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CREEP BUCKLING OF STEEL COLUMNS IN FIRE TEMPERATURES

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1. Introduction

Instability of compression members in elevated temperatures used to be investigated because of special industrial problems. First results on creep buckling were published by Freudenthal (1946), Rshanitsin (1946) and Ross (1946). Soon after those Życzkowski (1959) in Poland presented his first paper in this field.

But fire cases of buildings have been treated in a more practical manner. Conventional buckling curves have been recommended with some quantitative modifications for steel temperatures ν over 70 or 100°C. The Euler formula has been applied for the critical force P_{cr} in the elastic range of buckling with the elasticity modulus E dependent on temperature ν,

$$P_{cr} = (\pi/\lambda)^2 AE(\nu) \ , \tag{1}$$

where λ - column slenderness ratio,
A - cross-sectional area.

Uniform buckling coefficients $\phi(\lambda)$ have been defined in elasto-plastic range for all kinds of steel thanks to the concept of relative slenderness ratio $\bar{\lambda}$ (Murzewski, 1972). The coefficients $\phi(\bar{\lambda})$ are applied also to columns in elevated temperatures for a modified value

$$\bar{\lambda} = \sqrt{P_{pl}/P_{cr}} = (\lambda/\pi)\sqrt{R(\nu)/E(\nu)} \ , \tag{2}$$

where $P_{pl} = AR(\nu)$ - plastic resistance of the column in fire.

The initial modulus of elasticity $E(\nu) = d\sigma/d\varepsilon$ is decreasing in elevated temperatures. So is the yield strength $R(\nu)$. The proportion $R(\nu)/E(\nu)$

is variable. Separate reduction factors are defined:

$$\eta_E = E(\nu)/E_0 \quad , \qquad \eta_R = R(\nu)/R_0 \quad , \tag{3}$$

where E_0, R_0 - the Young's modulus and yield point at the room temperature $\nu = 20^\circ C$.

The factors η_E and η_R are estimated by many authors (Kosiorek et al., 1988; Murzewski and Domański 1989 etc.). Exponential functions of test temperatures are taken among athers to interpolate the experimental points. The boundary values are as follows for the temperature range of adequatly insulated structural members during fire

$$\begin{aligned} \eta_E &= 0.9 \quad , \ \eta_R = 0.8 \quad \text{at the creep limit } 320^\circ C, \\ \eta_E &= 0.5 \quad , \ \eta_R = 0.4 \quad \text{at temperatures } \nu = 600^\circ C. \end{aligned} \tag{4}$$

Stress-strain curves of steel in elevated temperatures are smooth for any kind of steel. The conventional yield strength $R = R_{0.2}$ is defined for a specified plastic strain 0.2%.

The Ramberg-Osgood formula defines the instantaneous elasto - plastic strain ε_i

$$\varepsilon_i = \sigma/E + 0.002\ (\sigma/R_{0.2})^m \quad , \tag{5}$$

$m = 2$ for temperatures ~ $600^\circ C$ (Fig. 1) is taken after an analysis of empirical curves of the European Convention of Constructional Steel (ECCS, 1982).

The secant coefficient $1/E_s = \varepsilon/\sigma$ increases with the tempe- rature ν,

$$\phi = E_o/E_s = 1/\eta_E + 0.002(\phi/\bar{\gamma}\varepsilon_o)^2/\eta_R^2 = 1/\eta_E + \eta_o/\eta_R^2 \ . \tag{6}$$

if the stress σ has reached its design limit $\phi R_o/\bar{\gamma}$,

where ϕ - column buckling factor,

$\bar{\gamma}$ - central resistance factor ; i.e. the proportion between mean yield strength R and the design strength R for tensile members.

Let take the frequent value $\eta_o = 0.9$. It is strict in the case of medium slender columns of a low carbon steel, when $\phi = 0,85$, $\bar{\gamma} = 1.33$ and

$\varepsilon_o = R_o/E_o = 0.0014$. The magnification factor ϕ is interpolated by an exponential formula which satisfies the boundery conditions (4):

$$\psi(\nu) = 1/\eta_E + 0.9/\eta_R^2 \approx 2.50 \quad \exp[(\nu \; - \; 320)/225]. \tag{7}$$

The Norton-Bailey law defines the creep strain rate

$$\dot{\varepsilon}_c = \partial\varepsilon/\partial t = k\sigma^n = (\sigma/R_c)^n. \tag{8}$$

Ponomariev (1958) gives some values of the Norton-Bailey's parameters, n and k $[cm^{2n}/kG^n h]$.

The values n = 3 and $k = 2 * 10^{-13} cm^6/kG^3 h$ are taken for a low-carbon steel and the temperature range under consideration (Fig.1). Hours [h] are replaced by minutes [min], i.e. $\dot{\varepsilon}$ $[min^{-1}]$ instead of $\dot{\varepsilon}$ $[h^{-1}]$ and the parameter R_e is expressed in SI units

$$R_e = \sqrt{60/2\ 10^{-13}/9.8} \approx 6000 \text{ MPa min}^{1/3} . \tag{9}$$

A question arrises how the temperature ν augments the deflections and precipitates the failure of a column and whether a critical temperature ν_{cr} can occur in the temperature range under consideration for normally dimensioned columns.

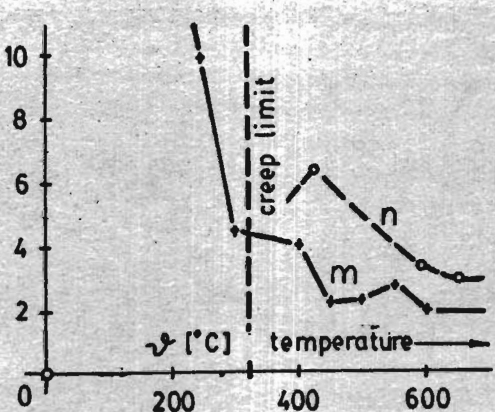

Fig.1. Eksponents m.n dependent on the temperature of steel.

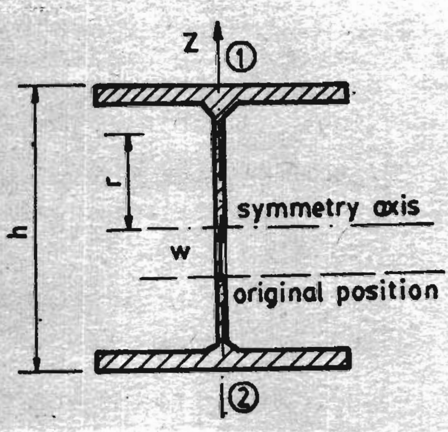

Fig.2. Displaced section of a column subject to creep buckling.

2. A solution

Let the temperature ν [°C] increase in time t[min] according to a formula specified by the International Organization for Standarization (ISO, 1975) for fire laboratory tests

$$\nu(t) = 20 + 345\ \lg(8t + 1) = 20 + 150\ \ln(8t + 1). \tag{10}$$

where 8 min^{-1}, 20°C, 345°C - the specificd constants. The geometric imperfection of the column is defined as an initial deflection of its axis

$$w(x,\ 0) = w_o \sin(\pi x/L)\ , \tag{11}$$

$w_o = \alpha\ r(\bar{\lambda} - 0,2)$ value recommended by the European Standards (Eurocode No.3, 1989) with various α parameters for particular profiles.

Second-order moments amplify the stress σ_1 in the upper flange as well as σ_2 - in the lower flange of an I profile (Fig.2)

$$\sigma_{1,2} = (P/A)[1 \pm w(x,t)/r \tag{12}$$

where r = 1.05 W/A - core of a wide-flange I section in elasto - plastic bending.

The elasto-plastic magnification factor (7) is now an algebraic function of time

$$\psi(t) = 0.66(8t + 1)^{2/3}. \tag{13}$$

The creep buckling equation (Życzkowski et al., 1988 p.322) is extended to the variable secant modulus, $E_s = E_o/\psi$ and it is applied to the specified ε-σ law (8)

$$-h\dot{w}'' = \frac{2P}{AE_o}\left[\dot{\psi}\ \frac{w}{r} + \psi\ \frac{\dot{w}}{r}\right] + \left[\frac{P}{AR_c}\right]^3 \left[3\ \frac{w}{r} + \left[\frac{w}{r}\right]^3\right]. \tag{14}$$

The equation (19) is linearized for small deflections the, solution.

Then we have the

$$w(x,\ t) = (w_t + w_o)\ \sin(\pi x/L), \qquad 0 < x < L\ . \tag{15}$$

The equation (14) becomes an ordinary differential equation with respect to the time t after getting rid of the term $\sin(\pi x/L)$ from its both sides:

$$\frac{\pi^2 hr}{L^2}\ \dot{w}_t = \frac{2P}{AE_o}\left[\dot{\psi}(w_t + w_o) + \psi\ \dot{w}_f\right] + 3\left[\frac{P}{AR_c}\right]^3 (w_t + w_o). \tag{16}$$

It is rearranged as follows

$$\frac{dw_t}{w_t + w_o} = \frac{\dot{\psi}\ P/P_{cr} + (3L^2/\pi^2 hr)(P/P_c)^3}{1 - \psi\ P/P_{cr}}\ dt\ , \tag{17}$$

where $P_{cr} = \pi^2 hrAE_o/2L^2 = 1.05\ \pi^2 E_o I/L^2$, $P_c = AR_c$, and it is integrated

$$\ln\frac{w_t + w_o}{w_o} = \ln\frac{1}{1 - \psi\ P/P_{cr}} + \\ + c\left[\ln\left(\frac{1 + \sqrt{\psi P/P_{cr}}}{1 - \sqrt{\psi\ P/P_{cr}}}\ \frac{1 - \sqrt{2.5P/P_{cr}}}{1 + \sqrt{2.5P/P_{cr}}}\right) - 2\sqrt{\frac{P}{P_{cr}}}\left[\sqrt{\psi} - \sqrt{2.5}\right]\right], \tag{18}$$

where

$c = (3L^2/\pi^2 hr)(P/P_c)^3 (3/16)(P_{cr}/0,66P)^{3/2} = (\lambda/3.067)^2(\sqrt{PP_{cr}}/P_c)^3$.

The mid-height deflection of the column is formulated briefly as follows

$$w = w_o \alpha_a \alpha_c\ , \tag{19}$$

where the amplification factor due to 2-nd order effects

$$\alpha_a = 1/(1 - \psi\ P/P_{cr})\ , \tag{20}$$

and the creep factor

$$\alpha_c = \left[\frac{(1 + \sqrt{\psi P/P_{cr}})}{(1 + 1.58\sqrt{P/P_{cr}})}\ \frac{(1 - 1.58\sqrt{P/P_{cr}})}{(1 - \sqrt{\psi\ P/P_{cr}})}\right]^c \exp\left[-2c\sqrt{\frac{P}{P_{cr}}}\left[\sqrt{\psi} - 1.58\right]\right], \tag{21}$$

for temperatures $\nu > 320°C$, i.e. $t > 0.8$min and $\psi = 0.66(8t+1)^{2/3} > 2.66$.

The identity of equations (18) and (19) can be checked by taking logarithmics of w, α_a and α_e and replacing 1.58 by $\sqrt{2.5}$ and w by $w_t + w_o$.

A collapse of the column occurs when $\alpha_a \longrightarrow \infty$. So we derive the stability condition from (20)

$$1 - 0.75\,\Phi\psi\bar{\lambda}^2 = 0 \quad \longrightarrow \quad \Phi\psi\bar{\lambda}^2 = 1.33 \tag{22}$$

and we get the critical temperature ν_{cr} from formula (7)

$$2.50\,\Phi\bar{\lambda}^2\exp[(\vartheta-320)/225] < 1.33 \longrightarrow \vartheta < 180-225\ln(\Phi\bar{\lambda}) \tag{23}$$

Some numerical calculations have been performed for welded column of low-carbon steel subject to a design stress $\sigma_d = R_o\phi/\bar{\gamma}$

$$P/P_{cr} = \frac{\phi\ \ 285}{\bar{\gamma}\ 200000}\left[\frac{\lambda}{\pi}\right]^2 = \phi\,\bar{\lambda}^2/1.33 = 0.75\,\phi\bar{\lambda}^2\ , \tag{24}$$

$$c = \left[\frac{\lambda}{3.07}\right]^2\left[\frac{\pi\ \sqrt{\phi}\ 285\cdot 200000}{\lambda\ \sqrt{1.33}\ 6800}\right]^3 = 2.976\ \sqrt{\phi}/\lambda = 0.36\sqrt{\phi}/\bar{\lambda}\ , \tag{25}$$

$$\alpha_a = 1/(1 - 0.75\phi\ \psi\ \bar{\lambda}^2)\ , \tag{26}$$

$$\alpha_c = \left[\frac{(1 + 0.866\sqrt{\phi\psi}\ \bar{\lambda})(1 - 1.370\ \sqrt{\phi}\ \bar{\lambda})}{(1 + 1.370\sqrt{\phi}\ \ \bar{\lambda})(1 - 0.866\ \sqrt{\phi\psi}\ \bar{\lambda})}\right]^{0.36\sqrt{\phi/\lambda}} * \exp\left[-\phi\frac{\sqrt{\psi} - 1.582}{16,14}\right]\ . \tag{27}$$

The Airton-Perry equation is taken from the Eurocode No.3 (1989).

$$\phi = 0.5(1 + w_o/r + \bar{\lambda}^2)/\bar{\lambda}^2 - 0.5\sqrt{\left[1 + w_o/r + \bar{\lambda}^2\right]^2 - 4\bar{\lambda}^2}/\bar{\lambda}^2 \tag{28}$$

$$w_o/r = 0.34(\bar{\lambda} - 0.2)\ \ , \quad \bar{\lambda} = \lambda/83.22\ . \tag{29}$$

The results of calculations are presented in Table 1.

TABLE 1. Amplification factors α_a creep factors α_c and the deflections w

$\bar{\lambda} = \lambda/\lambda_p$	0.2	0.4	0.6	0.8
w_o/r	0	0.068	0.136	0.204
$\phi(\bar{\lambda})$	1.000	0.926	0.837	0.724
θ_{cr}	904	610	450	356
t = 0.8 min	α_a α_c	α_a α_c	α_a α_c	α_a α_c
ν = 320°C	1.08 1.00	1.39 1.00	2.30 1.00	7.73 1.00
ψ = 2.50	w/r = 0	w/r=0.094	w/r=0.313	w/r=1.580
t = 1.5 min	α_a α_c	α_a α_c	α_a α_c	
ν = 400°C	1.12 1.21	1.66 1.29	5.19 1.68	
ψ = 3.57	w/r = 0	w/r=0.145	w/r=1.187	
t = 2.94 min	α_a α_c	α_a α_c		
ν = 500°C	1.20 1.65	2.63 2.18		
ψ = 5.57	w/r = 0	w/r=0.390		
t = 5.85 min	α_a α_c	α_a α_c		
ν = 600°C	1.35 2.54	29.2 20.3		
ψ = 8.69	w/r = 0	w/r=40.5		
t= 11.51 min	α_a α_c			
φ = 700°C	1.69 4.82			
ψ = 13.55	w/r = 0			

3. Conclusion

The creep effect should be remarkable even for short times of heating in fire resistance experiments. The temperature rate in real fire is much slower, because the steel columns in building are insulated . The creep effect is stronger then. These are new conclusions since the crep effects in structural analysis of fire situations have been usually neglected.

The quantitative evaluation of the fire resistance of columns is given as a preliminary and approximate result and their statistical characteristics are needed to make the results of analysis more reliable.

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Summary

WYBOCZENIE PEŁZAJĄCE SŁUPÓW STALOWYCH W TEMPERATURACH POŻAROWYCH

Przyjęto wzór Ramberga-Osgooda (8) dla odkształceń sprężysto-plastycznych ε_1 i prawo Nortona-Baileya (11) dla prędkości pełzania ε_c. Temperatura ν rośnie według formuły ISO (17). "Sieczny" współczynnik ψ odkształcenia sprężysto-plastycznego (18) jest więc funkcją czasu t. Wyprowadzono i scałkowano równanie wyboczenia pełzającego (21) przyjmując wstępne wygięcie słupa w_o według norm "Europejskich" (16). Określono współczynnik α_a amplifikacji (25) i α_c - pełzania (26). Zwiększają one wygięcie w - narastające w czasie pożaru. Oszacowano wartości w/r (Tablica 1) dla słupów ze stali nisko-węglowej.