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RESIDUAL STRESS ANALYSIS IN RAILROAD CAR WHEELS WORKING IN SERVICE CONDITIONS

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1. Introduction

Discussed is a concept of the analysis of residual stresses arising in solid railroad car wheels due to both service conditions and initial self-equilibrated stresses resulting from manufacturing.

In service conditions railroad car wheels are subjected, first of all, to cyclic contact loadings and to inertia forces caused by revolution. Moreover, some thermal effects may appear due to non-cyclic temperature changes in case of braking. Significant, though relatively slow, changes of temperature cause thermal stresses, essential variation of material constants, and in extreme cases even structural changes of the material.

Analysis of residual stresses in a railroad car wheel is usually done by experimental methods. A destructive technique is mainly applied then. A wheel is gradually cut along its radius and the width of the saw-cut opening is measured. That way we obtain the data used in further analysis [Lin and Huang (1988), Tong (1988)].

Theoretical analysis of residual stresses might be done by using one of the following approaches:

- an incremental one, when the whole loading process in subsequent cycles is traced until shakedown or destruction occurs;
- shakedown approach, when only thefinalstateofa wheel is considered;

The first approach yields a 3D, non-linear, time dependent boundary-value problem. Thus, this approach is practically non acceptable, even for very powerful mainframe computers .

That is why in the present paper the second approach has been chosen. An attempt has been made [see Orkisz (1989)] to adapt mechanical and numerical models recently proposed by the author for residual stress analysis at shakedown in a railroad rail [Orkisz al. et (1986,1989)]. That model, however, could not be directly applied here because of the presence of non-cyclic, slowly varied thermal effects, resulting in a car wheel from braking. These effects constitue the main difference between residual stress mechanism in a railroad rail and in a railroad wheel.

2. Mechanical model proposed

Following is the shakedown approach worked out for rails [Orkisz al. et (1989)]. In Tab 1. compared are some more important features of a real wheel, and its mechanical model is considered. Despite simplications made, one may describe, by means of this model, the basic effects influencing residual stresses with some secondary effects neglected.

We consider an elastic, perfectly plastic body subjected to cyclic loadings by surface tractions applied on a surface S_p

$$\hat{T}_i(\mathbf{x},t) = \hat{T}_i(\mathbf{x},t+nt_c) \quad , \tag{2.1}$$

body forces

$$\hat{F}_i(\mathbf{x},t) = \hat{F}_i(\mathbf{x},t+nt_c) \quad , \tag{2.2}$$

in the volume V , and displacements

$$\hat{u}_i(\mathbf{x},t) = \hat{u}_i(\mathbf{x},t+nt_c) \quad , \tag{2.3}$$

on a surface S_u. Here n denotes the number of cycles t-time, t_c-period of one cycle, i=1,2,3.

True contact loadings are simulated by the corresponding surface tractions $\hat{T}_i$. Moreover the body may be subjected to a slowly varying temperature field $\theta(\mathbf{x},t)_{|t=\tau}$, where τ is considered as a parameter. It is also assumed that after a number of loading cycles material properties are strain amplitude, and cycle independent, and are described by the values of material constants fixed for a given temperature. Such assumption is justified by the fact that stress variation due to temperature is much slower than due to wheel revolution. Material phase changes that may occur due to braking are neglected .

Finally, the problem is considered as three dimensional and parametr dependent (time) one. Rotational symmetry is assumed for residual stress distribution.

TABLE 1. Comparison of real a car wheel and its mechanical model.

Real wheel	Model
LOADING	
-dynamic	-quasistatic
-repetitive, non-cyclic	-cyclic
-rolling contact in the elastic-plastic range	-simulated Hertzian tractions
-stochastic	-deterministic
-non-steady high temperature	-temporary steady high temperature
MATERIAL	
-elastic-plastic, kinematic hardening strain amplitude and number of cycles dependent	-elastic perfectly plastic
-fatigue	-no
-cracks	-no
-initial non zero residual stresses	-yes
-structural changes resulting from high temperature	-no
-relaxation of stresses	-no

In order to analyse residual stresses at the full range of temperature variation, a sequence of problems should be solved for subseqent time steps. Each solution corresponds to different but temporarily fixed distribution of temperature. Residual stresses obtained this way in a time step t_{n-1} constitue initial residual stresses for a time step t_n.

Thus, taking into account the assumptions mentioned above, as well as the differences and analogies between work of a car wheel and a rail, one may propose the following way of analysis of residual stresses in a wheel, based on the approach already worked out for rails:

find

$$\min_{\underline{\sigma}^s} W(\underline{\sigma}^s - \underline{\sigma}^{Ro}) \quad , \tag{2.4}$$

satisfying equilibrium equations inside the body

$$\partial \underline{\sigma}^s = \underline{0} \qquad \text{in} V, \tag{2.5}$$

and static boundary conditions

$$\underline{\nu}^t \underline{\sigma}^s = 0 \qquad \text{on} S_\sigma \quad , \tag{2.6}$$

as well as yield conditions for the total stresses

$$\Phi\,(\,\sigma^E + \sigma^\vartheta + \sigma^s\,) \le 0 \qquad \text{in} V. \tag{2.7}$$

Here

$$W(\underline{\sigma}) = \frac{1}{2}\int_V \underline{\sigma}^t \underline{C}\ \underline{\sigma} d\underline{V} \quad , \tag{2.8}$$

denotes the total complementary energy of the body at stresses $\underline{\sigma}$. Due to rotational symmetry of residual stresses

$$W(\underline{\sigma}) = \pi \int \underline{\sigma}^t \underline{C}\ \underline{\sigma}\ r\ dA\ . \tag{2.9}$$

The following denotations have been made in formulas (2.4)-(2.9)

$\underline{\sigma}^{Ro} \doteq \underline{\sigma}^{Ro}(\underline{x})$ - initial residual stresses in a wheel,

$\underline{\sigma}^s = \underline{\sigma}^s(\underline{x})$ - self-equilibriated stresses,

$\underline{\sigma}^E = \underline{\sigma}^E(\underline{x}, t)$ - purely elastic solution to the given boundary value problem,takingintoaccount inertia forcesand wheel railrail contact effects with thermal effects excluded,

$\sigma^{\vartheta} = \sigma^{\vartheta}(x,\tau)$ - thermal stresses in a purely elastic wheel, caused by heat applied to the rim due to braking,

$\underline{\nu}$ - unit vector normal to the body,

$\phi = 0$ - yield surface,

A - wheel cros-section,

$\underline{c}$ - compliance matrix,

$\underline{x}$, t - space and time variables,

τ - a fixed time.

Stress field $\underline{\sigma}^{r}$ obtained from the above defined optimization problem provide an upper bound for the true residual stresses $\underline{\sigma}^{r}$ in the sense of bounding of the complementary work

$$W(\underline{\sigma}^{R} - \underline{\sigma}^{Ro}) \le W(\underline{\sigma}^{r} - \underline{\sigma}^{Ro}) \ . \qquad (2.10)$$

It is worth stressing, that if the work done on plastic deformations $\underline{\varepsilon}^{P}$ along a given loading path $c(\underline{\sigma})$ vanishes

$$\int_{c(\underline{\sigma})} (\underline{\varepsilon}^{P})^{t} \, d\underline{\sigma} = 0 \ , \qquad (2.11)$$

the Haar-Karman principle holds, and we obtain in this way the exact so lution, i.e., $\underline{\sigma}^{R} = \underline{\sigma}^{r}$. Otherwise, we have to assume, that we deal only with a bounding solution.

Though there are no solutions for a wheel obtained in this way as yet, it may be noticed that a variety of theoretical and numerical tests was done in order to verify and validate the mechanical model formulated above. Results of all these tests obtained, e.g., for a thick-walled cylindrical tube subjected to various combinations of symmetrical loads, show that within high numerical precision $\underline{\sigma}^{R} = \underline{\sigma}^{r}$, i.e., practically the exact solution was found in this way.

It is obvious from the formulation (2.4)-(2.8) that prior to any residual stress analysis the stresses $\underline{\sigma}^{E}$, $\underline{\sigma}^{\vartheta}$ and $\underline{\sigma}^{Ro}$ have to be found [Burczyński (1979)].

Initial residual stresses $\underline{\sigma}^{RO}$, existing in a wheel prior to a given cyclic loading program, may arise as a result of manufacturing, previous loading programs, material defects etc. They are assumed to be known either from calculations or from measurments (e.g saw-cut [Lin and Huang (1988), Tong (1988)]).

Nowadays analysis of stresses $\underline{\sigma}^{E}$ and $\underline{\sigma}^{\vartheta}$ presents a routine boundary value problem for discrete methods like FEM, FDM and BEM. In the case of the stress field $\underline{\sigma}^{E}$ we have, in fact, an elastic wheel-rail contact problem. However, a useful and much simpler result may be obtained when using a solution for the unit force and integrating it over a simulated Hertzian contact stress distribution.

Stress field $\underline{\sigma}^{\vartheta}$ again represents a solution of a purely elastic wheel, this time however, for heat applied to the rim due to braking. Both the thermal loading applied and resulting stresses $\underline{\sigma}^{\vartheta}$ are time dependent, though they are not cyclic.

We deal here with a linear thermoelastic problem that may be handled in a routine way. Moreover, if it is assumed that thermal and mechanical effects are decoupled, we may obtain a time dependent temperature distribution in the wheel solving at first an appropriate non-stationary heat conduction problem. In a simplified approach this temperature distribution in the wheel may be assumed in advance, e.g., as a linear or parabolic one [e.g., Lin and Huang (1988)].

Once the temperature distribution in a wheel is known, the resulting stresses $\underline{\sigma}^{\vartheta}$ may be found by means of routine discrete analysis.

It has to be stressed that since we deal with significantly high temperatures, changes of the material properties, especially yield stress $\sigma_{YIELD}(\vartheta)$ and Young modulus $E(\vartheta)$, have to be taken into account.

3. Numerical model

The mechanical model (2.4)-(2.8) proposed for the analysis of residual stresses in a railroad car wheel is similar to the one considered earlier for the railroad rails [Orkisz et al. (1989)]. Consequently, a numerical analysis may be done then in a similar way, e.q., by using the Finite

Element, Finite Difference or Boundary Element methods. There is, however, an essential qualitative difference between these two models. Due to variable temperature effects, all stresses, material constants, and quantities like energy, essentially depend on time, considered here as a parameter.

Thus, in order to take into account the full range of temperature changes resulting from braking, a sequence of optimization problems (2.4) (2.8) corresponding to subsequent time parameter values $t = \tau_0, \tau_1, \ldots, \tau_k, \ldots$ has to be considered, instead of a solution of only one problem, like in the case of a rail. Each time the relevant distributions of thermal stresses $\underline{\sigma}^{\vartheta}$ have to be found, and initial residual stresses $\underline{\sigma}^{Ro}(\tau_{k+1}) = \underline{\sigma}^{R}(\tau_k)$ are assumed. We select also values of the material constants (yield stress included) corresponding to the temperature considered.

The proposed solution algorithm and strategy for each optimization problem is similar to the one defined and tested before [Orkisz et al. (1989)]. It consists of three parts:

- discretization by means of the FE, FD or BE methods. It yields the following optimization problem:

$$\min_{\underline{x}} \ \frac{1}{2}\, \underline{x}^t \underline{A}\, \underline{x} + \underline{x}^t B \ , \qquad \underline{A} = \underline{A}^T \ , \quad x \in R^n \ , \tag{3.1}$$

- at linear equality constraints

$$\underline{c}_i^t\, \underline{x} = 0 \ , \qquad i = 1, 2, \ldots, m < n \ , \tag{3.2}$$

- and nonlinear inequality constraints in the case of von Mises yield criterion

$$\frac{1}{2}\, \underline{x}^t \underline{D}_j \underline{x} + \underline{x}^t \underline{E}_j + \underline{F}_j < 0, \ \underline{D} = \underline{D}^t, \ j = 1, 2, \ldots, k. \tag{3.3}$$

The particular form of the matrices depends on the discretization method used

- Reduction of the above given optimization problem, due to elimination of equality constraints, and elimination of decision variables, representing the elastic part of the wheel cross-section. That way the final optimization analysis is limited only to that part of the whole wheel, where plastic effects occured.

- Solution of the reduced optimization problem. The Feasible Directions Method proved to be effective here, both in its classical form [Vanderplaats (1985)], and especially in a new original version [Orkisz (1985)].

4. Final remarks

Presented is a concept (mechanical and numerical models) of evaluation of residual stresses that arise in a railroad car wheel in service conditions. The approach proposed here is based on the repetitive use of the formulation worked out and succesfully tested for railroad rails. Thus one may expect that practical application of that concept will be effective and provide us with a reasonable solution to the problem considered.

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Summary

ANALIZA NAPRĘŻEŃ RESZTKOWYCH WYWOŁANYCH W KOŁACH POJAZDÓW SZYNOWYCH WARUNKAMI EKSPLOATACJI

Przedmiotem pracy jest koncepcja analizy naprężeń resztkowych powstałych w monoblokowych kołach taboru kolejowego w wyniku eksploatacji oraz wstępnych naprężeń własnych, które pojawiły się w procesie wytwarzania koła. Zakłada się, że obciążenia kontaktowe oraz siły bezwładności są cyklicznie szyb ko zmienne, natomiast naprężenia termiczne wywołane tarciem w czasie hamowania koła nie są cykliczne i zmieniają się relatywnie wolno. Zagadnienie rozważa się w ramach teorii plastycznego przystosowania się materiału do obciążeń cyklicznie zmiennych. Koncepcja rozwiązania problemu opiera się na wielokrotnym wykorzystaniu sformułowania jakie ostatnio autor zaproponował dla analizy naprężeń resztkowych w ciałach idealnie sprężysto-plastycznych cyklicznie obciążonych. Ostatecznie obliczanie naprężeń resztkowych w kołach pojazdu szynowego sprowadza się do rozwiązania ciągu zadań nieliniowej optymalizacji w obszarze ograniczonym. Każde z nich odpowiada kolejnemu, chwilowo ustalonemu rozkładowi temperatury w kole. W ciągu tych zadań są analizowane kolejne stany naprężeń resztkowych powstających w kole od początku hamowania.