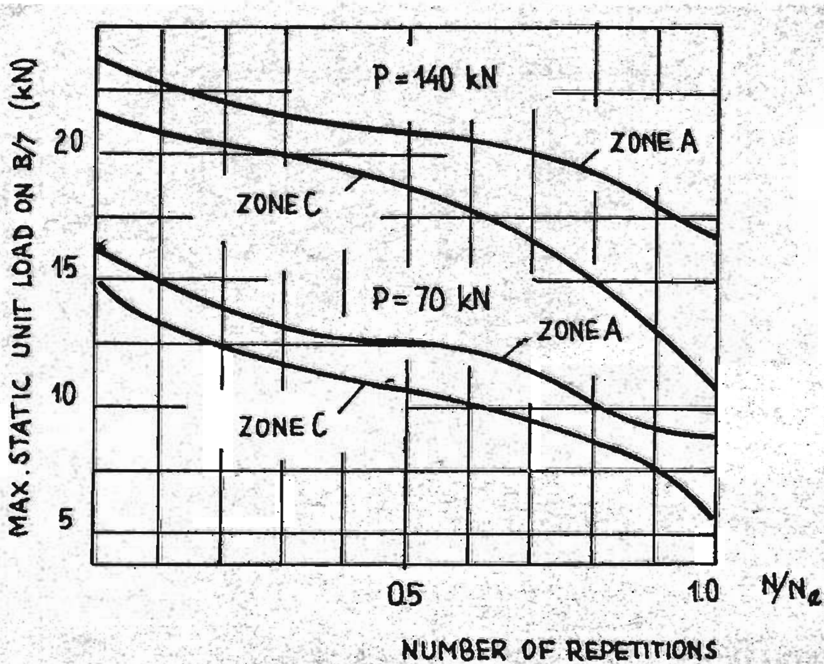

Fig. 6. Maximum static unit load as a function of number repetitions (P-external load).

Random errors change the value of maximum unit load P_A, P_B, P_C for any number N.

Fig. 6 summarizes the results of numerical computation of the gear :

$\varepsilon_a = 1.4$; $\varepsilon_b = 1.2$; $\varepsilon = 2.6$; $n = 5$; $m = 7$, $\delta_o = 10\ \mu m$;
$C_{i,j} = (0.7 \div 2.0)\ kN/\mu m$.

3. Dynamic unit load

Under the assumption stated before the equation of motion is based on Ryś and Stachoń theory (1987):

$$u'' + u' \frac{v' + 10^4 T \sqrt{20C/M}}{v} + u \frac{10^9 C}{Mv^2} = \frac{10^9 (P+D)}{M^2}$$

if $u > \mathrm{MIN}\{\delta_l\}$, $l = 1 \ldots k$, (3.1)

or

$$u'' = \frac{10^9 P}{Mv^2} \qquad \text{if } u \le \mathrm{MIN}\{\delta_l\} ,$$

where

- u - relative dynamic displacement (μm),
- u' - relative velocity (μm/s),
- $P = P(t)$ - external load (kN),
- $v = v(t)$ - velocity of teeth (tooth/s),
- $v' = v'(t)$ - acceleration of teeth (tooth/s^2),
- M - reduced mass (kg),
- T - coefficient of damping (-), $T = (0.2 \div 0.4)$ by Airapietow (1981),

$$\left.\begin{aligned} C &= \sum_{l=1}^{k}\sum_{j=1}^{m} C_{j,i+(l-1)n} \\ D &= \sum_{l=1}^{k}\sum_{j=1}^{m} \delta_l \, C_{j,i+(l-1)n} \end{aligned}\right\} \quad \text{for } i=1\ldots n , \qquad (3.2)$$

and $C_{j,i+(l-1)n} = 0$ if $u \le \delta_l$

Time is connected with i and N:

if $i = n$ then $N = N + 1$ and $n = 1$; $1 \le N \le N_e$,

$v_0 = v(N = 1,\ i = 1)$ - input velocity,

$a_0 = v'(N = 1,\ i = 1)$ - input acceleration,

$$v(N,i) = \sqrt{[v(N)]^2 + 2a_o(i-1)/n} \, ,$$
$$v(N) = \sqrt{[v(N-1)]^2 + 2a_o} \, , \; 1 \le i \le n \, , \tag{3.3}$$

if N = const and i = const interval of time is $\Delta t = 1/n\, v(N,i)$ and equations (3.1) are linear on step

$$\Delta\tau \ll \Delta t \, ,$$
$$u'' + 2\gamma\omega u' + \omega^2 u = S_1 \quad \text{and} \quad u > \min\{\delta_1\} \, , \tag{3.4}$$
$$u'' = S_2 \qquad\qquad \text{and} \quad u \le \min\{\delta_1\} \, .$$

Thus the whole calculation procedure is based on testing of analytical solution and restrictions on time $0 \le t \le \Delta t$. Solution u(t) is plotted for $N=1,2,\ldots,N_o$ and maximum unit load $P_A;P_B;P_C$ could be found with (2.6) on every step N, i.

The numerical examples obtained thus are presented below

DYNAMIC LOAD OF H.GEARS

max.ext.load(kN) =95.0;	min ext.load (kN) = 35.0
max.pitch error (mm) = 0.010;	reduced mass (kg) = 100.0
min.velocity (tooth/s) = 1;	max.velocity (tooth/s) =50
coef.of damping (-) =0.4;	input accel.(tooth/s^2) = 20

max.unit load

transient vibr.in zones			steady vibr.in zones		
A	B	C	A	B	C
34.851	50.947	39.851	16.854	23.396	19.507
20.352	31.224	26.164	11.770	16.690	14.851
15.182	21.987	18.820	7.170	13.036	10.892
9.602	14.942	13.785	3.170	8.340	6.931
3.743	8.597	6.968	0.929	3.373	3.935
0.076	1.897	2.035	0.000	0.535	0.841
0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000

Fig.7.shows the results, namely dynamic unit loads for the data: $\varepsilon = 2.6$; $n=5$; $m=7$; $\delta_o=10$ [μm]; $M=100$ [kg]; $T=0.4$; $C_{i,j}=(0.7\div2.0)$ [kN/μm]; $v=(1\div50)$[tooth/s] and three variants of start conditions are considered.

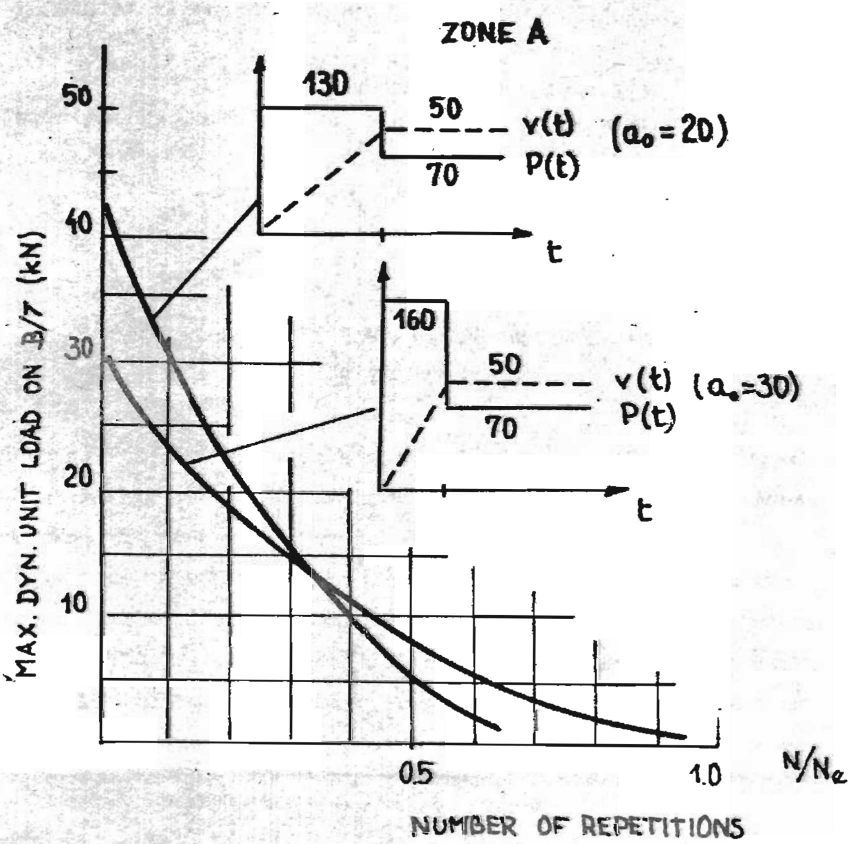

Fig. 7.Dynamic unit load as a function of number repetitions

The observed tendency is a direct effect of dynamic shocks and tooth separation if a_0=20 and smooth cooperation of teeth if a_0=10 or 30 tooth/s^2.

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Summary

NIEUSTALONE I USTALONE DRGANIA W PRZEKŁADNI ZĘBATEJ O ZĘBACH SKOŚNYCH

W pracy analizowano statyczne i dynamiczne obciążenie zębów przekładni z uwzględnieniem losowych odchyłek kinematycznych, zajęto się głównie problemem rozruchu z racji występowania wówczas zewnętrznych przeciążeń dynamicznych. Zaprezentowana metoda pozwala na określenie maksymalnych obciążeń na jednostkę szerokości koła dla określonych stref współpracy w ujęciu losowym. Praca stwarza możliwość obliczania współczynników bezpieczeństwa lub prawdopodobieństwa uszkodzeń (np. złomu zmęczeniowego, uszkodzenia powierzchni itp.) dla znanych warunków obciążenia przekładni.