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INTERNAL RESONANCE IN A KINEMATICALLY EXCITED SYSTEM OF RODS

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In this paper the method of analysis of internal resonance in the system of rods under vertical kinematic excitation is presented. The elements of the system are connected with articulated joints. The couplings of elements of the system through internal longitudinal forces, which are the parametrical ones are taken into account. The equations of motion are obtained from the Lagrange's equations and the harmonic balance method is applied. The small nonlinear forces connected with damping are necessary to get the stable amplitudes of vibrations. The considered problems may have the practical significance for the paraseismic phenomena when the weak excitation may couse the big effects because of the autoparametric resonances.

1. Introduction

In the papers of A.D.S. Barr [1,2,3] there were compiled the examples of problems connected with a phenomenon of parametric resonance in the systems of beams or rods which were kinematically forced. The phenomenon was analyzed in structural elements placed in the systems motion of which was determined. In many cases the resonance of autoparametric kind occurs due to the coupling of elements. For instance the system presented in Fig.1, analyzed Barr [1]. The system is made of two beams connected stiffly. Transverse harmonic load of angular frequency Ω acts on the horizontal beam. Therefore on the vertical beam periodic, axial load acts and plays the role of parametric excitation. If $\Omega = 2\omega_r$ (where ω_r is the natural angular frequency of vertical beam) the dynamic instability and autoparametric resonance occur (cf [4,5,6]).

A matter of this paper is a similar problem but in the system of three rods (Fig.2). Transverse, symmetrical vibration of the system is considered. The couplings of the rods through internal longitudinal forces are taken into account. The longitudinal forces are the transverse forces at the ends of neighbouring rods. The system is placed on a verticaly moving support, moreover the external harmonic

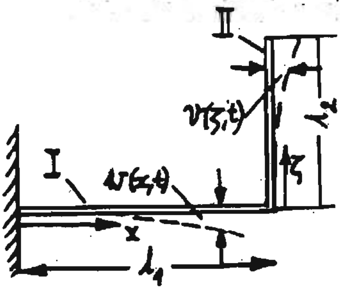

Fig. 1.

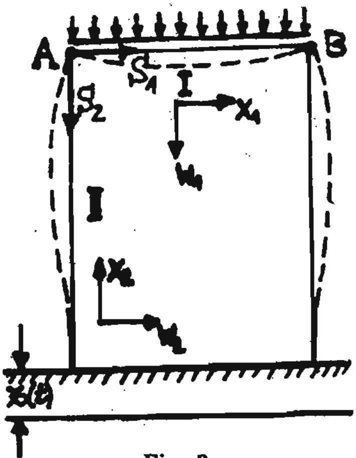

Fig. 2.

load acts on the horizontal element I. This load may occur due to machines or another devices acting in engine room.

The coupling has the autoparametric nature. The analysis of these problems in systems kinematicaly excited is important. Kinematic excitation of buildings and structures occurs for seismic or paraseismic excitations. The latter can be sometimes controlled, in terms of the possibility of evalution of their effects. Sources of paraseismic vibration can be: motion of vehicles, running of machines, shootings in quarries. Vibrations of support of buildings or structures gives a kinematic excitation. The response depends on dynamic properties and character of structures. The structures vibrate longitudinally and transversaly. First one can calculate inertial forces connected with the kinematic excitation.

In this paper we confine ourselsves to the vertical vibration of system of rods. Such system is an essential element of many buildings and structures e.g. engine rooms. Internal couplings between rods are important and are taken into account.

2. The model and the equation of motion

The analyzed system of rods is presented in Fig.2. The horizontal rod I is connected by means of two articulated joints (points A and B) with two identical elements II, stiffly connected with the support (points C and D).

We assume: the rods are prismatic and of square cross sections, the deflections of the rods are small, the elements are made of the Kelvin–Voigt linear viscoelastic material. The following notation is used: l_1, l_2 lengths of rods, m_1, m_2 linear mass densities, E_1, E_2 Young moduli, I_1, I_2 cross–sectional moments of inertia. The couplings of the rods through internal longitudinal forces S_i are taken into account. The transverse harmonic load acts on the horizontal element I. The system is placed on the vertically moving support and the function $z(t)$ represents. Transverse displacements are w_1, w_2 and longitudinal displacemets are u_1, u_2. The transverse, symmetrical vibrations of the elements of the system are considered. Because the exact description of analyzed vibrating system is difficult, the approximated method is used. We propose a certain model of the system of vibrating rods. The system vibrates in such a way that in the points of articulated joints the elements I and II are separated and the displacements of the ends of elements are small and following

$$u_i = \frac{1}{2}\int\limits_0^{l_i}\Big(\frac{\partial w_i}{\partial x_i}\Big)^2 dx_i$$

The rods are coupled with forces S_i $(i = 1,2)$ only. Equations of motion are obtained from the Lagrange's equations of the second order. Any external loadings, internal coupling forces, damping forces and non–linear forces are taken into account by means of corresponding generalized forces.

First we consider the vertical rod placed on the moving support (Fig.3). As a result of motion in direction of x axis – the longitudinal continuous load acting on the rod occurs

$$n(x,t) = -m\ddot{z} = \frac{dP}{dx} \tag{2.1}$$

where

$$z = \gamma \sin \omega t$$

The virtual work of $n(x,t)$ on the virtual displacement $u(x,t)$ is

$$\delta L = \int\limits_0^l n(x,t)\delta u(x,t)dx \tag{2.2}$$

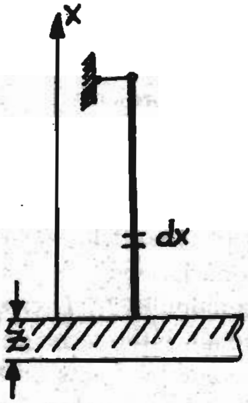

Fig. 3.

where $u(x,t)$ is determined by the following formula

$$u(x,t) = \frac{1}{2}\int\limits_0^x \Big(\frac{\partial w}{\partial \xi}\Big)^2 d\xi \tag{2.3}$$

We assume that the transverse displacement $w(x,t)$ can be expresed in the approximate form

$$w(x,t) = X(x)T(t) \tag{2.4}$$

According to Eqs (2.3) and (2.4) the expression (2.2) takes the form

$$\delta L = -m\ddot{z}\int\limits_0^l dx \int\limits_0^x \Big(\frac{\partial X}{\partial \xi}\Big)^2 d\xi T\delta T = Q_{zII}\delta T \tag{2.5}$$

where Q_{zII} is the generalized force coresponding to the longitudinal load generated by harmonic vertical displacement of support

$$Q_{zII} = -m\ddot{z}T\int\limits_0^l dx \int\limits_0^x \Big(\frac{\partial X}{\partial \xi}\Big)^2 d\xi = -\bar{G}\ddot{z}T \tag{2.6}$$

In the above formula the notation $\bar{G}$ is introduced

$$\bar{G} = m\int\limits_0^l dx \int\limits_0^x \Big(\frac{\partial X}{\partial \xi}\Big)^2 d\xi \tag{2.7}$$

When at the end of vertical rod the concentrated mass M is placed the generalized force is

$$Q_{zII} = -M\ddot{z}T\int_0^l \left(\frac{\partial X}{\partial \xi}\right)^2 d\xi$$

Next we consider the vibration of horizontal rod I. On each element of this rod acts the force

$$dP = -m_1\ddot{z}dx_1 \tag{2.8}$$

generated by the harmonic vibration $z(t)$ of the support. We calculate the virtual work of this force on the virtual displacement δw_1

$$\delta L = -\int_0^{l_1} m_1\ddot{z}\delta w_1 dx_1 = -\int_0^{l_1} m_1\ddot{z}X_1(x_1)\delta T_1 dx_1 = Q_{zI}\delta T_1 \tag{2.9}$$

So one gets the following formula for the generalized force Q_{zI}

$$Q_{zI} = -m_1\ddot{z}\int_0^{l_1} X_1(x_1)dx_1 = -G\ddot{z} = G\gamma\omega^2\sin(\omega t) \tag{2.10}$$

where

$$G = m_1\int_0^{l_1} X(x_1)dx_1$$

In the considered system we take into account the internal friction and the damping forces connected with the rates of displacements $\dot{u}_1$ of the articulated joints A and B (so called non linear damping).

Coresponding generalized forces are (cf [7,8,9])

$$Q_{D_i} = \eta_i\int_0^{l_i} I_i(x_i)X_i''^2(x_i)dx_i\dot{T}_i = D_i\dot{T}_i \tag{2.11}$$

where

$$D_i = \eta_i\int_0^{l_i} I_i(x_i)X_i''^2(x_i)dx_i$$

and the generalized forces coresponding to the forces P_i $(i = 1,2)$ connected with displacements u_i of the articulated joints A and B are

$$P_i = \Delta S_i = -k\dot{u}_i \tag{2.12}$$

where k is the coefficient of nonlinear damping.

The generalized forces connected with (2.12) are

$$Q_{\Delta S_i} = -k\Big\{\int\limits_0^{l_i} \big[X_i'(x_i)\big]^2 dx_i\Big\}^2 \dot{T}_i T_i^2 = -E_i \dot{T}_i T_i^2 \qquad i = 1,2 \tag{2.13}$$

where

$$E_i = k\Big\{\int\limits_0^{l_i} \big[X_i'(x_i)\big]^2 dx_i\Big\}^2$$

On the base of the Lagrange's equations (cf [7,8,9]) we get the system of differential equations, where the periodic longitudinal forces S_i are taken into account. The coresponding generalized forces connected with S_i are

$$Q_{S_1} = E_2 I_2 X_2'''(l_2) \int\limits_0^{l_1} \big[X_1'(x_1)\big]^2 dx_1 T_1(t) T_2(t)$$

$$Q_{S_2} = E_1 I_1 X_1'''(0) \int\limits_0^{l_2} \big[X_2'(x_2)\big]^2 dx_2 T_1(t) T_2(t)$$

The system of differential equations is

$$\begin{aligned} &A\ddot{T}_1 + BT_1 - CT_1T_2 + D\dot{T}_1 + E\dot{T}_1 T_1^2 + G\ddot{z} = \Gamma_1 \sin\omega t \\ &\bar{A}\ddot{T}_2 + \bar{B}T_2 + \bar{C}T_2T_1 + \bar{D}\dot{T}_2 + \bar{E}\dot{T}_2 T_2^2 + \bar{G}\ddot{z}T_2 = 0 \end{aligned} \tag{2.14}$$

The constants which appear in Eq (2.14) are as follows

$$A = \int\limits_0^{l_1} m_1(x_1) X_1^2(x_1)\,dx_1 \qquad \bar{A} = \int\limits_0^{l_2} m_2(x_2) X_2^2(x_2)\,dx_2$$

$$B = E_1 I_1 X_2'''(l_2) \int\limits_0^{l_2} (X_1'')^2 dx_1 \qquad \bar{B} = E_2 I_2 \int\limits_0^{l_2} (X_2'')^2 dx_2$$

$$C = E_2 I_2 X_2'''(l_2) \int\limits_0^{l_1} (X_1')^2 dx_1 \qquad \bar{C} = E_1 I_1 X_1'''(0) \int\limits_0^{l_2} (X_2')^2 dx_2 \tag{2.15}$$

D, $\bar{D}$, E, $\bar{E}$ according to Eq (2.11) and (2.13).

G, $\bar{G}$ according to Eq (2.10) and (2.7)

$$\Gamma = \int\limits_0^{l_1} \gamma_1 X_1(x_1)\,dx_1$$

where $D_1 = D$, $D_2 = \bar{D}$, $E_1 = E$, $E_2 = \bar{E}$ and γ_1 denotes the amplitude of external excitation whereas γ is the amplitude of kinematic excitation. In comparison with the previous papers (cf [7,8,9]) devoted to the internal resonance phenomenon the two new constants G and $\bar{G}$ connected with the kinematic excitation appear.

Because of the assumptions that the coupling has a little influence on the modes of transverse vibrations, these modes are obtained as the solutions to partial differential equations describing the transverse vibrations of the seperate rods; solving the proper boundary value problems one gets

$$X_1(x_1) = \sin \frac{\pi}{l_1} x_1$$
$$X_2(x_2) = -\cos\lambda_1 \left[\sin\lambda_1 \frac{x_2}{l_2} - \sinh\lambda_1 \frac{x_2}{l_2} - \tan\lambda_1 \left(\cos\lambda_1 \frac{x_2}{l_2} - \cosh\lambda_1 \frac{x_2}{l_2}\right)\right] \tag{2.16}$$

where $\lambda_1 = 3.9266$. Because our further considerations are confined to the main parametric internal resonance which occurs when the following relations hold

$$\omega_{01} = \omega \qquad \omega_{02} = \frac{1}{2}\omega_{01}$$

the eigenfunctions for the first eigenvalues were used. Calculating the integrals in relations (2.15) we get

$$\begin{aligned}
&A = 0.500 l_1 m_1 && \bar{A} = 0.499 l_2 m_2 \\
&B = 48.71 E_1 I_1 \frac{1}{l_1^3} && \bar{B} = 118.8 E_2 I_2 \frac{1}{l_2^3} \\
&\bar{C} = -172.8 E_1 I_1 \frac{1}{l_1^3 l_2} && \Gamma_1 = 0.489 E_1 I_1 \frac{1}{l_1^2} \\
&D = 48.71 I_1 \eta_1 \frac{1}{l_1^3} && \bar{D} = 118.8 I_2 \eta_2 \frac{1}{l_2^3} \\
&E = 24.35 k \frac{1}{l_1^2} && \bar{E} = 33.09 k \frac{1}{l_2^2} \\
&G = 0.6366 l_1 m_1 && \bar{G} = 2.2836 m_2
\end{aligned} \tag{2.17}$$

3. Internal resonance

Next we consider an internal resonance in the system of rods described by the set of ordinary differential equations (2.14). In modificated form the equations of motion are

$$\ddot{T}_1 + \omega_{01}^2 T_1 = \frac{C}{A}T_2T_1 - \frac{D}{A}\dot{T}_1 - \frac{E}{A}\dot{T}_1T_1^2 + \Big(\frac{G}{A}\omega^2\gamma + \frac{\Gamma_1}{A}\Big)\sin(\omega t)$$
$$\ddot{T}_2 + \omega_{02}^2 T_2 = -\frac{\bar{C}}{\bar{A}}T_1T_2 - \frac{\bar{D}}{\bar{A}}\dot{T}_2 - \frac{\bar{E}}{\bar{A}}\dot{T}_2T_2^2 + \frac{\bar{G}}{\bar{A}}\omega^2\gamma\sin(\omega t)T_2 \qquad (3.1)$$

where

$$\omega_{01} = \sqrt{\frac{B}{A}} \qquad \omega_{02} = \sqrt{\frac{\bar{B}}{\bar{A}}}$$

Solving Eqs (3.1) we assume that we seek only the for non-zero real solutions which we analyse. The second equation in the set (3.1) has also trivial solution $T_2 = 0$ which is of no interest to us.

We introduce the following notations

$$F_1 = \frac{C}{A}T_2T_1 - \frac{D}{A}\dot{T}_1 - \frac{E}{A}\dot{T}_1T_1^2$$
$$F_2 = -\frac{\bar{C}}{\bar{A}}T_1T_2 - \frac{\bar{D}}{\bar{A}}\dot{T}_2 - \frac{\bar{E}}{\bar{A}}\dot{T}_2T_2^2 + \frac{\bar{G}}{\bar{A}}\omega^2\gamma\sin(\omega t)T_2$$
$$p_1 = \Big(\frac{G}{A}\omega^2\gamma + \frac{\Gamma_1}{A}\Big)\sin(\omega t)$$
$$p_2 = 0 \qquad (3.2)$$

Now we introduce the complex variables U_s and V_s $(s = 1,2)$ as follows

$$\frac{\dot{T}_s}{i\omega_{0s}} + T_s = 2U_s$$
$$-\frac{\dot{T}_s}{i\omega_{0s}} + T_s = 2V_s \qquad (3.3)$$

Then the equations of motion (3.1) take the form

$$\dot{U}_s - i\omega_{0s}U_s = \frac{1}{2i\omega_{0s}}\Big(F_s + p_s\Big)$$
$$\dot{V}_s + i\omega_{0s}V_s = -\frac{1}{2i\omega_{0s}}\Big(F_s + p_s\Big) \qquad (s = 1,2) \qquad (3.4)$$

where

$$F_1(U_1,V_1,U_2,V_2) = \frac{C}{A}(U_1 + V_1)(U_2 + V_2) - i\omega_{01}\frac{D}{A}(U_1 - V_1)\cdot$$

$$\cdot\Big[1+\frac{E}{D}(U_1+V_1)^2\Big] \tag{3.5}$$

$$F_2(U_1,V_1,U_2,V_2) = -\frac{\bar{C}}{A}(U_1+V_1)(U_2+V_2) - \mathrm{i}\omega_{02}\frac{\bar{D}}{A}(U_1-V_1)\cdot$$
$$\cdot\Big[1+\frac{\bar{E}}{D}(U_2+V_2)^2\Big] + \frac{\bar{G}}{A}\omega^2\gamma(U_2+V_2)\sin(\omega t)$$

We analyse the steady state of internal resonance with the external hamonic load acting on element I. Then the following relations hold

$$\omega_{01} \cong \omega \qquad\qquad \omega_{02} \cong \frac{1}{2}\omega_{01} \tag{3.6}$$

So we seek for approximated solutions to Eqs (3.4) in the form

$$U_1 = A_1 e^{\mathrm{i}\omega t} \qquad\qquad V_1 = A_1^* e^{-\mathrm{i}\omega t}$$
$$U_2 = A_2 e^{\frac{\mathrm{i}\omega t}{2}} \qquad\qquad V_2 = A_2^* e^{-\frac{\mathrm{i}\omega t}{2}} \tag{3.7}$$

Inserting Eqs (3.7) into (3.4) and equaling coefficients at proper harmonics we get equations.

$$\Big[2(\omega_{01}-\omega) + \mathrm{i}\frac{D}{A}\Big(1+\frac{E}{D}A_1A_1^*\Big)\Big]A_1 = \frac{1}{\omega_{01}}\Big[\frac{G}{A}\omega^2\gamma + \frac{\Gamma_1}{A}\Big]\frac{1}{2\mathrm{i}}$$
$$\Big[2(\omega_{01}-\omega) - \mathrm{i}\frac{D}{A}\Big(1+\frac{E}{D}A_1A_1^*\Big)\Big]A_1^* = -\frac{1}{\omega_{01}}\Big[\frac{G}{A}\omega^2\gamma + \frac{\Gamma_1}{A}\Big]\frac{1}{2\mathrm{i}} \tag{3.8}$$

From Eqs (3.8) we get the formula for the quantity $x_1 = A_1A_1^*$ which is proportional to the second power of amplitude R_1 $(R_1 = 2\sqrt{A_1A_1^*})$. After some rearrangements one gets

$$ax_1^3 + bx_1^2 + cx_1 + d_1 = 0$$

where

$$a = \Big(\frac{E}{A}\Big)^2 \qquad\qquad c = 4(\omega_{01}-\omega)^2 + \Big(\frac{D}{A}\Big)^2$$
$$b = 2\frac{DE}{A^2} \qquad\qquad d = -\frac{1}{4\omega_{01}^2A^2}\Big(G\omega^2\gamma + \Gamma_1\Big)^2 \tag{3.9}$$

On the other hand from the second and fourth equations of the set (3.4) we get

$$\Big[2\Big(\omega_{02}-\frac{\omega}{2}\Big) + \mathrm{i}\frac{\bar{D}}{A}\Big(1+\frac{\bar{E}}{D}A_2A_2^*\Big)\Big]A_2 = \frac{1}{\omega_{02}}\Big[\frac{\bar{G}}{A}\frac{\gamma}{2\mathrm{i}}\omega^2 - \frac{\bar{C}}{A}A_1\Big]A_2^*$$
$$\Big[2\Big((\omega_{02}-\frac{\omega}{2}\Big) - \mathrm{i}\frac{\bar{D}}{A}\Big(1+\frac{\bar{E}}{D}A_2A_2^*\Big)\Big]A_2^* = \frac{1}{\omega_{02}}\Big[-\frac{\bar{G}}{A}\frac{\gamma}{2\mathrm{i}}\omega^2 - \frac{\bar{C}}{A}A_1^*\Big]A_2 \tag{3.10}$$

After some transformation and taking into account the relation

$$A_1 - A_1^* = \frac{1}{2i\omega_{01}} \frac{4(\omega_{01}-\omega)\left(\frac{G}{A}\omega^2\gamma + \frac{\Gamma_1}{A}\right)}{4(\omega_{01}-\omega)^2 + \left(\frac{D}{A}\right)^2\left(1+\frac{E}{D}A_1A_1^*\right)^2} \tag{3.11}$$

one gets the following formula for the quantity $x_2 = A_2A_2^*$ which is proportional to the second power of amplitude R_2

$$ax_2^2 + bx_2 + c = 0$$

where

$$a = \left(\frac{\bar{E}}{A}\right)^2 \qquad b = 2\frac{\bar{D}\bar{E}}{A^2}$$

$$c = 4\left(\omega_{02} - \frac{\omega}{2}\right) + \left(\frac{\bar{D}}{A}\right)^2 - \frac{1}{\omega_{02}^2}\left\{\left(\frac{\bar{G}}{A}\frac{\gamma}{2}\omega^2\right)^2 + \left(\frac{\bar{C}}{A}\right)^2 A_1A_1^* - \right. \tag{3.12}$$

$$\left. - \frac{\bar{G}\bar{C}\gamma}{A^2}\frac{\omega^2}{\omega_{01}} \frac{(\omega_{01}-\omega)\left(\frac{G}{A}\omega^2\gamma + \frac{\Gamma_1}{A}\right)}{4(\omega_{01}-\omega)^2 + \left(\frac{D}{A}\right)^2\left(1+\frac{E}{D}A_1A_1^*\right)^2}\right\}$$

4. Numerical calculation and results

We assume that the rods are made of steel and are of square cross-sections. For calculations the following numerical values of parameters and material constants are adopted: the coefficients of internal damping $\eta_{1,2} \in \{1\cdot 10^5, 1\cdot 10^6, 1\cdot 10^7\}$ [Ns/m^2], the coefficients of nonlinear damping $k \in \{1\cdot 10^3, 1\cdot 10^4\}$ [kg/s], the cross-sections sides $a_1 = a_2 = 6\cdot 10^{-2}$ [m], the length of element I equals $l_1 = 8$ [m], the natural angular frequency of I is $\omega_{01} = \sqrt{B/A} = 14.47$ [s^{-1}]. The remaining constants are the same as in [9].

The sizes of rods are selected so that the static strains caused by their own weights are so small that one can employ the linear geometric theory. We can change the value ω_{02}/ω_{01} (the tuning) by means of changing the length l_2 of element II, so that

$$\omega_{02} = 0.50\omega_{01} \qquad l_2 = 14.14[\mathrm{m}]$$
$$\omega_{02} = 0.40\omega_{01} \qquad l_2 = 15.81[\mathrm{m}]$$
$$\omega_{02} = 0.60\omega_{01} \qquad l_2 = 12.90[\mathrm{m}] \qquad \text{(cf Fig.5)}$$

In our calculations the value of amplitude of kinematic excitation is $\gamma \in\{0.1$ [m], 1 [m]$\}$ – so that it is many times greater than the amplitudes coming

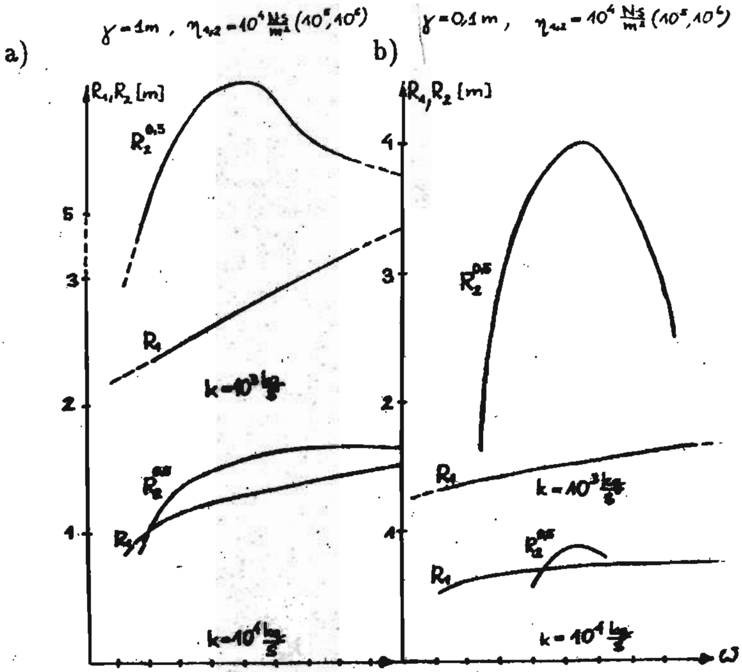

Fig. 4. Harmonic and kinematic excitation $\gamma_1 \neq 0$, $\gamma \neq 0$, $w_1(l_1/2)_{max} = R_1$, $w_2(l_2/2)_{max} = 1.02R_2$

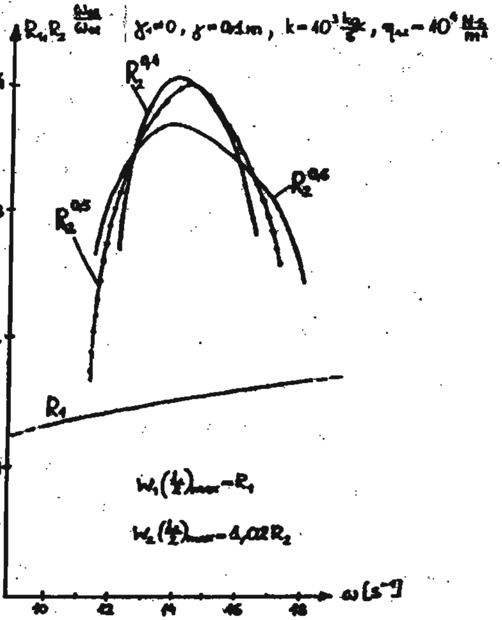

Fig. 5.

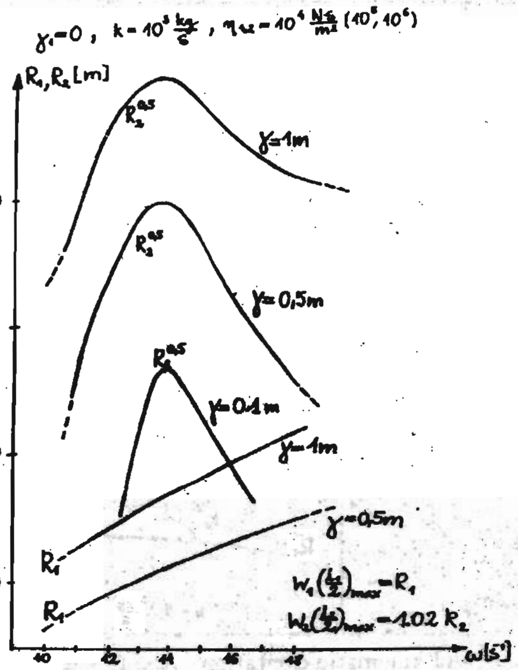

Fig. 6. Kinematic excitation $\gamma \neq 0$

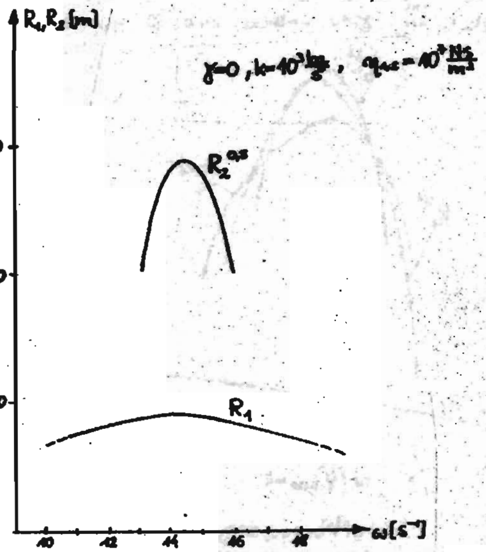

Fig. 7. Harmonic excitation $\gamma_1 \neq 0$

from the real paraseismic sources of vibration. These values of amplitudes emphasize the existance of phenomenon connected with the specifisity of the model. The results are presented in Fig. 4 ÷ 7 for different values of the tuning. On graphs the plots of the amplitudes $R_1(\omega)$ for the horizontal element I as well as the amplitudes $R_2(\omega)$ for the vertical elements II are presented. On the element II the parametric excitation coming from the vertical motion of the support acts (cf (3.1)).

In the figures the parametric resonances in element II are conspicuous. From the analysis of equations of motion (3.10) we can see the reason for adding the non–linear damping. For $k = 0$ we get only the trivial solution of to Eqs (3.10). For $k \neq 0$ we get non–trivial steady response of the element II in certain interval of ω near to ω_{01} and this response is typical for parametrically excited systems (cf Fig.7). In the case of non–linear damping the resonance curves are stable (cf [11]). One can see how the externally excited systems tuning into the internal resonance are close to the systems parametrically excited.

Analysing the results one can distinguish two kinds of resonance curves. The first kind – when the kinematic excitation $G\gamma\omega^2 \sin(\omega t)$ is greater than the harmonic load $\Gamma_1 \sin(\omega t)$ and the second kind when the harmonic load is greater than the kinematic excitation. The results depend on the value of damping coefficients. For $G\gamma\omega^2 > \Gamma_1$ and for big value of damping the amplitudes are presented in Fig.4a and 6. Their patterns are identical with those presented by Osiński (cf [12]). The amplitude of element II is non–zero in the large interval of frequencies near ω_{01}. For $G\gamma\omega^2$ near Γ_1 the resonance curves are presented in Fig.4b and 5 (cf [12]). In this situation the amplitude R_2 is non–zero in the smaller interval of frequencies. We point out that on the above figures the dashed lines represent non–zero resonance amplitudes which are not taken into account because of the relation (3.6).

5. Conclusions

In the present paper the internal resonance in the system of three rods placed on the moving support was considered. The internal interactions between neighbouring rods were taken into account. The considered problems may have the practical significance. The system is the typical element of many devices and constructions. The analysis of such object under kinematic excitation may have the essential significance at the study of paraseismic phenomena. The fundamental problem is to choose the appropriate model of the described object (structures, buildings, mechanical devices, mechanisms). The model should include some important properties of the object in the particular situation. Most often the physical models of investigate objects are discrete or discrete–continuous models with concentrated masses. In the papers [10,13,14] the influence of paraseismic phenomena on buildings and another objects was investigated adopting the simplification that they are the rods, bars or floor plates connected together with articulated joints. In this analysis the internal forces were not taken into account and thereby the autoparametric phenomena were neglected. How fundamental these effects are we estimate in each case separatly. The autoparametric phenomena can play an essential role in processes of destruction of described objects. In considerations presented in the paper some simplifications were adopted. The results seem to be encouraging to further analysis of this kind of phenomena. The description should be more precise (without some simplifications encountered).

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Rezonans wewnętrzny w układzie prętów przy wymuszeniu kinematycznym

Streszczenie

Celem pracy jest analiza rezonansu wewnętrznego w układzie prętów przy pionowym wymuszeniu kinematycznym. Elementy układu są połączone przegubowo. W opisie są uwzględnione wewnętrzne siły sprzędające o charakterze parametrycznym. Na podstawie równań Lagrange'a otrzymuje się układ rwnań różniczkowych zwyczajnych, które rozwiązano metodą bilansu harmonicznego. Uwzględnienie małych sił nieliniowych pozwala uzyskać ograniczone amplitudy. Omawiane tutaj zagadnienie może mieć istotne znaczenie przy badaniu zjawisk parasejsmicznych, gdy słabe siły wymuszające mogą powodować znaczne efekty na skutek rezonansu autoparametrycznego.

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