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ON METHODS OF MODELING ANISOTROPIC COMPRESSED WOOD FAILURE

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In the paper the methods of finding strength criterion for compressed wood with orthotropic properties have been discussed and analyzed. Besides, we also have taken into account the tensile strength in the directions of the arthotropy axes.
As a point of departure we take three terms of the polynomial tensorial criterion for anisotropic strength in the form:

$$P_{ij}\sigma_{ij} + R_{ijkl}\sigma_{ij}\sigma_{kl} + Q_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn} + ... = 1.$$

The following particular cases are discussed: 1) the first term, 2) the first and second terms, 3) the second term, 4) the second and third terms. The best description has been obtained in the case of the second term criterion, then the minimum number of tests was necessary.
In the paper, the description proposed by E.K.Ashkenazi based on the similarity of the strength tensor to the elasticity tenosr is also compared with our models.
For the generalization of results of the failure investigation, additional investigations of biaxial compression were carried out. On considering only the first two terms of the criterion it was found that in the planes of orthotropy, three failure elipses were displaced and turned in the system of the orthotropy axes.

1. Introduction

Wood based materials are anisotropic in general and orthotropic in particular. For description of the orthotropic material mechanical properties, different criteria are applied among others those of: E.K.Ashkenazi [1], A.K.Malmeyster [2], J.I.Goldenblat and W.A.Kopnov [3], J.Marin [4], K.W.Zakharov [5].

The aim of the paper was to study the compression strength of anisotropic materials by the criterion of failure given by A.K.Malmeyster [2] in the form

$$P_{ij}\sigma_{ij} + R_{ijkl}\sigma_{ij}\sigma_{kl} + Q_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn} + ... = 1, \tag{1.1}$$

where: P, R, Q,... anisotropy tensors, and to find the most appropriate case of the criterion for the accuracy of the description with, at the same time, a minimalization number of tests.

In this paper the description given by E.K.Ashkenazi [1], based on the similarity of the strength tensor to the elasticity tensor is also proposed.

2. Experimental data

2.1. Samples and method of investigations

2.1.1. Samples and investigation method with uniaxial compression

The compression strength tests were carried out according to standard PN-65/D-04215, on rectangular prisms measuring $10 \times 10 \times 15$ mm (the longer dimension being, in the direction of the action of the force). The test samples were cut out of bars of compressed wood for which the technical specifications of production was given in the paper by M.Czech [6]. The directions in which the samples were cut out are shown in fig.1.

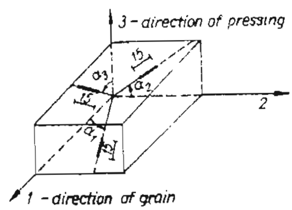

Fig. 1. Direction of cutting samples from compressed wood slats

The tests were carried out on samples of compressed beech wood of a density of $\rho = 990$ kg/m^3, determined by standard PN-78/D-04210, humidity of 8.7% and a compression ratio $n = 1.45$. For every direction 10 samples were taken. Before the tests, the samples were conditioned at a humidity of 65 ± 5 % and a temperature of 293 ± 3 K. The compression strength tests were carried out under identical conditions.

2.1.2. Samples and experimental method for uniaxial tension

The tensile strength tests were carried out according to standard PN-65/D-04218. The samples for tensile tests were prepared as shown in fig.2. The experimental conditions were the same as those for the compression strength experiments.

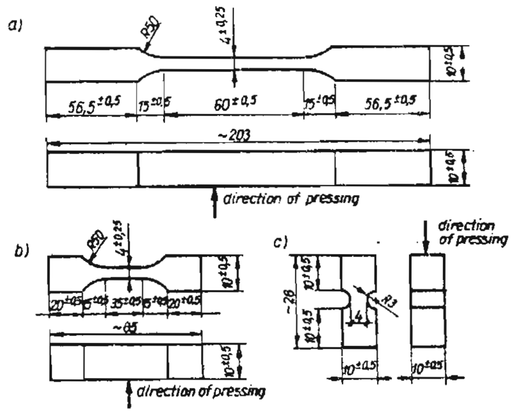

Fig. 2. Diagrams of samples for the tensile strength tests; a – in direction 1 (in direction of grain), b – in direction 2, c – in direction 3 (in direction of pressing)

2.1.3. Samples and experimental method with biaxial stress states

The biaxial compression tests were carried out on samples 15 * 15 * 7.5 mm in dimension (the longer dimensions being in the direction of the force action). The samples were cut out in the principal orthotropy directions. The experimental conditions were the same as those for uniaxial compression.

Because of great deformations in the plane 23, the strength was assumed to be equal to the proportionality limit.

2.2. Results of tests

2.2.1. Results of the uniaxial compression tests

The results of the tests are presented in Tab.1.

The standard deviation calculated from the following

$$S_{n-1} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(R_i - \bar{R})^2}, \tag{2.1}$$

and teh coefficient of variation from:

$$v = \frac{S_{n-1}}{\bar{R}} \cdot 100\%, \tag{2.2}$$

where: R_i – compression strength of the ith sample, $n = 10$ – number of samples for given angle.

Table 1. Results of the compression strength test of compressed wood in different directions from orthotropy axes

α	In plane 12					In plane 23					In plane 31				
[°]	$\bar{R}^c$ [MPa]	S_{n-1} [MPa]	v [%]	R^c_{min} [MPa]	R^c_{max} [MPa]	$\bar{R}^c$ [MPa]	S_{n-1} [MPa]	v [%]	R^c_{min} [MPa]	R^c_{max} [MPa]	$\bar{R}^c$ [MPa]	S_{n-1} [MPa]	v [%]	R^c_{min} [MPa]	R^c_{max} [MPa]
0	99.5	2.04	2.05	96.9	104.0	44.7	3.02	6.76	39.1	47.9	70.0	6.30	9.00	60.3	78.7
22.5	94.1	2.60	2.76	90.4	96.9	43.5	2.91	6.69	39.5	48.8	62.4	2.86	4.58	57.2	66.0
30	68.5	3.09	4.51	63.7	73.2	46.2	5.91	12.79	39.1	53.5	56.8	4.81	8.47	51.5	61.9
45	61.7	3.49	5.66	56.2	66.9	50.3	3.66	7.28	47.1	57.4	49.0	3.40	6.94	43.5	53.4
60	46.5	2.45	5.27	42.6	49.9	57.9	5.27	9.10	55.0	65.4	46.4	3.56	7.67	41.9	52.8
67.5	53.4	2.63	4.93	49.3	57.7	60.2	7.83	13.01	50.4	73.2	63.2	3.62	5.28	58.0	65.8
90	44.7	3.02	6.76	39.1	47.9	70.0	6.30	9.00	60.3	78.7	99.5	2.04	2.05	96.9	104.0

2.2.2. Results of the uniaxial tensile tests

The results of the tests are shown in the Tab.2. The standard deviation was calculated from eq.(2.1) and the coefficient of variation from eq.(2.2), where: R_i – tensile strength for ith sample, $n = 10$ samples for given direction.

Table 2. Results of the tensile strength tests in the directions of the orthotropy axes

Direction of the sample cutting	R_m [MPa]	S_{n-1} [MPa]	v [%]	R_{min} [MPa]	R_{max} [MPa]
1	195.1	20.94	10.73	162.8	216.5
2	28.0	2.31	8.30	24.6	31.7
3	8.6	0.32	3.70	8.1	9.0

2.2.3. Results of tests in biaxial stress states

The results of the tests are shown in Tab.3.

2.3. Estimate of confidence intervals of the mean value and single observations of uniaxial compresssion

Before calculating the confidence intervals, the variance homogeneity was verified by means of the Cochran test in the form:

$$G = \frac{S^2_{n-1,max}}{\sum\limits_{j=1}^{7} S^2_{n-1,j}}, \tag{2.3}$$

$$G_{rel.} = \frac{S^{2,rel.}_{n-1,max}}{\sum\limits_{j=1}^{7} S^{2,rel.}_{n-1,j}}, \tag{2.4}$$

where S_{n-1} with eq.(2.1) and $S^{2,rel.}_{n-1}$ – from the following equation

$$S^{rel.}_{n-1} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\frac{(R_i - \bar{R})^2}{\bar{R}}}. \tag{2.5}$$

The results of the calculations are given in tab.4

Table 3. Results of the failure tests on biaxial compression in direction of orthotropy axes

(∗ ∗)	In plane 12					In plane 23					In plane 31				
	$\bar{\sigma}^c_{ii}$ [MPa]	S_{n-1} [MPa]	v [%]	$\sigma^c_{ii_{min}}$ [MPa]	$\bar{\sigma}^c_{ii_{max}}$ [MPa]	$\bar{\sigma}^c_{ii}$ [MPa]	S_{n-1} [MPa]	v [%]	$\sigma^c_{ii_{min}}$ [MPa]	$\sigma^c_{ii_{max}}$ [MPa]	$\bar{\sigma}^c_{ii}$ [MPa]	S_{n-1} [MPa]	v [%]	$\sigma^c_{ii_{min}}$ [MPa]	$\sigma^c_{ii_{max}}$ [MPa]
1	48.8	2.43	4.98	44.8	51.6	–	–	–	–	–	29.0	4.09	14.10	23.8	35.4
2	49.2	2.46	5.00	45.1	52.2	49.0	3.33	6.80	45.2	52.6	–	–	–	–	–
3	–	–	–	–	–	20.2	1.33	6.58	18.6	21.9	71.6	10.76	15.03	62.6	88.0

(∗ ∗) – Direction of the principle orthotropy axis i.

Table 4. Results of calculations of the G and $G_{rel.}$

Values G, $G_{rel.}$	Plane		
	12	23	31
G	0.2231	0.3150	0.1987
$G_{rel.}$	0.2816	0.2649	0.2158

On comparing the values G and $G_{rel.}$ with the critical value $g(0.05; 7; 9) = 0.3259$ (0.05 – level of confidence, 7 – number of sample cutting angles in a given plane, $9 = n - 1$ – number of degrees of freedom for given angle) it is been that there is no basis for rejection of the hypothesis that variances are homogeneous.

For homogenity of variances, the confidence intervals are calculated from the equations:

– for single observations with absolute error

$$\bar{R} - \Delta_1 < R < \bar{R} + \Delta_1, \tag{2.6}$$

– for single observations with relative error

$$\bar{R}(1 - \Delta_2^{rel.}) < R < \bar{R}(1 + \Delta_2^{rel.}), \tag{2.7}$$

– for the mean population with absolute error

$$\bar{R} - \frac{\Delta_1}{\sqrt{n}} < R < \bar{R} + \frac{\Delta_1}{\sqrt{n}}, \tag{2.8}$$

– for the mean population with relative error

$$\bar{R}\left(1 - \frac{\Delta_2^{rel.}}{\sqrt{n}}\right) < R < \bar{R}\left(1 + \frac{\Delta_2^{rel.}}{\sqrt{n}}\right), \tag{2.9}$$

where:

$$\Delta_1 = t_{\alpha,\nu}\sqrt{\frac{1}{\nu}\sum_{i=1}^{7}\sum_{j=1}^{10}(R_{ij} - \bar{R}_i)^2}, \tag{2.10}$$

$$\Delta_2^{rel.} = t_{\alpha,\nu}\sqrt{\frac{1}{\nu}\sum_{i=1}^{7}\sum_{j=1}^{10}\frac{(R_{ij} - \bar{R}_i)^2}{\bar{R}_i}}, \tag{2.11}$$

$\alpha = 0.05$ – level of confidence, $\nu = 70 - 7 = 63$ – degrees of freedom. The calculations according to eqs.(2.6) – (2.11) are given in tab.5.

Table 5. Results of calculations of the confidence intervals of mean values and single observations

Intervals of confidence	Plane		
	12	23	31
Δ_1 [MPa]	5.5800	10.530	8.150
$\frac{\Delta_1}{\sqrt{n}}$ [MPa]	1.7600	3.330	2.580
$\Delta_2^{rel.}$	0.0960	0.191	0.146
$\frac{\Delta_2^{rel.}}{\sqrt{n}}$	0.0304	0.060	0.046

3. General of strength criterion and its variants

The general strength criterion proposed by A.K.Malmeyster [2], is presented in eq.(1.1). We shall now examine the seperate terms of this criterion and its variants.

3.1. Description of strength by means of the first term criterion, eq.(1.1)

From eq.(1.1) we obtain

$$P_{ij}\sigma_{ij} = 1. \tag{3.1}$$

For uniaxial compression $i = j = 1$. The transformation formula for a tensor of rank two will be

$$P'_{11} = P_{11}\cos^2\alpha + P_{12}\sin\alpha + P_{22}\sin^2\alpha. \tag{3.2}$$

For $\alpha = 45^o$ eq.(3.2) will be

$$P_{11}^{45} = P_{11}\frac{1}{2} + P_{12} + P_{22}\frac{1}{2}. \tag{3.3}$$

Then substituting P_{12} from eq.(3.3) into eq.(3.2) we have

$$\begin{aligned} P'_{11} &= P_{11}\cos\alpha(\cos\alpha - \sin\alpha) + P_{11}^{45}\sin 2\alpha + \\ &+ P_{22}\sin\alpha(\sin\alpha - \cos\alpha). \end{aligned} \tag{3.4}$$

From eq.(3.1) we obtain

$$P_{11} = \frac{1}{R_0^c}, \qquad P_{22} = \frac{1}{R_{90}^c}, \qquad P'_{11} = \frac{1}{R_\alpha^c}. \tag{3.5}$$

where $\bar{R}_0^c$ and $\bar{R}_{90}^c$ – mean experimental strengths for principal orthotropy axes. On substituting eq.(3.5) into eq.(3.4) we obtain

$$\begin{aligned} \frac{1}{R_\alpha^c} &= \frac{1}{\bar{R}_{45}^c}\sin 2\alpha + \frac{1}{\bar{R}_0^c}\cos\alpha(\cos\alpha - \sin\alpha) + \\ &+ \frac{1}{\bar{R}_{90}^c}\sin\alpha(\sin\alpha - \cos\alpha). \end{aligned} \tag{3.6}$$

On determining

$$\begin{aligned} &\frac{1}{R_\alpha^c} = y, \qquad \frac{1}{\bar{R}_{45}^c} = a, \\ &-\frac{1}{\bar{R}_0^c}\cos\alpha(\cos\alpha - \sin\alpha) - \frac{1}{\bar{R}_{90}^c}\sin\alpha(\sin\alpha - \cos\alpha) = A, \\ &\sin 2\alpha = x, \end{aligned} \tag{3.7}$$

we obtain

$$y = ax - A. \tag{3.8}$$

On calculating coefficient a by the least square method (LSM), its value is the first aproximation in the nonlinear regression of the eq.(3.8) transformed into:

$$R_\alpha^c = \frac{1}{a\sin 2\alpha - A}. \tag{3.9}$$

3.2. Description of strength using the second term of eq.(1.1)

From eq.(1.1) we obtain the criterion

$$R_{ijkl}\sigma_{ij}q_{kl} = 1. \tag{3.10}$$

After similar transformations as at the point 3.1. we obtain the equations:

$$\begin{aligned} R'_{1111} &= R_{1111}\cos^2\alpha\cos 2\alpha + R_{1111}^{45}\sin^2 2\alpha + \\ &- R_{2222}\sin^2\alpha\cos 2\alpha, \end{aligned} \tag{3.11}$$

$$\begin{aligned} \frac{1}{R_\alpha^2} &= \frac{1}{(\bar{R}_0^c)^2}\cos^2\alpha\cos 2\alpha + \frac{1}{(\bar{R}_{45}^c)^2}\sin^2 2\alpha + \\ &- \frac{1}{(\bar{R}_{90}^c)^2}\sin^2\alpha\cos 2\alpha. \end{aligned} \tag{3.12}$$

On determining in (3.12)

$$\begin{aligned} &\frac{1}{(R_\alpha^c)^2} = y, \qquad \frac{1}{(\bar{R}_{45}^c)^2} = a, \\ &-\frac{1}{(\bar{R}_0^c)^2}\cos^2\alpha\cos 2\alpha + \frac{\sin^2\alpha\cos 2\alpha}{(\bar{R}_{90}^c)^2} = A, \\ &\sin^2 2\alpha = x, \end{aligned} \tag{3.13}$$

we obtain an equation in the form (3.8). The value of coefficient a calculated by the least square method constitutes the first aproximation in the nonlinear regression of eq.(3.12). On taking into account eq.(3.13), we obtain from eq.(3.12)

$$R_\alpha^c = \frac{1}{\sqrt{a\sin^2 2\alpha - A}}. \tag{3.14}$$

3.3. Strength criterion by Ashkenazi method

The method of E.K.Ashkenazi is based on the assumption that the strength of orthotropic materials are transformed like similarity to the anisotropy tensor coordinates in the linear elasticity theory, i.e. as in eq.(3.11), but with coefficients R_{iiii} which are the inverse of strength in a given direction. Hence this criterion takes the form

$$\frac{1}{R_\alpha^c} = \frac{1}{R_0^c}\cos^2\alpha\cos 2\alpha + \frac{1}{R_{45}^c}\sin^2 2\alpha - \frac{1}{R_{90}^c}\sin^2\alpha\cos 2\alpha. \tag{3.15}$$

Determining:

$$\begin{aligned} &y = \frac{1}{R_\alpha^c} + A, \qquad \frac{1}{R_{45}^c} = a,\\ &A = -\frac{1}{R_0^c}\cos^2\alpha\cos 2\alpha + \frac{1}{R_{90}^c}\sin^2\alpha\cos 2\alpha,\\ &\sin^2 2\alpha = x, \end{aligned} \tag{3.16}$$

we obtain the equation in the form (3.8). The value of coefficient calculated by the least square method is the first aproximation in the nonlinear regression of eq.(3.15). On taking into account the results of (3.16), eq.(3.15) takes the form

$$R_\alpha^c = \frac{1}{a\sin^2 2\alpha - A}. \tag{3.17}$$

3.4. Description of strength by means of the first two terms of eq.(1.1)

From the eq.(1.1) we obtain

$$P_{ij}\sigma_{ii} + R_{ijkl}\sigma_{ij}\sigma_{kl} = 1. \tag{3.18}$$

For uniaxial comression ($i = j = k = l = 1$) we put together egs.(3.4) and (3.11):

$$P_{11}' = P_{11}\cos\alpha(\cos\alpha - \sin\alpha) + P_{11}^{45}\sin 2\alpha + P_{22}\sin\alpha(\sin\alpha - \cos\alpha), \tag{3.19}$$

$$R'_{1111} = R_{1111}\cos^2\alpha\cos 2\alpha + R^{45}_{1111}\sin^2 2\alpha - R_{2222}\sin^2\alpha\cos 2\alpha. \qquad (3.20)$$

Assuming that eq.(3.18) is satisfied for the strength of the principal orthotropy axes 1 and 2, then for $i = j = k = l = 1, 2$ from this equation we obtain:

$$P_{11}\bar{R}^c_0 + R_{1111}(\bar{R}^c_0)^2 = 1,$$
$$P_{22}\bar{R}^c_{90} + R_{2222}(\bar{R}^c_{90})^2 = 1. \qquad (3.21)$$

Substituting eqs.(3.19) into eq.(3.18) and taking eq.(3.21) into consideration we obtain:

$$\begin{aligned} &P_{11}\Big[\cos\alpha(\cos\alpha - \sin\alpha)R^c_\alpha - \frac{(R^c_\alpha)^2}{\bar{R}^c_0}\cos^2\alpha\cos 2\alpha\Big] + \\ &+P^{45}_{11}R^c_\alpha\sin 2\alpha + R^{45}_{1111}(R^c_\alpha)^2\sin^2 2\alpha + \\ &+P_{22}\Big[R^c_\alpha\sin\alpha(\sin\alpha - \cos\alpha) + \frac{(R^c_\alpha)^2}{\bar{R}^c_{90}}\sin^2\alpha\cos 2\alpha\Big] + \\ &+\frac{(R^c_\alpha)^2}{(\bar{R}^c_0)^2}\cos^2\alpha\cos 2\alpha - \frac{(R^c_\alpha)^2}{(\bar{R}^c_{90})^2}\sin^2\alpha\cos 2\alpha = 1. \end{aligned} \qquad (3.22)$$

Solving the system of eqs.(3.22) (for five different α) since two angles were taken into account in (3.21), by the least square method we obtain P_{11}, P^{45}_{11}, R^{45}_{1111}, R_{2222}. Having the values of the above coefficient from (3.19) and (3.20), we calculate P'_{11} and R'_{1111}, and then R^c_α from the equation obtained from (3.18), i.e.

$$R^c_\alpha = \frac{-P'_{11} + \sqrt{P'^2_{11} + 4R'_{1111}}}{2R'_{1111}}. \qquad (3.23)$$

3.5. Description of strength using second and third terms of eq.(1.1)

From eq.(1.1) we have:

$$R_{ijkl}\sigma_{ij}\sigma_{kl} + Q_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn} = 1. \qquad (3.24)$$

For uniaxial compression ($i = j = k = l = m = n = 1$) we combine eq.(3.11) and the equation obtained from the transformation formula for the tensor of rank six, we have:

$$\begin{aligned} R'_{1111} &= R_{1111}\cos^2\alpha\cos 2\alpha + R^{45}_{1111}\sin^2 2\alpha - R_{2222}\sin^2\alpha\cos 2\alpha, \\ Q'_{111111} &= Q_{111111}(\cos^6\alpha - \frac{10}{3}\cos^4\alpha\sin^2\alpha + \cos^2\alpha\sin^4\alpha) + \\ &+ Q^{30}_{111111}(8\cos^4\alpha\sin^2\alpha - \frac{8}{3}\cos^2\alpha\sin^2\alpha) + \end{aligned} \qquad (3.25)$$

$$+ \quad Q^{60}_{111111}(-\frac{8}{3}\cos^4\alpha\sin^2\alpha + 8\cos^2\alpha\sin^4\alpha) +$$
$$+ \quad Q_{222222}(\sin^6\alpha + \cos^4\alpha\sin^2\alpha - \frac{10}{3}\cos^2\alpha\sin^4\alpha).$$

Assuming that eq.(3.24) is satisfied for compression strength and tensile strength in the direction of the principal orthotropy axes 1 and 2, then for $i=j=k=l=m=n=1,2$ from eq.(3.24) we obtain

$$\begin{aligned} R_{1111}(\bar{R}^m_{100})^2 + Q_{111111}(\bar{R}^m_{100})^3 &= 1,\\ R_{1111}(\bar{R}^c_{100})^2 - Q_{111111}(\bar{R}^c_{100})^3 &= 1,\\ R_{2222}(\bar{R}^m_{200})^2 + Q_{222222}(\bar{R}^m_{200})^3 &= 1,\\ R_{2222}(\bar{R}^c_{200})^2 - Q_{222222}(\bar{R}^c_{200})^3 &= 1, \end{aligned} \tag{3.26}$$

where:
$\bar{R}^m_{100}$ – tensile strength in direction of axis 1,
$\bar{R}^m_{200}$ – tensile strength in direction of axis 2,
$\bar{R}^c_{100}$ – compression strength in direction of axis 1,
$\bar{R}^c_{200}$ – compression strength in direction of axis 2.
Inserting eq.(3.25) into eq.(3.24) we have:

$$\begin{aligned} &R^{45}_{1111}(\bar{R}^c_\alpha)^2\sin^2 2\alpha - Q^{30}_{111111}(8\cos^4\alpha\sin^2\alpha - \frac{8}{3}\cos^2\alpha\sin^4\alpha)\cdot\\ &\cdot(\bar{R}^c_\alpha)^3 - Q^{60}_{111111}(-\frac{8}{3}\cos^4\alpha\sin^2\alpha + 8\cos^2\alpha\sin^4\alpha)(\bar{R}^c_\alpha)^3 =\\ &= 1 - (R_{1111}\cos^2\alpha\cos 2\alpha - R_{2222}\sin^2\alpha\cos 2\alpha)(\bar{R}^c_\alpha)^2 +\\ &+[Q_{111111}(\cos^6\alpha - \frac{10}{3}\cos^4\alpha\sin^2\alpha + \cos^2\alpha\sin^4\alpha) +\\ &+Q_{222222}(\sin^6\alpha + \cos^4\alpha\sin^2\alpha - \frac{10}{3}\cos^2\alpha\sin^4\alpha)](\bar{R}^c_\alpha)^3. \end{aligned} \tag{3.27}$$

On solving the linear system of equations (3.27) – LSM – for five different α, since two angles were taken into consideration in eq.(3.26), on determining R_{1111}, Q_{111111}, R_{2222}, Q_{222222} we obtain the values of coefficients: R^{45}_{1111}, Q^{30}_{111111}, Q^{60}_{111111}. Having the values of the these coefficients we calculate R'_{1111} and Q'_{111111} from eq.(3.25) and then the R^c_α from the equation which we obtain from eq.(3.24), on substitution of $i=j=k=l=m=n=1$,

$$R'_{1111}(R^c_\alpha)^2 + Q'_{111111}(R^c_\alpha)^3 = 1. \tag{3.28}$$

The eqs.(3.9), (3.14), (3.18) and (3.28) are also applied for the orthotropy planes 23 and 31 by cyclic index change.

The description of strength presented by eqs.(3.9), (3.14) (3.18) and (3.28), with the mean experimental data, is shown: for plane 12 in fig.3, for plane 23 in fig.4 and for plane 31 in fig.5.

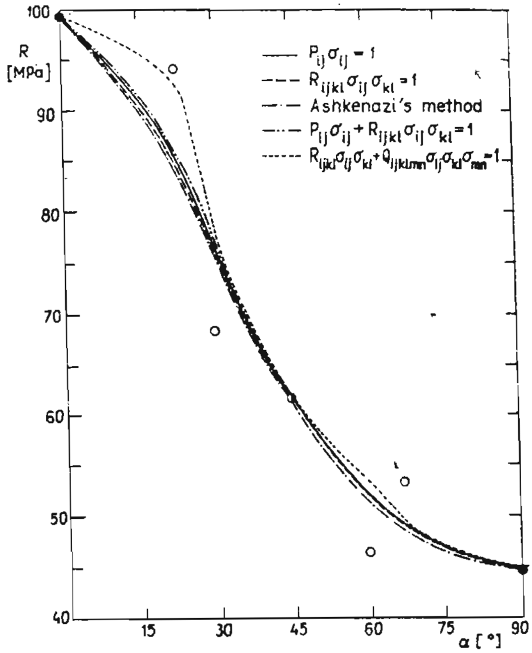

Fig. 3. Comparison of the strength description in plane 12 by different methods

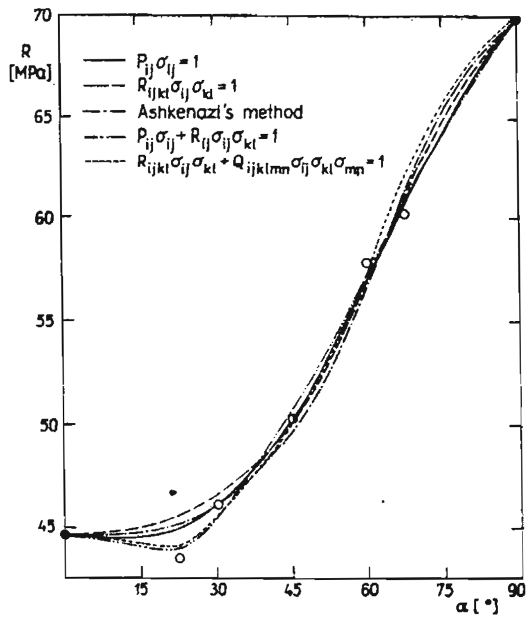

Fig. 4. Comparison of the strength description in plane 23 by different methods

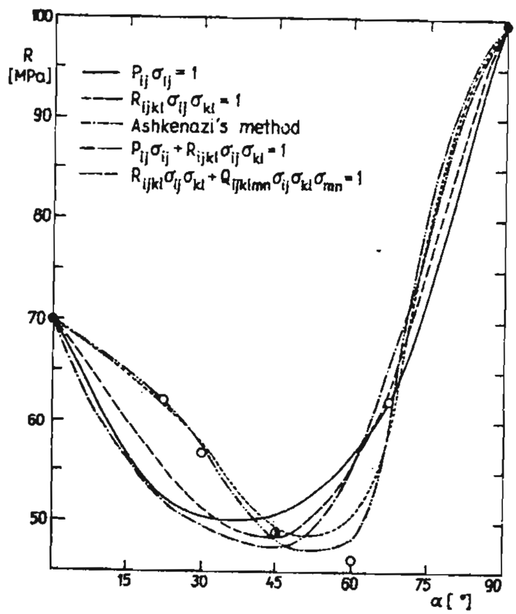

Fig. 5. Comparison of the strength description in plane 31 by different methods

In these figures, the black points are the mean experimental values of the strength on the principal orthotropy axes, through which all the curves pass. These points are the reference points for all three planes.

4. Choice of criterion for a minimal number of measuring points

In calculating the coefficients from equations containing only one term (eqs.(3.6), (3.11), (3.15)), three coeffcients are to be determined, two of which are determined on assuming curve transition through the mean values of strength in the principal orthotropy axes. One coefficient is, however, determined by the least square method. For a given orthotropy plane, this requires investigation of the strength in at least four directions. Where the least square method is not applied, in order to reduce the investigation range, one can select three directions only and on the basis of the strength in these directions determinre the constants (usually the principal orthotropy axes and the directions for angle $\alpha = 45^\circ$ are chosen).

4.1. Description of compression strength by the use the first term of eq.(1.1) in the minimal test number method (MTNM)

Transforming eq.(3.6), we have

$$R^c_\alpha = \frac{\bar{R}^c_0}{\cos\alpha(\cos\alpha - \sin\alpha) + b\sin 2\alpha + c\sin\alpha(\sin\alpha - \cos\alpha)}, \tag{4.1}$$

where:

$$b = \frac{\bar{R}^c_0}{\bar{R}^c_{45}}, \qquad c = \frac{\bar{R}^c_0}{\bar{R}^c_{90}}. \tag{4.2}$$

4.2. Description of compression strength by the use the second term of eq.(1.1) in of the minimal test number method (MTNM)

On transforming eq.(3.10), we obtain

$$R^c_\alpha = \frac{\bar{R}^c_0}{\sqrt{\cos^4\alpha + b_1\sin^2 2\alpha + c_1\sin^4\alpha}}, \tag{4.3}$$

where:

$$b_1 = \frac{\bar{R}^c_0}{\bar{R}^c_{45}} - \frac{1+c_1}{4}, \qquad c_1 = \frac{\bar{R}^c_0}{\bar{R}^c_{90}}. \tag{4.4}$$

4.3. Ashkenazi's method of compression strength description in the minimal test number method (MTNM)

On transforming eq.(3.15) we obtain

$$\bar{R}^c_\alpha = \frac{\bar{R}^c_0}{\cos^4\alpha + b_2\sin^2 2\alpha + c_2\sin^4\alpha}, \tag{4.5}$$

where:

$$b_2 = \frac{\bar{R}^c_0}{\bar{R}^c_{45}} - \frac{1+c_2}{4}, \qquad c_2 = \frac{\bar{R}^c_0}{\bar{R}^c_{90}}. \tag{4.6}$$

The compression strength described by eqs.(4.1), (4.3), (4.5) is shown in figs.6,7,8 (full line). In these figures, the curves (broken line) which were calculated from eqs.(3.9), (3.14) and (3.18) are presented. The equations were obtained by the least square method (LSM) for the single terms of eq.(1.1), i.e. for:

$$P_{ij}\sigma_{ij} = 1, \qquad R_{ijkl}\sigma_{ij}\sigma_{kl} = 1$$

and for Ashkenazi's method.

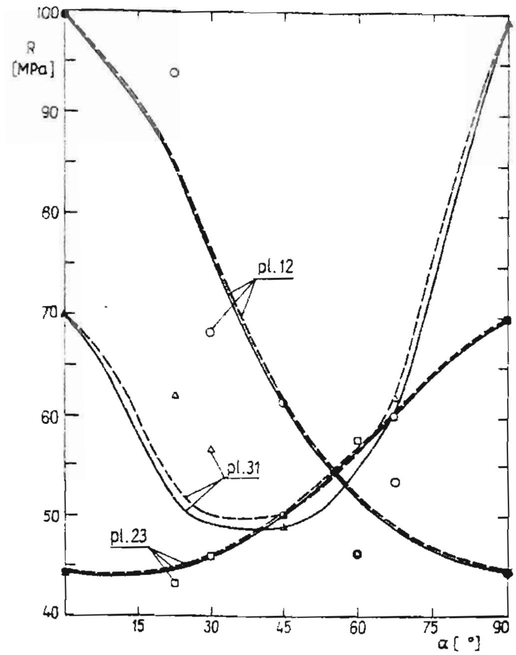

Fig. 6. Comparison of the strength description using the first term ($P_{ij}\sigma_{ij} = 1$): MTMN – full line, LSM – broken line

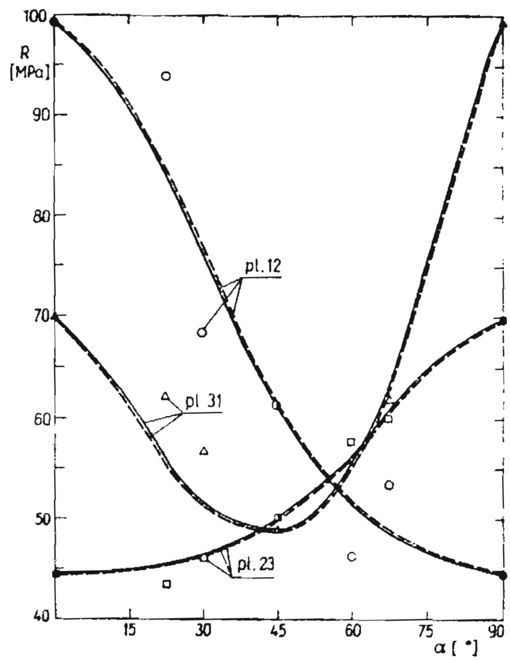

Fig. 7. Comparison of the strength description using the second term ($R_{ijkl}\sigma_{ij}\sigma_{kl} = 1$): MTMN – full line, LSM – broken line

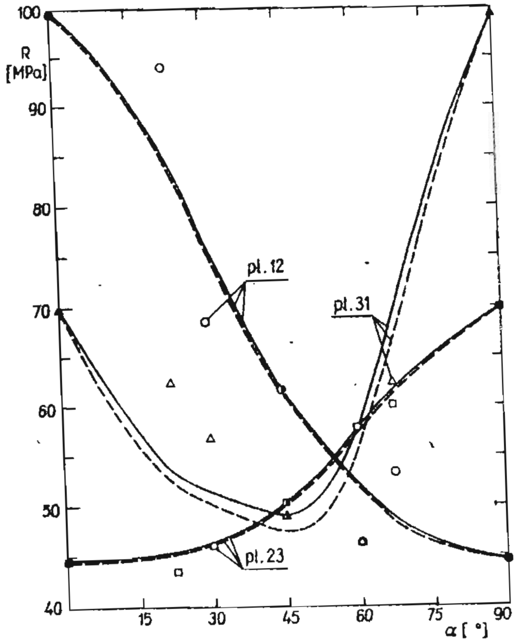

Fig. 8. Comparison of the strength description using of Ashkenazi's method: MTMN – full line, LSM – broken line.

In figs.6,7,8 the full dark points are the mean experimental values of the strength in the principal orthotropy axes. The curves obtained by both methods, LSM and MTMN pass through these points. The half dark points are the mean values of strength through which only curves calculated by the MTMN pass.

5. Analysis of the results

5.1. Analysis of the results of uniaxial failure on compression

For comparison of the different methods of description and for assessment of correctness of this description, we calculate the mean square absolute and relative error from the eqs.:

$$r_1 = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(R_{ei} - R_t)^2}, \qquad r = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(R_{ei} - R_t)^2}{R_t}}. \tag{5.1}$$

where: R_{ei} – experimental strength, R_t – theoretical strength, $N = 70$ – number of measurements. The results are shown in tab.6.

Table 6. Results of calculations of r_1 and r

Method of description	Plane					
	12		23		31	
	r_1 [MPa]	r	r_1 [MPa]	r	r_1 [MPa]	r
LSM $P_{ij}\sigma_{ij} = 1$ eq.(3.9)	6.000	0.0906	5.057	0.0919	7.020	0.1278
MTNM $P_{ij}\sigma_{ij} = 1$ eq.(4.1)	6.060	0.0915	5.088	0.0924	7.021	0.1319
LSM $R_{ijkl}\sigma_{ij}\sigma_{kl} = 1$ eq.(3.14)	6.017	0.0911	5.089	0.0926	6.099	0.1091
MTNM $R_{ijkl}\sigma_{ij}\sigma_{kl} = 1$ eq.(4.3)	6.060	0.0915	5.107	0.0925	6.099	0.1082
LSM Ashkenazi's meth. eq.(3.17)	6.043	0.0914	5.088	0.0923	7.180	0.1321
MTNM Ashkenazi's meth. eq.(4.5)	6.060	0.0915	5.101	0.0921	7.348	0.1289
$P_{ij}\sigma_{ij} + R_{ijkl}\cdot$ $\cdot\sigma_{ij}\sigma_{kl} = 1$ eq.(3.23)	5.773	0.0888	5.022	0.0913	3.961	0.0676
$R_{ijkl}\sigma_{ij}\sigma_{kl}+$ $+Q_{ijklmn}\sigma_{ij}\sigma_{kl}\cdot$ $\cdot\sigma_{mn} = 1$, eq.(3.28)	5.508	0.0884	5.050	0.0912	4.312	0.0741

5.2. Analysis of the failure investigation results in biaxial stress states

For generalization of the failure investigation results which are given as mean strengths in tabs.1,2,3 it is advisable to destribe the failure ellipses in the three orthotropy planes: 12, 23 and 31. For this purpose the first two terms from eq.(1.1) must be taken into consideration. Thus for failure in biaxial stress states, limited to the directions of the orthotropy axes, the general equation will be taken the form

$$P_{ij}\sigma_{ii} + P_{jj}\sigma_{jj} + R_{iiii}\sigma_{ii}^2 + R_{jjjj}\sigma_{jj}^2 + 2R_{iijj}\sigma_{ii}\sigma_{jj} = 1, \tag{5.2}$$

where for planes: 12, $i = 1$, $j = 2$, 23, $i = 2$, $j = 3$, 31, $i = 3$, $j = 1$.

Assuming that eqs.(5.2) are satisfied for strength on compression and tension in the direction of the orthotropy axes, and in biaxial compression stresses in the direction of these axes, for every plane of orthotropy, we obtain five equations from which we determine the constants. The values of coefficients of eqs.(5.2) are given in tab.7. On the basis of these of the coefficient values it can be shown that eqs.(5.2) in our a case are ellipses.

Table 7. Values of the orthotropy coefficients for eqs.(5.2)

Orthotropy coefficients	Coefficient values		
	Plane 12	Plane 23	Plane 31
P_{ii} [MPa^{-1}]	- 4.9247 ·10^{-3}	0.013343	0.10199
P_{ij} [MPa^{-1}]	0.013343	0.10199	- 4.9247·10^{-3}
R_{iiii} [MPa^{-2}]	5.1513·10^{-5}	7.9898·10^{-4}	1.6611·10^{-3}
R_{jjjj} [MPa^{-2}]	7.9898·10^{-4}	1.6611·10^{-3}	5.1513·10^{-5}
R_{iijj} [MPa^{-2}]	- 1.3307·10^{-4}	0.5656·10^{-3}	- 0.9644·10^{-4}

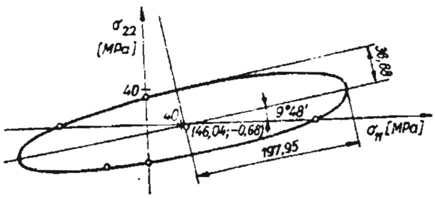

Fig. 9. Ellipse of failure in orthotropy plane 12

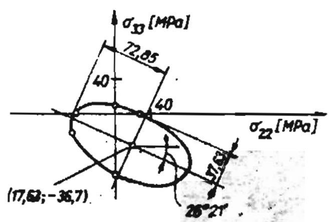

Fig. 10. Ellipse of failure in orthotropy plane 23

The description of the compressed wood failure according to eqs.(5.2) are shown in the form of ellipses with their parameters: for plane 12 in fig.9, for plane 23 in fig.10, for plane 31 in fig.11.

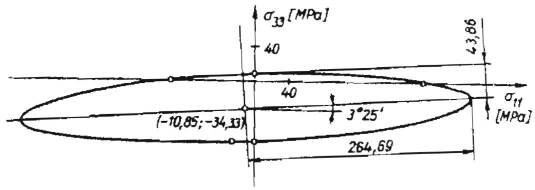

Fig. 11. Ellipse of failure in orthotropy plane 31

6. Conclusions

6.1. Uniaxial failure on compression and tension

On comparing the values r_1 and Δ_1, r and $\Delta_2^{rel.}$ (tab.6 and 5) the following conclusions can be drawn:

1. All the descriotion methods provide correct results (for three planes $r < \Delta_2^{rel.}$).

2. The best description is obtained on applying both the second and the third terms of eq.(1.1) because the conditions $r < \Delta_2^{rel.}$ and $r_1 < \Delta_1$ for all planes are satisfied. In this method, the number of angles must be greater than seven. However where there are seven the tensile strength in the principle orthotropy axes should be investigated. In this study the latter method of approach was applied. For other methods, these two conditions are satisfied only for the planes 23 and 31. This is confirmed by the data in figs.3–8. From tab.6 it is also seen that slightly better results are obtained by methods which have two terms (r_1 and r are less than those in other methods). However, for these methods it is necessary to investigate the failure for a greater of number of angles because there are more constants to determine.

3. From tab.6 and figs.6–8 it is seen that reducing the number of angles to three and taking into account only single terms of eq.(1.1) (first, second or Ashkenazi's method), we obtain a corret description with, a minimal test number in all these methods. However, the best results were obtained from equation containing the anisotropy tensor of the fourth rank.

6.2. Failure in biaxial stress states

1. The correctness of the failure description by the hypothesis (5.2) was verified above (the first conclusion under p. 6.1).

2. From figs.9–11 it is seen that in our case, the hypothesis (5.2) describes the failure ellipses which are displaced and turned in all, the orthotropy planes. The

greates displacements in the ellipse midpoint occur in the direction of orthotropy axis 1 in plane 12 and in the direction of orthotropy axias 3 in plane 23 and 31. The largest angle of the ellipse turn occurs in the plane of orhotropy 23.

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Streszczenie

W pracy badane są możliwości stosowania różnych opisów anizotropii wytrzymałości na ściskanie materiału drewnopochodnego, o własnościach ortotropowych i następnie poddanie ich krytycznej analizie. Badania wytrzymałości na ściskanie uzupełniono koniecznymi badaniami na rozciąganie w kierunkach osi ortotropii. Przy rozpatrywaniu tylko trzech pierwszych wyrazów wielomianowego tensorowego kryterium anizotropowej wytrzymałości w postaci:

$$P_{ij}\sigma_{ij} + R_{ijkl}\sigma_{ij}\sigma_{kl} + Q_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn} = 1,$$

zbadano opis: 1) przy pomocy pierwszego członu, 2) pierwszego i drugiego, 3) drugiego, 4) drugiego i trzeciego. Stwierdzono, że najodpowiedniejszy opis ze względu na liczbę prób doświadczalnych uzyskuje się tylko z drugiego członu, kiedy stosowano minimalną liczbę prób.

W pracy proponuje się również opis zaproponowany przez E.K.Aszkenazi, polegający na podobieństwie tensora wytrzymałości do tensora stałych sprężystych.

Dla uogólnienia wyników badań zniszczenia przeprowadzono dodatkowe badania przy dwuosiowym ściskaniu. Dla pierwszych dwóch członów tego kryterium otrzymano w płaszczyznach ortotropii trzy elipsy zniszczenia. Elipsy te są przesunięte i obrócone w układzie osi ortotropii.

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