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LIMIT ANALYSIS DUE TO FAILURE PROCESS IN CREEP CONDITIONS

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1. Introduction

In creep conditions the concept of limit loading has to be replaced by the limit time of structure exploitation, since the failure occurs at any load level provided the working time is long enough. This fundamental observation was made already in fifties and supported by long-term experiments (Robinson, 1952). After it was proved that the final rupture is a result of gradual material degradation (cf. e.g. Rosenberg, 1967), a new material variable was introduced by Kachanov (1958), with an appropriate rate equation for this variable. This gave impetus for the development of a new branch of fracture mechanics: Continuum Damage Mechanics (the term coined by Jansson, Hult, 1977), which brought a number of effective methods to evaluate the growth of defects in materials (cf. Euromech Colloquiua, 1981, 1989, and IUTAM Symposium ,1985).

However, the analysis is normally limited to two stages. In the first one the damage variable is less than its critical value (normally set to 1) at each point of a body. The time at which damage variable reaches its critical value at least at one point of a body is denoted by t_I. For the time $t > t_I$ the degradation process develops in a structure to come to the state when a surface on which damage is equal to 1 spans all the body at time t_{II}. This time is usually considered as an ultimate time of structure exploitation. In fact it corresponds only to the state when failure occurs on "semi-integral" level $\mathcal{S}$ (according to the classification proposed by Życzkowski, 1981) related to the cross-section of structure, whereas time t_I corresponds to the local level $\mathcal{P}$. One can consider that in the fractured cross-section a "super-hinge" is formed without any load bearing capacity. If the structure is statically indeterminate, the existence of such a hinge not necessarily means the total loss of loading capacity of a structure. The loading may be transferred to those parts of a structure which still can work, until it will become kinematically unstable as the result of consecutive appearance of "super-hinges". This time will be denoted as t_{III} and the state of a structure failure process will be related to the integral level $\mathcal{B}$ in Życzkowski's classification.

The works concerned with the above problems, known to the authors, were limited only to the analysis of a ratio t_{II}/t_I (Kachanov, 1960, Piechnik and Chrzanowski 1970). It is the main goal of the present paper to perform an analysis of a simple structure to evaluate all times t_i(i=I,II,III) and to analyse the relation between them.

2. General assumptions and governing equations

The damage description was assumed to have the simplest form, i.e. scalar representation of damage variable ω ($0\geq\omega\geq1$), and stress governed damage rate law:

$$\dot{\omega} = A \langle\sigma_{eq}/(1-\omega)\rangle^m, \quad \omega(0) = 0, \tag{2.1}$$

where A and m are material constants and σ_{eq} is the equivalent stress according to Hayhurst et al. (1984):

$$\sigma_{eq} = \alpha\sigma_1 + (1-\alpha)\sigma_e, \tag{2.2}$$

where σ_1 is maximum principal stress and σ_e is effective stress. The brackets $\langle\rangle$ in (2.1) denote that the damage accumulation occurs only for positive σ_{eq}.

For such a formulation the stress redistribution in time plays an essential role in damage cumulation and therefore the general theory of nonstationary creep was adopted (Hult, 1966):

$$\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^c_{ij} \quad , \tag{2.3}$$

$$\varepsilon^{e}_{ij} = D_{ijkl}\sigma_{kl} \quad , \tag{2.4}$$

$$\dot{\varepsilon}^{c}_{ij} = \gamma\,[\sigma_e/(1-\omega)]^{n}\, s_{ij} \quad , \quad \varepsilon^{c}_{ij}(0)=0, \tag{2.5}$$

where superscript e and c refer to elastic and creep deformation, respectively, s_{ij} is deviatoric stress, and γ and D are material functions. As it can be seen from (2.5) that so called coupled theory of creep is used. Therefore the stress redistribution will be an effect of two processes: redistribution connected with a transition from initial elastic solution at t=0 to the quasi stationary creep (for $t \to \infty$), and that associated with the damage growth. The latter will cause the stress to diminish at the points where damage develops, to drop finally to zero when $\omega = 1$ at time t_I.

For time $t>t_I$ all above equations will hold but the domain of integration will be reduced only to these points of the body at which $\omega < 1$. This will imply the change in structure dimensions, which in turn will result in strain and damage localization. This effect will be shown by the results of example calculations.

The behavior of a structure for time $t > t_{II}$ depends first of all on the kinematic boundary conditions (statically determinate or indeterminate structures) and on the type of a structure (beams, plates, shells etc.). For statically determinate structures $t_{II}=t_{III}$ and the further analysis is impossible. In the following a simple statically indeterminate beam will be analysed to show the effect of the damage development for $t > t_{II}$.

3. Example

The beam of redundancy equal to 1 (Fig.1a.) with the rectangular cross-section 2h × b and loaded by a concentrated force P was chosen as the simplest example of a structure which behaviour will cover all three stages of damage development.

In the analysis the assumption of plane cross-section deformation was made and shear stresses were neglected. In the consequence the equation

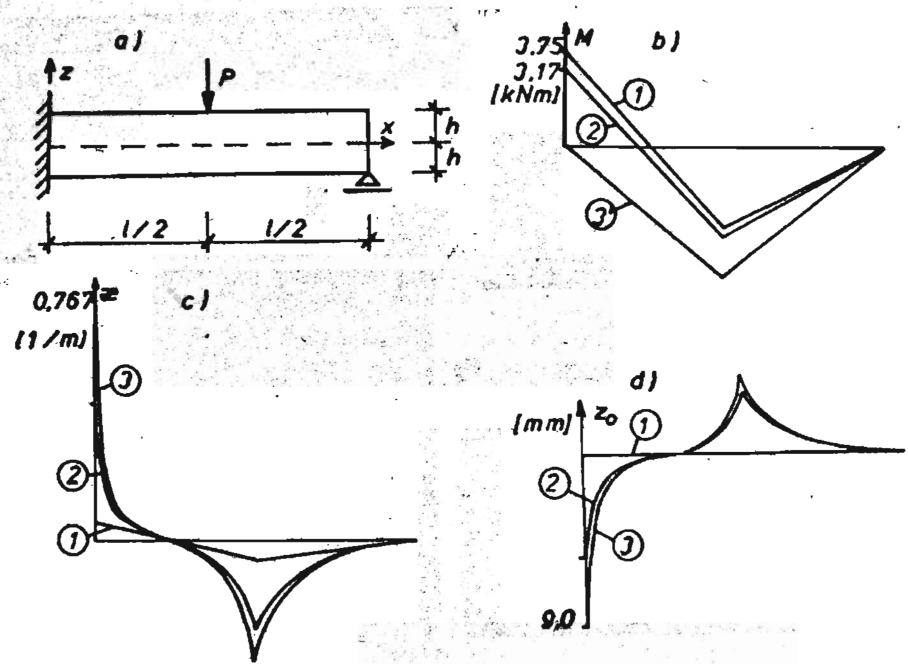

Fig.1. Bending moment (b), curvature (c) and neutral axis position (d) for analysed beam (a).

(2.2) reduces to:

$$\sigma_{eq} = \sigma_x , \tag{3.1}$$

i.e. $\sigma_1 = \sigma_x$, and eq. (2.4) and (2.5) become:

$$\varepsilon_x^e = E\sigma_x , \tag{3.2}$$

$$\dot{\varepsilon}_x^c = K[\sigma_x/(1-\omega)]^n , \qquad \varepsilon_x^c(0)=0 , \tag{3.3}$$

where E, K , and n are material constants.

The above relations and the internal-external force system equivalent conditions in any cross-section yield the formulae for neutral axis position $z_0(x)$ and its curvature $\kappa(x)$. For time $t < t_I$ these formulae are:

$$z_0(x) = \int_{-h}^{h} \varepsilon_x^c \,(z,x)\, dz \,/\{2h\, \kappa(x)\} , \tag{3.4}$$

$$\kappa(x) = \{3/2h_3^3\} \{ M(x)/bE + \int_{-h}^{h} \varepsilon_x^c (z,x) z \, dz \} , \tag{3.5}$$

where M(x) is bending moment distribution at time t.
The equations (3.4) and (3.5) are a special case of more general one when upper limit of integrals h becomes a function of time. This situation has been considered for $t_I<t<t_{II}$, and is not shown here because of equations' complexity. Numerical integration of governing equations was performed for the following boundary conditions:

$$w(x=0) = 0, \; dw/dx(x=0) = 0, \; w(x=l) = 0, \tag{3.6}$$

with time step limited by a criterion

$$\Delta t = \{4(1+\nu)\} / \{3EKn \max_{x,z} [\sigma^{n-1}(x,z)]\} , \tag{3.7}$$

proposed by Cormeau (1975).

For material constants equal to: E = 120 [GPa], ν=0.33, K= 0.013 [$GPa^{-3}h^{-1}$], n = 3, A = 0.208 [$GPa^{-2}h^{-1}$], m = 2 the redistribution of bending moments in time, curvature changes and position of neutral axis is shown in Figs 1b through 1d, where all curves denoted by 1 correspond to time t=0, by 2 - to time t_I, by 3 to $t = t_{II} - \Delta t$, and by 3' to $t > t_{II}$.

For time $t > t_{II}$ bending moments distribution remains constant, and curvature at x=l/2 tends to infinity at time $t = t_{III}$. Distributions of stress, damage and creep strain

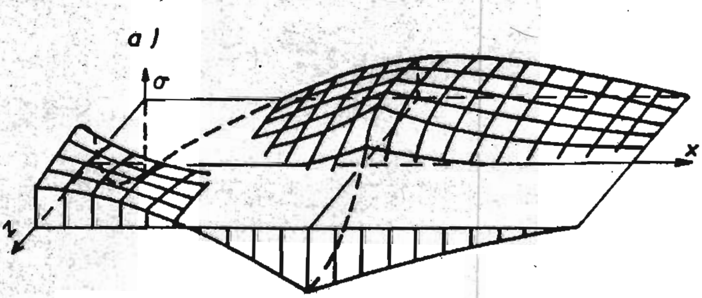

Fig.2a. Distribution of stress for $t = 0.8\, t_I$.

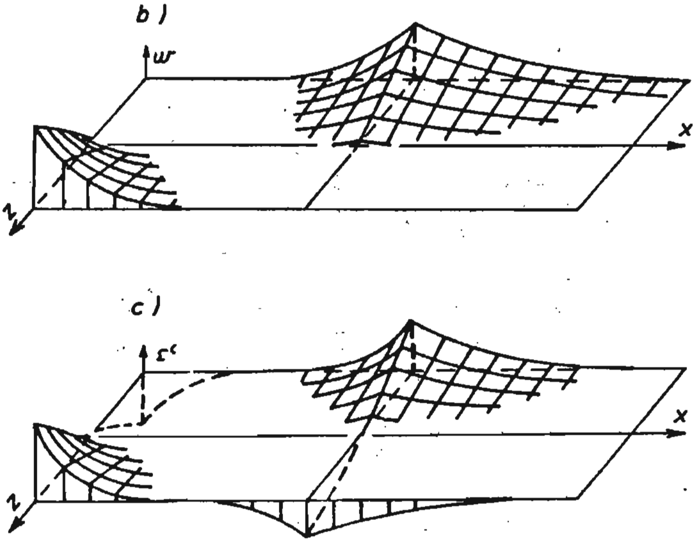

Fig.2b,c. Distribution of damage (b) and creep-strain (c) for $t = 0.8\ t_I$.

are shown in Figs 2 and 3 (a, b, and c respectively) for two time instances: $t=0.8t_I$ and $t=t_{II}-\Delta t$. The fractured area of a cross-section at x=0 is marked as a dashed one. The essential damage and creep strain localization can be observed in the areas where first crack appears (x=0, z=h) and final rupture occurs (x=l/2, z=-h).

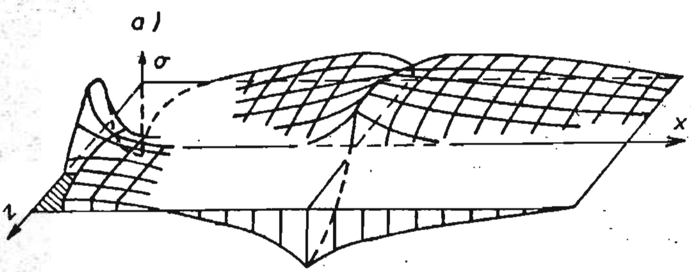

Fig.3a. Distribution of stress for $t = t_{II} - \Delta t$.

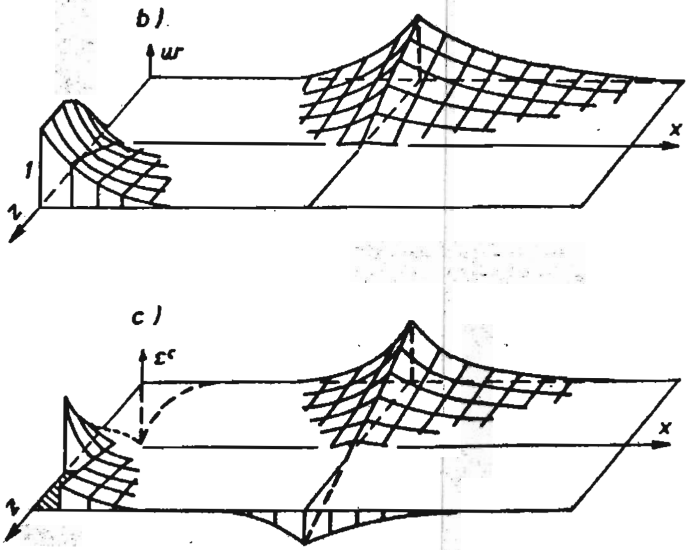

Fig. 3b,c. Distribution of damage (b) and creep-strain (c) for $t=t_{II}-\Delta t$.

4. Conclusions

The results of calculation give the following values of t_i: t_I=94 [h], t_{II}= 103 [h], t_{III}= 109 [h], and show that the safety margin related to the time t_I is low for this example (t_{II}/t_I= 1.09 and t_{III}/t_I= 1.16). Undoubtedly, time t_{III} depends heavily on structure redundancy, and will grow with it. However, the most important factors influencing structure behavior are redistributions of bending moments and stress. These effects are connected with assumptions made. Table 1 summarize the comparison of t_I and t_{II} values for the following simplifying assumptions:

1. Elastic moment distribution was assumed to be constant, and corresponding elastic stress was used in Kachanov's formula for t_I= $[A(m+1)\sigma^m(x=0,z=h/2)]^{-1}$,
2. Structure was considered to be statically determinate but stress redistribution due to damage growth was taken into account,
3. Both effects of moment and stress redistribution were considered, as well as cross-section height reduction for $t > t_I$ (actual solution).

TABLE 1

	Redistribution of			t_I	t_{II}	t_{II}/t_I	Lines in Figs 4&5
	M	σ	h				
1	-	-	-	37	-	-	-------
2	-	+	+	83	89	1.06	-x-x-x-
3	+	+	+	94	103	1.09	———
4	-	+	+	141	236	1.67	-o-o-o-
5	-	-	-	141	293	2.06	-•-•-•-

4. Steady-state creep theory for statically indeterminate structure was applied, with stress redistribution due to damage growth.
5. Same as 4, but without stress redistribution.

In Table 1 different types of lines are used to identify different above cases, and the same type of lines apply to Figs 4 and 5.

Fig. 4 depicts the bending moment and stress redistribution for time $0 \leq t \leq t_I$, whereas Fig. 5 shows the stress redi- stribution and reduction of beam height h.

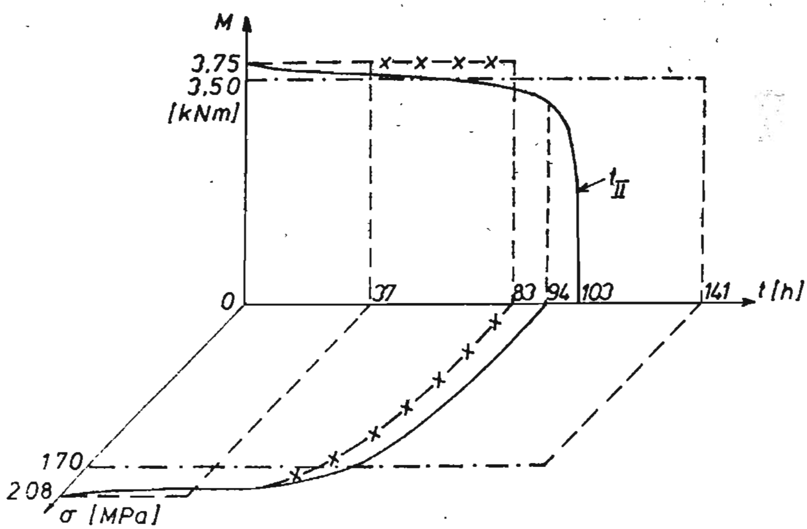

Fig. 4. Bending moment and stress redistribution for $t < t_I$.

The general conclusion is that the evaluation of structure safety on the base of uncoupled steady-state creep theory is non-conservative (except the lower bound obtained for pure elastic moment distribution), and only full analysis with bending moments and stress redistribution taken into account should be used for a proper assessment.

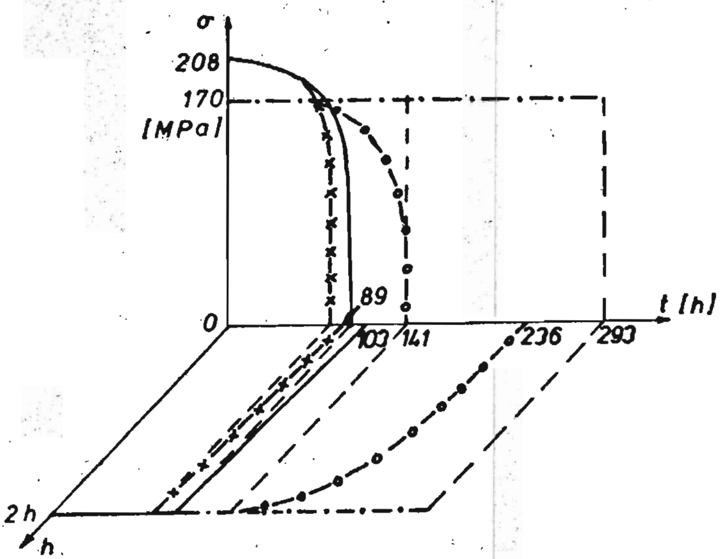

Fig.5. Stress redistribution and beam height reduction.

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Summary

ANALIZA NOŚNOŚCI GRANICZNEJ ZWIĄZANEJ Z PORCESEM ZNISZCZENIA W WARUNKACH PEŁZANIA

W pracy dokonano analizy zniszczenia konstrukcji, wyróżniając trzy etapy, odpowiadające poziomom analizy, zaproponowanym przez M.Życzkowskiego (1981): na poziomie punktu (*P*), przekroju (*S*) i ciała (*B*). W warunkach pełzania odpowiada to wyznaczaniu czasów do: pojawienia się pierwszego pęknięcia t_I, propagacji pęknięcia w przekroju poprzecznym t_{II} i czasu t_{III} zniszczenia konstrukcji na skutek zamiany w mechanizm, w wyniku zniszczenia odpowiedniej liczby przekrojów. Znajomość wartości tych czasów pozwala na oszacowanie zapasu bezpieczeństwa konstrukcji pracujących w warunkach pełzania.