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BIFURCATING SELF-EXCITED VIBRATIONS OF A JOURNAL BEARING ROTOR

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1. Introduction

The dynamics of rotors suppored in journal bearings has been studied for more than 60 years. Rich literature on the subject from the early paper by Newkirk and Taylor (see [1]) through more advanced research done by Hori [2] and Tondl [3] to the recent results obtained by Akkok and Ettles [4], Malik and Hori [5] and Muszyńska [6,7] gives a survey of methods and approaches to the problem of stability, critical rotational speed and postcritical whirling of shafts rotating in journal bearings. A comprehensive study of transverse vibrations of an unloaded flexible rotor/bearing system was presented in [6] and then developed in [7]. There are, however, structures in which rotors work under significant transverse loadings and their eccentric equilibrium stability and self excited vibration create a research problem. Provided that an adequate journal bearing model is admitted, a critical speed of rotation can be found by applying Hurwitz stability criterion to the linearized system. That is often enough to design a reliable rotor/bearing system operating far from the stability limit. One of the important problems of a nonlinear approach is the behaviour of a rotating system in a neighbourhood of criticality. On the one hand, sometimes rotors have to operate under conditions close to criticality and a vibration of small amplitude is acceptable. On the other hand, it is important to be able to exclude subcritical self-excited vibrations which correspond to a catastrophic loss of stability and are much more dangerous than usually considered supercritical ones.

A satisfactory work of a rotor/bearing system in an unstable region of equilibrium determined on a linearized theory was experimentally confirmed by Akkok and Ettles [4]. Malik and Hori [5] have recently obtained nonlinear trajectories in an unstable region of a linearized system using an approximate nonlinear analysis.

The present paper offers a new approach to the above mentioned problem based on the theory of Hopf bifurcation which has been developed by Hassard [8], Iooss and Joseph [9] and others. The Hopf bifurcation theory provided a mathematical tool for a nonlinear analysis and, moreover, it enabled us to discover some interesting qualitative features of the system near critically. It is well known that rotors supported in journal bearings exhibit critical speeds of rotation. However, their near critical behaviour (super – as well as subcritical) is still a problem of question. Self-excited vibrations which occur at criticality and develop with further changes in rotation speed correspond to the Hopf bifurcation in the differential equations of motion. This paper gives an explanation to the phenomenon of self-excitation of a rigid rotor vibration due to an oil-film action. First it is shown that the considered system satisfies the assumptions of the Hopf theorem and the bifurcation theory can be used. A theory formulated by Iooss and Joseph [9] is applied which enables one to construct a bifurcating periodic solution in a parametric form of a series and to determine its stability.

2. Equations of motion

The considered system is a rigid rotor of mass m supported in journal bearings and statically loaded by a transverse force Q of constant direction. The system is symmetric and its motion is assumed to be plane. Applying a plane journal

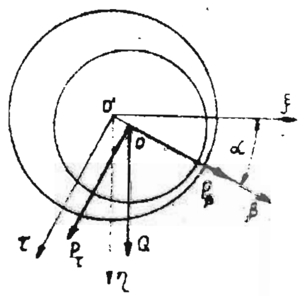

Fig. 1. Coordinate system

bearing model based on superposition of hydrodynamical forces corresponding to the wedge and squeeze effects treated separately, we obtain the following equations of motion in the polar coordinate system shown in fig.1 (see Appendix):

$$
\begin{aligned}
m(\ddot{\beta} - \beta\dot{\alpha}^2)\varepsilon' &= F_\beta(\beta,\alpha,\dot{\beta},\dot{\alpha};\omega,Q), \\
m(\beta\ddot{\alpha} + 2\dot{\beta}\dot{\alpha})\varepsilon' &= E_\tau(\beta,\alpha,\dot{\beta},\dot{\alpha};\omega,Q),
\end{aligned}
\tag{2.1}
$$

where:

$$F_\beta = P_\beta + Q\sin\alpha, \qquad F_\tau = P_\tau + Q\cos\alpha,$$

$$P_\beta = -2CR\left\{\frac{\beta^2(\omega - 2\dot{\alpha})}{(1-\beta^2)(2+\beta^2)} + \dot{\beta}\left[\frac{\beta}{1-\beta^2} + \frac{2}{(1-\beta^2)^{\frac{3}{2}}}\mathrm{arctg}\sqrt{\frac{1+\beta}{1-\beta}}\right]\right\}, \quad (2.2)$$

$$P_\tau = \Pi CR\frac{\beta(\omega - 2\dot{\alpha})}{(1-\beta^2)(2+\beta^2)},$$

where in turn:

$$C = \frac{6\mu L}{m\delta^3}$$

and μ – oil viscosity, L – bearing total length, R – journal radius, ε' – bearing clearance, $\delta = \varepsilon'/R$ – clearance ratio, ω – angular speed of rotation, m – rotor mass, Q – transverse static load, (β, α) – polar coordinates of journal center. Equations of motion (2.1) contain two parameters ω and Q which are essential for the bifurcation parameter.Puting $\dot{\beta} = \dot{\alpha} = 0$ and $\ddot{\beta} = \ddot{\alpha} = 0$ in (2.1) leads to the expressions:

$$\frac{2C\beta^2\omega}{(1-\beta^2)(2+\beta^2)} = \frac{Q}{m\varepsilon'}\sin\alpha,$$

$$\frac{C\pi\beta\omega}{(1-\beta^2)^{\frac{1}{2}}(2+\beta^2)} = \frac{-Q}{m\varepsilon'}\cos\alpha, \quad (2.3)$$

which determine the set of equilibrium points (β_0, α_0):

$$\mathrm{tg}\alpha_0 = -\frac{2\beta_0}{\pi(1-\beta_0^2)^{\frac{1}{2}}}. \quad (2.4)$$

For every fixed Q and ω, equilibrium coordinates β_0 and α_0 can be calculated using relations (2.3), therefore $\beta_0 = \beta_0(Q,\omega)$ and $\alpha_0 = \alpha_0(Q,\omega)$. Introducing new variables:

$$\begin{aligned} u_1 &= \beta - \beta_0, \\ u_2 &= \dot{u}_1, \\ u_3 &= \alpha - \alpha_0, \\ u_4 &= \dot{u}_3, \end{aligned} \quad (2.5)$$

we obtain the following matrix equation of motion:

$$\dot{\mathbf{u}} = \mathbf{f}(\omega, \mathbf{u}; Q), \quad (2.6)$$

where:

$$\mathbf{u} = [u_1, u_2, u_3, u_4]^\top, \qquad \mathbf{f} = [f_1, f_2, f_3, f_4]^\top,$$

and

$$
\begin{aligned}
f_1 &= u_2,\\
f_2 &= (u_1+\beta_0)u_4^2 + \frac{1}{m\varepsilon'}[P_\beta(u_1+\beta_0, u_3+\alpha_0, u_2, u_4; Q) +\\
&\quad +Q\sin(u_3+\alpha_0)],\\
f_3 &= u_4,\\
f_4 &= -\frac{2u_2u_4}{u_1+\beta_0} + \frac{1}{m\varepsilon'(u_1+\beta_0)}[P_\tau(u_1+\beta_0, u_3+\alpha_0, u_2, u_4; \omega, Q) +\\
&\quad +Q\cos(u_3+\alpha_0)].
\end{aligned}
\tag{2.7}
$$

3. Stability of equilibrium

We shall determine the region of stability of the trivial solution of equation (2.6) applying the Routh-Hurwitz criterion to the linearized equation of motion:

$$\dot{\mathbf{u}} = \mathbf{A}(\omega, Q)\mathbf{u}, \tag{3.1}$$

where:

$$A_{ij} = \left.\frac{\partial f_i}{\partial u_j}\right|_{\mathbf{u}=0}.$$

The characteristic equation for a nondimensional eigen-value r can be written as follows:

$$r^4 + b_3r^3 + b_2r^2 + b_1r + b_0 = 0, \tag{3.2}$$

where:

$$
\begin{aligned}
b_0 &= \frac{1}{\omega^4}(A_{21}A_{43} - A_{41}A_{23}),\\
b_1 &= \frac{1}{\omega^3}(A_{22}A_{43} + A_{44}A_{21} - A_{24}A_{41}),\\
b_2 &= \frac{1}{\omega^2}(-A_{43} - A_{21} + A_{22}A_{44}),\\
b_3 &= \frac{1}{\omega}(-A_{44} - A_{22}).
\end{aligned}
$$

Calculating derivatives A_{ij} and introducing $\psi = \frac{C}{6\omega}$, we obtain the following expressions for coefficients of equation (3.2):

$$b_0 = \frac{36\psi^2}{B^3D^3}\left\{8\beta_0^2(2+\beta_0^4) + \pi^2B(2\beta_0^4 - \beta_0^2 + 2)\right\},$$

$$\begin{aligned} b_1 &= \frac{144\psi^2\beta_0}{B^{\frac{5}{2}}D^2}(\pi + W), \\ b_2 &= \frac{12\psi}{B^2D^2}\left\{\beta_0(6-\beta_0^2+\beta_0^4) + 12\pi\psi W\right\}, \\ b_3 &= \frac{12\psi}{B^{\frac{3}{2}}D}, \end{aligned} \tag{3.3}$$

where:

$$B = 1-\beta_0^2, \quad D = 2+\beta_0^2, \quad W = D\left(\beta_0 B^{\frac{1}{2}} + 2\text{arctg}\sqrt{\frac{1+\beta_0}{1-\beta_0}}\right).$$

Since $b_i > 0$ $(i = 0,1,2,3)$, boundary of the stability region on the (β_0, ψ) – plane is described by the following equality (on the Routh-Hurwitz criterion):

$$b_1b_2b_3 - b_0b_3^2 - b_1^2 = 0, \tag{3.4}$$

which can be solved with respect to ψ:

$$\psi = h(\beta_0), \tag{3.5}$$

where $h(\beta_0)$ is a monotonically decreasing function shown in fig.2. The region

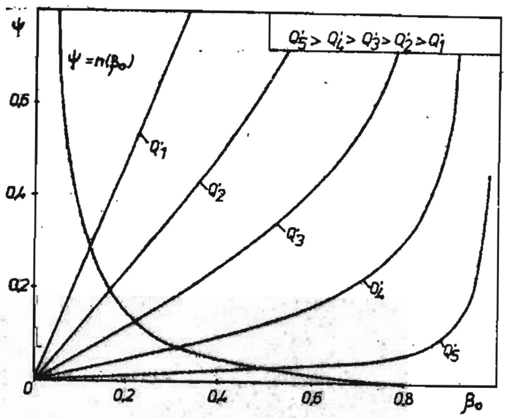

Fig. 2. Boundary of the stability region and curves corresponding to increase in rotation speed under constant nondimensional load

above the curve is the region of stability. Fig.2 also presents a set of curves representing operating points for various journal loadings with increasing rotation speed ω. They form a family of curves derived from (2.3) and described as follows:

$$\psi = \frac{1}{Q'}\frac{\beta_0}{BD}(4\beta_0^2 + \pi^2 B)^{\frac{1}{2}}, \tag{3.6}$$

where $Q' = 6Q/(m\varepsilon' C^2)$ is the family parameter.

At points of intersection of curves (3.5) and (3.6) there always exists a pair of imaginary eigen-values and that is one of the necessary conditions for the Hopf bifurcation. A dominant role is played by the first eigen-value (with maximum real part):

$$r_1(\omega, Q') = \xi(\omega, Q') + i\eta(\omega, Q'),$$

which becomes imaginary at the criticality, i.e:

$$r_1(\omega_{cr}, Q') = i\Omega_0(Q').$$

It was examined by solving the eigen-problem that the trajectory of the first eigen-value r_1 on the complex plane (ξ, η) intersected the imaginary axis for every Q'. It implies the following inequality:

$$\frac{d\xi}{d\omega}(\omega_{cr}, Q') > 0,$$

which expresses the second condition for the Hopf bifurcation.

4. Bifurcating solution

Bifurcating at the critically periodic solution of equation (2.6) is sought in the following parametric form of a series due to Iooss and Joseph [9]:

$$\begin{aligned} u(s,\varepsilon) &= \sum_{n=1}^{\infty}(n!)^{-1}\varepsilon^n u^{(n)}(s), \qquad s = \Omega(\varepsilon)t, \\ \omega &= \omega_{cr} + \sum_{n=1}^{\infty}(n!)^{-1}\varepsilon^n \omega_n, \\ \Omega &= \Omega_0 + \sum_{n=1}^{\infty}(n!)^{-1}\varepsilon^n \Omega_n, \end{aligned} \tag{4.1}$$

where Ω is the periodic solution frequency, ε is a parameter interpreted as a distance between periodic and constant solutions, $u^{(n)}(s)$ are 2π – periodic functions and ω_n, Ω_n are constants to be determined.

Let $P_{2\pi}$ denotes a space of continuous, differentiable 2π – periodic functions with the scalar product:

$$[a(s), b(s)] \stackrel{\text{def}}{=} \frac{1}{2\pi}\int_0^{2\pi} < a(s), b(s) > ds = \frac{1}{2\pi}\int_0^{2\pi} a_i \bar{b}_i ds. \tag{4.2}$$

Then unknown functions $u^{(n)}(s)$ belong to $P_{2\pi}$.

Introducing (4.1) into (2.6), expanding the right hand side of (2.6) into Maclaurin's series and using the notion of the multilinear operator:

$$\underbrace{f_{uu...u}}_{k}(\omega, u_0|a_1|...|a_k) \stackrel{\text{def}}{=} \lim_{\delta_1,...,\delta_k \to 0} \frac{f(\omega, u_0 + \delta_1 a_1 + ... + \delta_k a_k; Q)}{\partial\delta_1 ... \partial\delta_k},$$

We obtain the following linear recurent system of equations:

$$\begin{aligned} I_0 u^{(1)} &= 0, \\ I_0 u^{(n)} &= g_n(s) \quad \text{for } n > 1, \end{aligned} \tag{4.3}$$

where:

$$I_0(\cdot) \stackrel{\text{def}}{=} -\Omega_0 \frac{d(\cdot)}{ds} + f_u(\omega_{cr}, 0|(\cdot)),$$

$$g_n(s) = g_n(s + 2\pi) = n\Omega_{n-1}\frac{du^{(1)}}{ds} - n\omega_{n-1} f_{u\omega}(\omega_{cr}, 0|u^{(1)}) - R_{n-1}$$

and R_{n-1} contains terms of order lower than n.

Introduce harmonic functions:

$$z = \xi_0 e^{is}, \qquad z^* = \xi_0^* e^{is}, \tag{4.4}$$

where ξ_0 and ξ_0^* are orthonormal eigen-vectors of $\mathbf{A}(\omega, Q)$ and $\mathbf{A}^{\mathsf{T}}(\omega, Q)$ corresponding to the first eigen-value at criticality. Assume also the following condition:

$$[u, z^*] = \varepsilon, \tag{4.5}$$

which implies that the first harmonic is contained in $u^{(1)}$ and functions $u^{(n)}$ for $n > 1$ do not contain it.

It is easy to examine that $I_0 z = I_0 \bar{z} = 0$. As z and $\bar{z}$ are linearly independent, they can be superposed to form the steady solution for $u^{(1)}$ satysfying condition $[u^{(1)}, z^*] = 1$, which yields from (4.5).

Therefore:

$$u^{(1)} = z + \bar{z}. \tag{4.6}$$

The n-th equation of (4.3) has a 2π – periodic solution when the following orthogonality condition (Fredholm alternative) is satisfied:

$$[g_n, z^*] = 0. \tag{4.7}$$

It enables to eliminate secular terms from solutions for $n > 1$ and it can be transformed to two real equations for unknown coefficients ω_{n-1} and Ω_{n-1}:

$$\begin{aligned} n\omega_{n-1}\xi_\omega(\omega_{cr}) + \text{Re}[R_{n-1}, z^*] &= 0, \\ n\Omega_{n-1} + n\omega_{n-1}\eta_\omega(\omega_{cr}) + \text{Im}[R_{n-1}, z^*] &= 0. \end{aligned} \tag{4.8}$$

For $n = 2$ we have $R_1 = f_{uu}(\omega_{cr}, 0|u^{(1)}|u^{(1)})$. Using (4.6) and (4.4) we can show that $[R_1, z^*] = 0$ and then $\omega_1 = \Omega_1 = 0$. Moreover, applying the mathematical induction, one can prove that $\omega_{2k-1} = \Omega_{2k-1} = 0$ for $k \in N$.

Putting $n = 3$, we obtain ω_2 and Ω_2 from (4.8):

$$\begin{aligned} \omega_2 &= -\frac{\mathrm{Re}[R_2, z^*]}{3\xi_\omega(\omega_{cr})}, \\ \Omega_2 &= 3\omega_2\eta_\omega(\omega_{cr}) + \frac{1}{3}\mathrm{Im}[R_2, z^*], \end{aligned} \tag{4.9}$$

where:

$$R_2 = \frac{3}{2}f_{uu}(\omega_{cr}, 0|u^{(1)}|u^{(2)}) + f_{uuu}(\omega_{cr}, 0|u^{(1)}|u^{(1)}|u^{(1)}). \tag{4.10}$$

Periodic function $u^{(2)}(s)$ is described by equation of type (4.3). Its right hand side has the following form after applying the Fredholm alternative:

$$g_2(s) = S + Pe^{i2s} + \bar{P}e^{-i2s}, \tag{4.11}$$

where:

$$P = -f_{uu}(\omega_{cr}, 0|\xi_0|\xi_0) \quad \text{and} \quad S = -2f_{uu}(\omega_{cr}, 0|\xi_0|\bar{\xi}_0).$$

Solving (4.3) and using (4.11), we obtain:

$$u^{(2)} = K + Y + \bar{Y}, \tag{4.12}$$

where:

$$\begin{aligned} Y &= Le^{i2s}, \\ K &= \left\{\mathsf{A}^{-1}(\omega_{cr}, Q)\right\}S, \\ L &= \left\{\mathsf{A}(\omega_{cr}, Q) - 2i\Omega_0\mathsf{I}\right\}^{-1}P \end{aligned}$$

and I is the unit 4×4 matrix.

Required in (4.9) scalar product of R_2 and z^* after some algebra can be expressed as follows:

$$\begin{aligned} [R_2, z^*] &= \frac{3}{2}\langle (f_{uu}(\omega_{cr}, 0|\xi_0|K) + f_{uu}(\omega_{cr}, 0|\xi_0|L)), \xi_0^*\rangle + \\ &+ 3\langle f_{uuu}(\omega_{cr}, 0|\xi_0|\xi_0|\xi_0), \xi_0^*\rangle. \end{aligned} \tag{4.13}$$

Now we have determined everything which is necessary to build the periodic solution of the second order approximation:

$$\boldsymbol{u}(s,\varepsilon) = \varepsilon(\boldsymbol{\xi}_0 e^{is} + \bar{\boldsymbol{\xi}}_0 e^{-is}) + \frac{1}{2}\varepsilon^2(\boldsymbol{K} + \boldsymbol{L}e^{i2s} + \bar{\boldsymbol{L}}e^{-i2s}),$$

$$\omega = \omega_{cr} + \frac{1}{2}\omega_2\varepsilon^2, \qquad (4.14)$$

$$\Omega = \Omega_0 + \frac{1}{2}\Omega_2\varepsilon^2,$$

where $i\Omega_0 = r_1(\omega_{cr}), \boldsymbol{\xi}_0$ is the normal eigen-vector described by the equation $\{\mathsf{A}(\omega_{cr}, Q) - i\Omega_0 \mathsf{I}\}\boldsymbol{\xi}_0 = 0$ and ε is proportional to the norm of $\boldsymbol{u}$ in $P_{2\pi}$.

5. Stability of the bifurcating solution

As the equilibrium is unstable in postcritical situations, one could expect that the bifurcating periodic solution corresponding to the self-excited vibration of the rotor is alvays stable. However, looking at expressins (4.14) we can suspect that things can be different. In some cases the subcritical bifurcation is possible in which the bifurcating solution exists for $\omega < \omega_{cr}$. So, stability of the periodic solution is not a trivial problem.

Let $\boldsymbol{U}(s,\varepsilon)$ denotes the bifurcating solution, so that:

$$\dot{\boldsymbol{U}} \equiv \boldsymbol{f}(\omega, \boldsymbol{U}; Q).$$

Consider the linearized equation in the neighbourhood of U:

$$\dot{\boldsymbol{v}} = \boldsymbol{f}_{\boldsymbol{u}}(\omega, \boldsymbol{U}(s,\varepsilon)|\boldsymbol{v}), \qquad (5.1)$$

where $\boldsymbol{v} = \boldsymbol{u} - \boldsymbol{U}$.

Stability of the trivial solution of (5.1) depends on the Floquet exponents of this equation. It can be shown that periodic function $\boldsymbol{U}$ satisfies equation (5.1), so one of the Floquet exponents is equal to 0.

On the factorization theorem (after Iooss and Joseph [9]) the second real Floquet exponent determining orbital stability of the bifurcating solution can be expressed in the following form:

$$\sigma(\varepsilon) = \hat{\sigma}(\varepsilon)\frac{d\omega}{d\varepsilon}, \qquad (5.2)$$

where $\hat{\sigma}(\varepsilon)$ is a smooth function in a neighbourhood of $\varepsilon = 0$, such that $\hat{\sigma}_\varepsilon(0) = -\xi_\omega(\omega_{cr})$ and $\frac{1}{\varepsilon}\hat{\sigma}(\varepsilon)$ is an even function. Since $\frac{d\omega}{d\varepsilon} = \varepsilon\omega_2 + 0(\varepsilon^4)$ and $\hat{\sigma}(\varepsilon) = -\xi_\omega(\omega_{cr})\varepsilon + 0(\varepsilon^2)$ then:

$$\sigma(\varepsilon) = -\xi_\omega(\omega_{cr})\omega_2\varepsilon^2 + 0(\varepsilon^3). \qquad (5.3)$$

Therefore, the condition for the asymptotic orbital stability of the bifurcating periodic solution is:

$$\xi_\omega(\omega_{cr})\omega_2 > 0. \qquad (5.4)$$

We shall examine this condition for different transverse loadings Q in the next part of the paper.

6. Exemplary numerical results

In the numerical calculations the transverse loading was represented by the nondimensional force Q'.

For every Q' selected from a set corresponding to a wide loading range the following quantities were calculated:

ω_{cr}	–	critical rotation speed,
β_{0cr}, ψ_{cr}	–	critical eccentricity and dynamic journal parameter,
$\xi_\omega(\omega_{cr}), \eta_\omega(\omega_{cr})$	–	ω - derivative of the first eigenvalue at criticality,
Ω_0	–	initial flutter frequency,
ω_2, Ω_2	–	first coefficients in $\omega(\varepsilon)$ and $\Omega(\varepsilon)$ series,
$\mathbf{K}$	–	vector of the constant component in the second approximation solution,
$\mathbf{L}$	–	second harmonic vector,
ξ_0, ξ_0^*	–	orthonormal eigen-vectors at criticality.

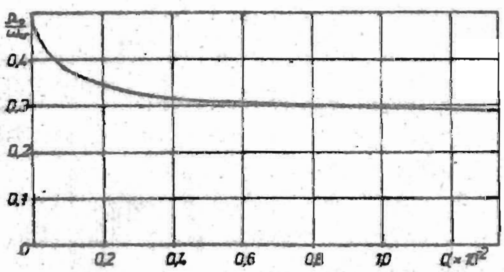

Fig. 3. Initial frequency of flutter vs. nondimensional static load

Fig.2 shows the curve $\psi = h(\beta_0)$ separating the regions of stability (above) and instability (below) of the equilibrium on the (β_0, ψ) – plane. When ω increases, the representing point moves down the curve corresponding to the selected static loading Q'. It enters the instability region for $\omega = \omega_{cr}$. It can be seen that self-excited vibration may occur in relatively large range of eccentricities. Relation

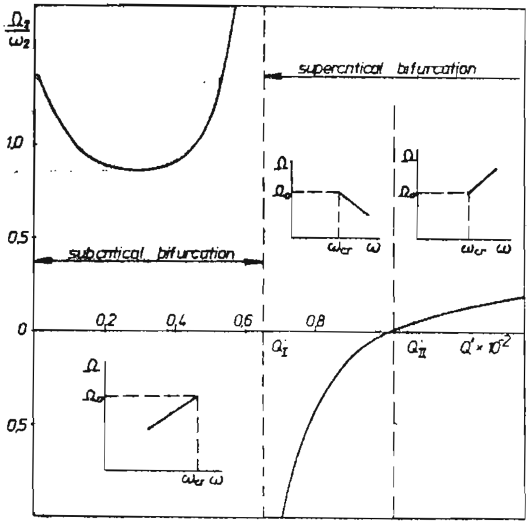

Fig. 4. Regions of sub-and supercritical Hopf bifurcation and corresponding graphs of flutter frequency vs. rotation speed

between the initial frequency of vibration and the static loading is presented in fig.3.

One of the most important results can be observed in fig.4 exhibiting existence of two loading domains. For small static forces Q', say $Q < Q'_I$ we deal with the subcritical bifurcation in which unstable vibration exists for $\omega < \omega_{cr}$. For $Q' > Q'_I$ the Hopf bifurcation is supercritical. Fig.4 shows also small fragments of $\Omega - \omega$ relation which is linear at the present approximation $(\Omega = \Omega_0 + \frac{\Omega_2}{\omega_2}(\omega - \omega_{cr}))$.

Two limit cycles projected on the plane of displacements – one unstable corresponding to a small nondimensional load Q' and the other stable for much greater Q' are shown in fig.5. Both limit cycles were obtained for the same relative distance from the criticality.

7. Final remarks and conclusions

The main results of the present paper are as follows

1. An approximate periodic solution describing small self-excited vibration of a journal bearing rotor has been constructed in a parametric form based on the Hopf bifurcation theory. Formulae derived in the paper enable one to determine amplitudes, frequency and phase displacements for both radial

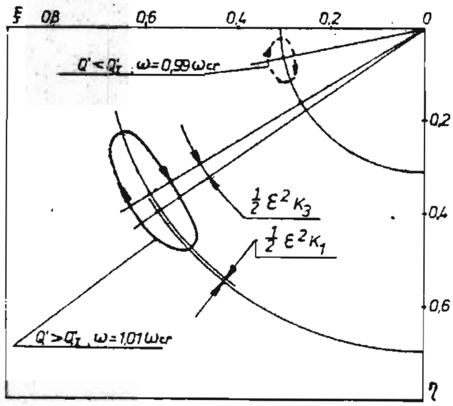

Fig. 5. Stable and unstable limit cycles projected on the plane of displacements

and circumferential components of a rotor motion as well as to determine its orbital stability.

2. Self-excited vibrations may occurs in large range of static loadings and respectively in large range of eccentricities. There is no characteristic eccentricity as mentioned in some previous works.

3. In case of small static loads we deal with a subcritical bifurcation to an unstable limit cycle for $\omega < \omega_{cr}$ and this corresponds to a catastrophic loss of equilibrium stability. When the static force is sufficiently great, a stable vibration bifurcates from the equilibrium that becomes unstable itself. This theoretical result coincides with some experimental data obtained by Tondl [3] and graphically presented without theoretical explanation.

4. The initial frequency of the self-excited vibration (flutter) considerably depends on the nondimensional static load. Devided by ω_{cr} it gives a fraction which is found to be much smaller than usually reported 0.5 (in the region of supercritical bifurcation it can be even smaller than 0.3).

5. The second order solution $\mathbf{u}^{(2)}(s)$ contains a constant component which depends on loading. It means that the center of vibration is slightly displaced from the initial equilibrium position.

6. The coefficient of inclination of the linear function $\Omega(\omega)$ also depends on loading. In the subcritical bifurcation domain it is positive. In the region of supercritical bifurcation it increases with the nondimensional loading from negative to positive.

7. The method of analysis applied is general and efficient enough to be used in variety of problems of elastic and viscoelastic rotor/bearing systems.

References

1. NEWKIRK B.L., GROBEL L.P., *Oil-film whirl - a non-whirling bearing,* Transactions of the ASME, 56, 1934, pp. 607-615

2. HORI Y., *A theory of oil whip,* Transactions of the ASME, Journal of Applied Mechanics, 26, 1959, pp. 189-198

3. TONDL A., *Some problems of rotor dynamics,* Chapman and Hall, London, 1965

4. AKKOK M., ETTLES C.M.M., *Journal bearing response to excitation and behavior in the unstable region,* ASLE Transactions, 27, 1984, pp. 341-351

5. MALIK M., HORI Y., *An approximate nonlinear transient analysis of journal bearing in unstable region of linearized system,* International Conference on Rotordynamics, Tokyo, 1986, pp. 217-220

6. MUSZYŃSKA A., *Whirl and whip - rotor/bearing stability problems,* Journal of Sound and Vibration, 110, 1986, pp. 443-462

7. MUSZYŃSKA A. RM ET AL., *Numerical and analytical study of fluid dynamic forces in seals and bearings,* ASME Design Engineering Division Conference on Vibration and Noise, Boston, 1987

8. HASSARD B.D., KAZARINOF N.D., WAN Y.H., *Theory and applications of Hopf bifucation,* Cambridge University Press, New York, 1981

9. IOOSS G., JOSEPH D.D., *Elementary stability and bifurcation theory,* Springer Verlag, New York, 1980

10. OSIŃSKI Z., *The nonlinear force of the oil film pressure in a radial bearing,* Prace Instytutu Podstaw Konstrukcji Maszyn Politechniki Warszawskiej, no 2, 1972, pp. 5-23

11. KURNIK W., STARCZEWSKI Z., *Hydrodynamical forces in a journal bearing corresponding to combined plane journal motion,* Machine Dynamics Problems, no 4, 1984, pp. 89-102

12. BRINDLEY J., ELLIOT L., MC CAY J.T., *The role of cavitation in whirl instability,* Journal of Applied Mechanics, 50, 1983, pp. 877-890

Appendix

The hydrodynamical pressure in a transverse journal bearing corresponding to a plane journal motion can be described by the well known Reynolds equation (see

[10] for the assumptions and derivation):

$$\frac{\partial}{\partial\varphi}(H^3\frac{\partial p^*}{\partial\varphi}) = \omega\frac{\partial H}{\partial\varphi} + 2\frac{\partial H}{\partial t}, \tag{0.1}$$

where $p^* = p/(6\mu\delta^2)$ – dimensionless pressure, $H = 1 - \beta\cos\Theta$ – normalized film thickness, t – time, φ, Θ – angular coordinates ($\Theta = \varphi - \alpha$).

The bearing is assumed to be sealed which means that the pressure is uniformly distributed along the bearing. The problem is how to determine boundary conditions for the pressure function. It can be shown that Gümbel's approach (so called $0 - \pi$ model, [2]) is not valid in cases when squeeze has to be taken into account [11]. The idea of superposition of the wedge and squeeze effects treated separately consists in decomposition of the combined journal motion into three elementary motions – pure rotation (ω), circumferential motion (α) and radial motion (β) [11]. The rotation of the journal and its circumferential motion create the wedge effect and the corresponding pressure function is equal to zero at the points of minimum and maximum oil film thickness. In the case of pure squeeze (radial journal motion) points at which the pressure equals zero belong to the diameter perpendicular to the radial velocity. Thus, the pressure distribution can be expressed as follows:

$$p^*(\Theta) = \frac{2 - \beta\cos\Theta}{(1 - \beta\cos\Theta)^2}\left\{\frac{\beta(2\dot{\alpha}) - \omega}{2 + \beta^2}\sin\Theta + \dot{\beta}\cos\Theta\right\} = p^*_w + p^*_s. \tag{0.2}$$

Expression (0.2) differs from that used by Hori [2] and by Brindley et al. [12] (see [11] for the comparison). The radial and circumferential components of the hydrodynamical force are determined as follows:

$$P_{\beta w} = -RL\int_0^{\pi} p_w(\Theta)\cos\Theta d\Theta, \quad P_{\beta s} = -RL\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} ps(\Theta)\cos\Theta d\Theta, \tag{0.3}$$

$$P_{\tau w} = -RL\int_0^{\pi} p_w(\Theta)\sin\Theta d\Theta, \quad P_{\tau s} = -RL\int_{\frac{-\pi}{2}} p_s(\Theta)\sin\Theta d\Theta = 0.$$

Calculating the integrals in (0.3) and superposing the effects of wedge and squeeze, we obtain expressions (0.2).

Streszczenie

Praca poświęcona jest analizie małych drgań samowzbudnych sztywnego, poprzecznie obciążonego wirnika łożyskowanego ślizgowo. W analizie zastosowano teorię bifurkacji Hopfa. Założono, że wirnik jest układem o dwóch stopniach swobody. Równania ruchu wynikają z płaskiego medelu łożyska ślizgowego, w którym superponowane są siły hydrodynamiczne odpowiadające efektom klina smarnego i wyciskania smaru. Rozwiązanie okresowe powstające z nietrywialnego położenia równowagi poszukiwane jest, według teorii Ioossa i Josepha, w postaci parametrycznej. Wyprowadzono rozwiązania pierwszego i drugiego przybliżenia. Rozważania teoretyczne zilustrowano obliczeniami liczbowymi, które wskazują, że obciążenie poprzeczne może znacznie zmienić zachowanie się układu w sąsiedztwie punktu krytycznego.

Резюме

В работе проводится анализ малых автоколебаний жёсткого ротора вращающегося в подшипниках скальжения в присутствии поперечной нагрузки. В анализе применяется теория бифуркации рождения цикла. Предполагается, что колебательная система обладает двумя степеньями свободы. Уравнения дбижения следуют из плоской модели подшипника, в которой отдельные эффекты смазочного клина и выдавливания смазки подвергаются сложению. Периодическое решение возникающие из нетривиального состояния равновесия пазыскивается в виде параметрического ряда по теории Иосса и Джозефа. Выведены решения первого и второго приближения. Теоретические рассуждения иллюстрируются численными примерами, которые указуют, что поперечная нагрузка может значительно изменить поведение системы вблизи критической точки.

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