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MODEL OF DYNAMICS OF AIRCRAFT FLIGHT IN NAVIGATIONAL COORDINATE SYSTEMS

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Equations, algorithm and computer programme illustrating a controlled aircraft flight dynamics in navigational coordinate systems have been presented. The being formulated equations make the use of classical equations of aircraft motion to develop the aforementioned model possible. Results of exemplary calculations have been inserted, as well.

1. Introduction

Models of flight dynamics of a controlled aircraft have been discussed by Dżygadło [1] and the others (cf [4] ÷ [6]). The aircraft motion is considered there in a stationary cartesian coordinate system related to the Earth. The foregoing models during have been formulated using results obtained in a wind–tunnel (cf [1,2]) and during flight tests (cf [4]). Modelling of a modern, controlled aircraft leads to a substantial simplification of a physical object and its environment and to inadequancy of control principles model for real, automatic flight control systems.

The present contribution discusses the aircraft motion considered to be a spheroid (cf [2,3]) in relation to the Earth. The aircraft flying model is controlled either manually or by an automatic control system (cf [3,4,6]). A preprogrammed en-route flight, homing flight to any selected point on Earth, return flight to the preprogrammed route, etc. are some examples of many possible flight modes to be examined. They particularly correspond with the modern military aircraft flights.

The aforementioned models of aircraft flight in the stationary coordinate system have been used for the purposes of this paper. Additional coordinate systems enabling accomplishment of aircraft flight in relation to the Earth have been introduced. Suitable equations and algorithms of numerical solution to the problem have been included, followed by exemplary calculation results.

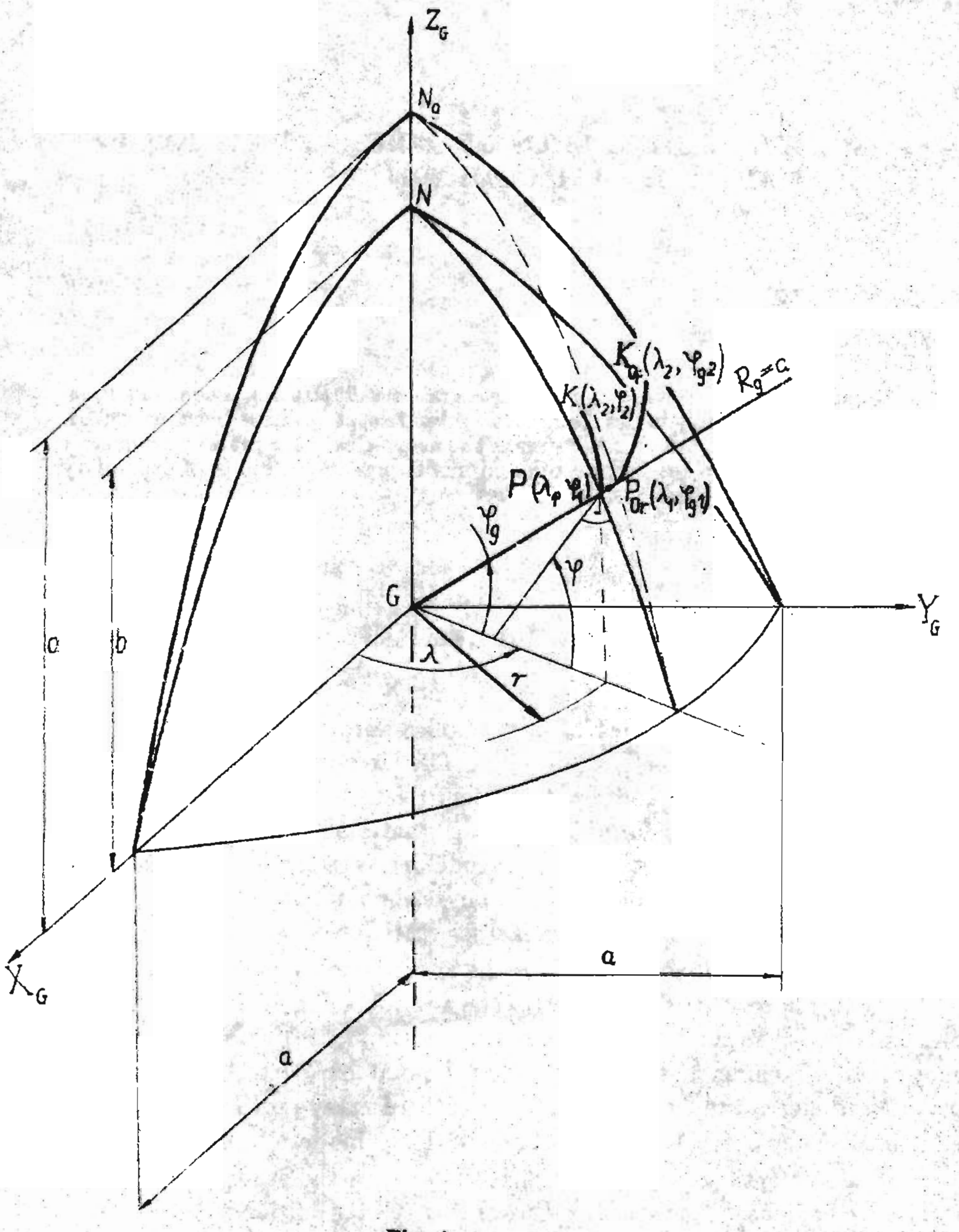


Fig. 1.

2. Coordinate systems. Geometric and kinematic relations

The Earth has been assumed to be a spheroid with the semimajor and semiminor axes a and b, respectively (according to Krassowski [3], $a = 6378245$ m, $b = 6356863$ m, see Fig.1).

Following coordinate systems have been introduced (Fig.1, 2).

$GX_GY_GZ_G$ – cartesian geocentric coordinate system. Point G coincides with the spheroid centre,

$PXYZ$ – related to the spheroid centre. The axis PX lies on the meridian plane and is tangent to a geographical meridian at point P,

$0xyz$ – related to the aircraft itself,

$P_{0r}X_{0r}Y_{0r}Z_{0r}$ – related to a sphere of radius $R_g = a$ (Fig.1). The axis $P_{0r}X_{0r}$ lies on the meridian plane and is tangent to a meridian at point P_{0r}. P_{0r} lies on the straight line that runs through the points G and P.

Coordinate systems $PXYZ$ and $0xyz$ correspond with the systems used in the classical flight mechanics, (cf [1,4,5]). The position of $0xyz$ system relative to the $PXYZ$ one has been determined with the aid of Eulerian angles Ψ, Θ, and Φ (Fig.2).

Relationships between unit vectors of the aforementioned systems have been determined to define geometric and kinematic variables. Assuming that the vectors are of the following form, (Fig.2)

$\boldsymbol{i}_G$ – unit vector for the system $GX_GY_GZ_G$, is defined as follows

$$\boldsymbol{i}_G = [\boldsymbol{i}_G, \boldsymbol{j}_G, \boldsymbol{k}_G]^{\mathsf{T}}$$

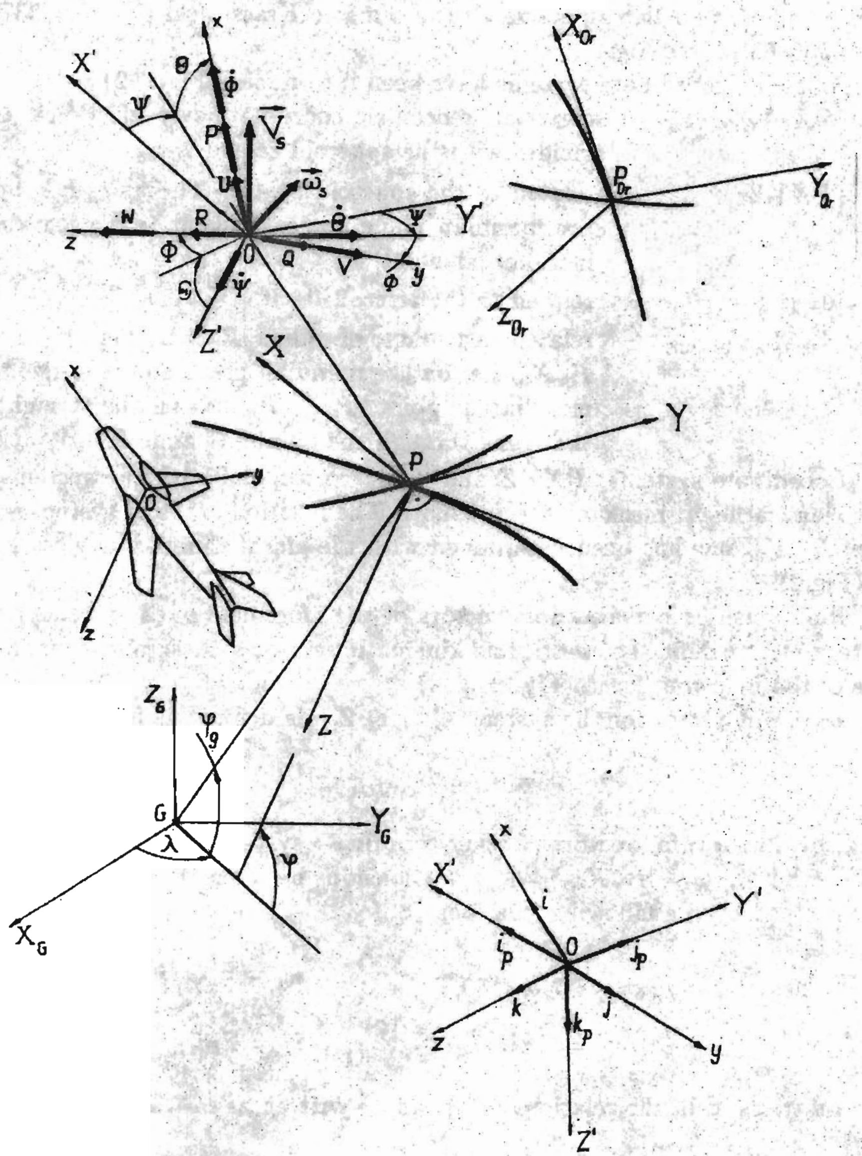
and, by analogy, for vectors of other coordinate systems we have $\boldsymbol{i} - 0XYZ$, $\boldsymbol{i}_P - PXYZ$, $\boldsymbol{i}_{0r} - P_{0r}X_{0r}Y_{0r}Z_{0r}$, relationships between the foregoing vectors are determined by the following relations

$$\begin{aligned} \boldsymbol{i}_{0r} &= \mathbf{E}_{0r}\boldsymbol{i}_G & \qquad \boldsymbol{i}_{0r} &= \mathbf{E}_{P0r}\boldsymbol{i}_P \\ \boldsymbol{i} &= \mathbf{E}\boldsymbol{i}_P & \qquad \boldsymbol{i}_P &= \mathbf{E}_P\boldsymbol{i}_G \end{aligned} \tag{2.1}$$

Matrices $\mathbf{E}$ in the relations (2.1) can be written as

$$\mathbf{E}_{0r} = \begin{bmatrix} -\sin\varphi_g\cos\lambda & -\sin\varphi_g\sin\lambda & \cos\varphi_g \\ -\sin\lambda & \cos\lambda & 0 \\ -\cos\varphi_g\cos\lambda & -\cos\varphi_g\sin\lambda & -\sin\varphi_g \end{bmatrix}$$

$$\mathbf{E}_P = \mathbf{E}_{0r}\Big|_{\varphi_g=\varphi} \tag{2.2}$$


x
X'
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V_s
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λ
φ
X_G
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Fig. 2.

$$\mathbf{E}_{P0r} = \begin{bmatrix} \cos\varepsilon & 0 & \sin\varepsilon \\ 0 & 1 & 0 \\ \sin\varepsilon & 0 & \cos\varepsilon \end{bmatrix} \qquad \varepsilon = \varphi - \varphi_g$$

where

λ, φ – longitude and latitude, respectively,

φ_g – geocentric latitude.

The foregoing matrices are the orthonormal ones, having the following features

$$\begin{aligned} \mathbf{E}_{P0r} &= \mathbf{E}_{0r} E_P^{\top} \\ \mathbf{E}_{0r} &= \mathbf{E}_{P0r} E_P \\ \mathbf{E}_P &= \mathbf{E}_{P0r}^{\top} E_{0r} \end{aligned} \tag{2.3}$$

Taking Fig.1 and spherical trigonometry relations into consideration, interrelations between linear and angular coordinates are determined by the following formulas

$$\tan\varphi = \frac{a^2}{b^2}\tan\varphi_g \tag{2.4}$$

$$\mathbf{X}_{GP} = \frac{a}{\sqrt{1 - e^2\sin^2\varphi}} \begin{bmatrix} \cos\varphi\cos\lambda \\ \cos\varphi\sin\lambda \\ (1-e^2)\sin^2\varphi \end{bmatrix} \tag{2.5}$$

where

$$\begin{aligned} &\mathbf{X}_{GP} = [X_{GP}, Y_{GP}, Z_{GP}]^{\top} \\ &e^2 = \frac{a^2 - b^2}{a^2} \\ &\lambda = \begin{cases} \lambda_a & \text{for } Y_{GP} \geq 0 \\ 2\pi - \lambda_a & \text{for } Y_{GP} < 0 \end{cases} \qquad \lambda_a = \arccos\frac{X_{GP}}{\sqrt{X_{GP}^2 + Y_{GP}^2}} \\ &\varphi_g = \arcsin\frac{Z_{GP}}{\sqrt{X_{GP}^2 + Y_{GP}^2 + Z_{GP}^2}} \end{aligned} \tag{2.6}$$

3. Flight route characteristics. Aircraft control principles

Position lines (cf [2]) are used to solve aircraft navigational problems. An orthodrome (also called: great circle, i.e. the shortest line on a sphere of radius

R_g (Fig.1, 2) between two points P_{0r} and K_{0r}) is one of them, used mainly in the air force.

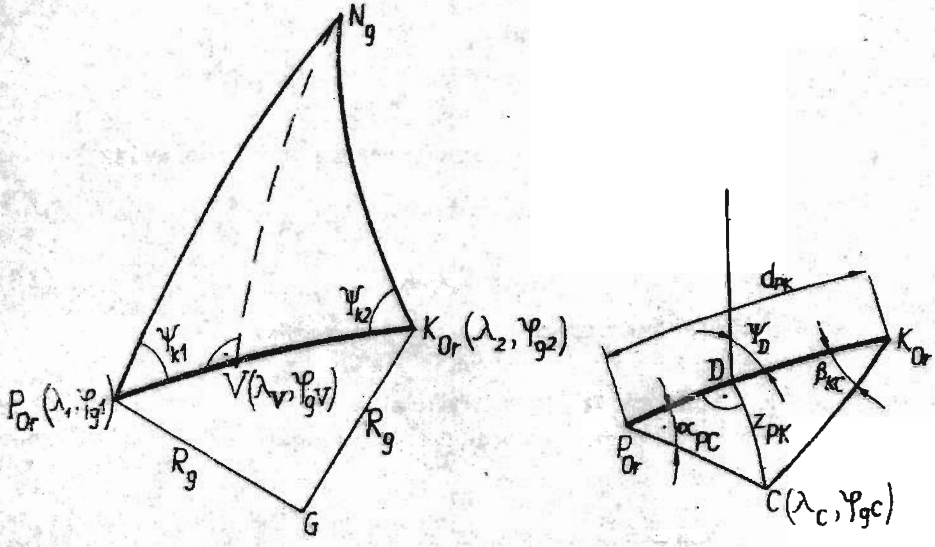


Fig. 3.

Again, with primary relations of spherical trigonometry applied, the following relations (Fig.3) determine the orthodrome lenght (arc measure) – d_{PK}, and both the track angles – initial angle (Ψ_{k1}) and final one (Ψ_{k2})

$$d_{PK} = \arccos(\cos\varphi_{g1}\cos\varphi_{g2}\cos\Delta\lambda_m + \sin\varphi_{g1}\sin\varphi_{g2})$$

$$\psi_{k1} = \arcsin\left(\frac{\sin\Delta\lambda_m}{\sin d_{PK}}\cos\varphi_{g2}\right) \tag{3.1}$$

$$\psi_{k2} = \arcsin\left(\frac{\sin\Delta\lambda_m}{\sin d_{PK}}\cos\varphi_{g1}\right)$$

where: $\Delta\lambda_m = |\lambda_2 - \lambda_1|$

The course–line deviation along the orthodrome from P_{0r} to K_{0r} (with actual aircraft position: λ_c and φ_{gc}, and assumption that lines $P_{0r}C$ and CK are orthodromes) equals to

$$z_{PK} = \arcsin(\sin P_{0r}C\sin\alpha_{PC}) \tag{3.2}$$

where

$$\alpha_{PC} = \arccos\frac{\cos CK_{0r} - \cos d_{PK}\cos P_{0r}C}{\sin d_{PK}\sin P_{0r}C}$$

$P_{0r}C$ and CK_{0r} can be determined from the relation that defines the orthodrome lenght (3.1). The orthodrome lenght DK_{0r} equals to

$$DK_{0r} = \arctan(\cos\beta_{KC}\tan CK_{0r}) \tag{3.3}$$

where

$$\beta_{KC} = \arcsin\frac{\sin|z_{PK}|}{\sin CK_{0r}}$$

The longitude and geocentric latitude, respectively of point D can be expressed as

$$\begin{aligned}\varphi_{gD} &= \arcsin[\cos(s_{PV} - P_{0r}D)\sin\varphi_{gV}] \\ \lambda_D &= \lambda_{P_{0r}} + \arccos\frac{\cos P_{0r}D - \sin\varphi_{g1}\sin\varphi_{gD}}{\cos\varphi_{g1}\cos\varphi_{gD}}\end{aligned} \tag{3.4}$$

where

φ_{gV} – geocentric latitude of orthodrome vertex

$$\varphi_{gV} = \arccos(\cos\varphi_{g1}\sin\psi_{k1}) \tag{3.5}$$

s_{PV} – arc length $P_{0r}V$

$$s_{PV} = \arctan\frac{\cos\psi_{k1}}{\tan\varphi_{g1}} \tag{3.6}$$

The foregoing relations that determine flight characteristics once can directly applyd to determine the aircraft roll (cf [3]) which enables the aircraft to return to the preprogrammed route. The aircraft roll results from the aileron deflection. Established below one can find a general form of the equation of aileron control during the flight under consideration

– the required aircraft roll

$$\dot{\Phi}_n = a_n\Phi_n + b_n z_{PK} + c_n\dot{z}_{PK} \tag{3.7}$$

– aileron deflection

$$\begin{aligned}\dot{\kappa}_l &= a_l\kappa_l + b_l\delta_l + c_l\Delta\ddot{\Phi} + d_l\Delta\dot{\Phi} + e_l\Delta\Phi + f_l\ddot{P} + g_l\dot{P} + e_lP \\ \dot{\delta}_{la} &= \kappa\end{aligned} \tag{3.8}$$

where

$\Delta\Phi$	–	difference between the required (Φ) and actual (Φ) aircraft rolls, respectively,
P	–	aircraft in–roll angular velocity,
$a_n \div c_n$, $a_l \div e_l$	–	equation factors, in general case dependent on flight characteristics (Mach number, speed, altitude, etc.).

The aircraft control principles are complementary to the equations of elevator deflection δ_h, and render the following possible: the aircraft angle of pitch stabilization, the geometric or pressure flight altitudes determination, etc. The aforementioned equations are discussed in detail by J.Manerowski [4] and R.C.Nelson [6].

4. Numerical algorithm. Calculation results

The foregoing relations have been used to develop equations of motion as well as the algorithm and computer programme of the flying model flight in navigational coordinate systems.

The after–inserted equations make accomplishment of the flight consideration possible. At the same time they constitute a numerical algorithm adjusted to classical methods of numerical integration of equations. A flowchart of calculation stages is presented.

(1) Initial conditions: λ, φ, λ_1, φ_1, λ_2, φ_2, flight characteristics.

- *Loop of successive stages of equation integration*

(2) Geometric and kinematic relations in the navigational coordinate systems

$$\varphi = f(\varphi_g) \qquad \mathbf{E}_P = f(\lambda, \varphi) \qquad \mathbf{E}_{P0r} = f(\varepsilon)$$
$$\dot{\mathbf{X}}_G = \mathbf{E}_P^{\mathrm{T}}\dot{\mathbf{X}}$$
$$\lambda,\ \varphi_g = f(X_G, Y_G, Z_G) \qquad \mathbf{E}_{0r} = f(\lambda, \varphi_g)$$
$$\dot{\mathbf{X}}_{0r} = \mathbf{E}_{0r}\dot{\mathbf{X}}_G$$
$$\dot{\mathbf{X}} = \mathbf{E}_{P0r}^{\mathrm{T}}\dot{\mathbf{X}}_{0r}$$
$$\Psi = f(\dot{X}, \dot{Y}, \dot{Z}) \qquad \Psi_{0r} = f(\dot{X}_{0r}, \dot{Y}_{0r}, \dot{Z}_{0r})$$
$$\mathbf{E} = f(\Phi, \Theta, \Psi)$$
$$\dot{x} = \mathbf{E}\dot{\mathbf{X}}$$

(3) Aircraft control principles

$$\Psi_D = f(\lambda_D, \varphi_{gD}, \lambda_2, \varphi_{g2})$$
$$\dot{z}_{PK} = \dot{X}_{0r} \sin \Psi_D - \dot{Y}_{0r} \cos \Psi_D$$
$$\dot{\delta}_{la} = f(\dot{z}_{PK}, z_{PK}, \text{flight characteristics})$$
$$\delta_l = \delta_{la} + \delta_{lr}$$
$$\dot{\delta}_{ha} = f(\text{flight characteristics})$$
$$\delta_h = \delta_{ha} + \delta_{hr}$$

where indices denote the controls deflections: r – manual control, a – automatic control, respectively.

(4) Equations of the aircraft motion – classical form of the flight mechanics (cf [1,4,5]), see Fig.2

$$\begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \\ \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix} = \begin{array}{l} f(\text{parameters of: engine work,} \\ \quad \text{aircraft movements etc.}) \end{array}$$

$$\dot{\mathbf{X}} = f(\Psi, \Theta, \Phi, U, V, W)$$

$$\begin{bmatrix} \dot{\Phi} \\ \dot{\Theta} \\ \dot{\Psi} \end{bmatrix} = f(\Theta, \Phi, P, Q, R)$$

(5) Illustration of the flight trajectory characteristics on maps (applying various map projection techniques, e.g. conical projection (cf [2]))

– linear coordinates of aircraft position on a map of the aforementioned conical projection (x_m, y_m)

$$x_m = r \sin d_\lambda \qquad\qquad y_m = r_0 - r \cos d_\lambda$$

where

$d_\lambda = (\lambda - \lambda_0) \sin \varphi_0$

$r_0 = R_s \cot \varphi_0$

$r = r_0 - R_s \tan(\varphi - \varphi_0)$

and

λ_0, φ_0	–	longitude and latitude characteristic of a map,
R_s	–	mean radius of curvature of the terrestrical spheroid at the point of latitude φ_0,

$$R_s = \frac{a\sqrt{1 - \mathrm{e}^2}}{1 - \mathrm{e}^2 \sin^2 \varphi_0}$$

- *end of loop of equation integration*

Stated below one can find exemplary calculation results obtained from the computer programme being developed. The present equations of motion refer to

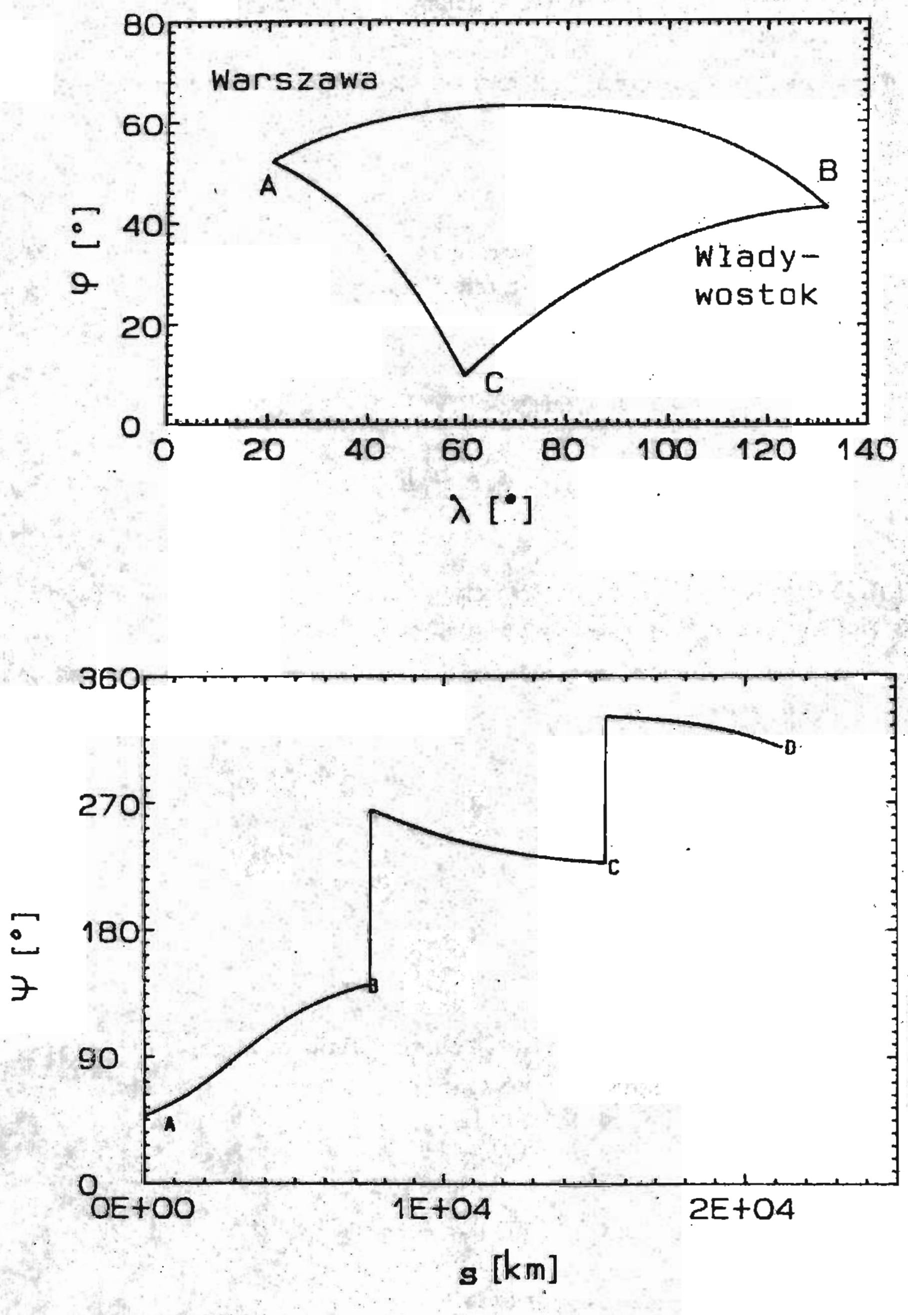
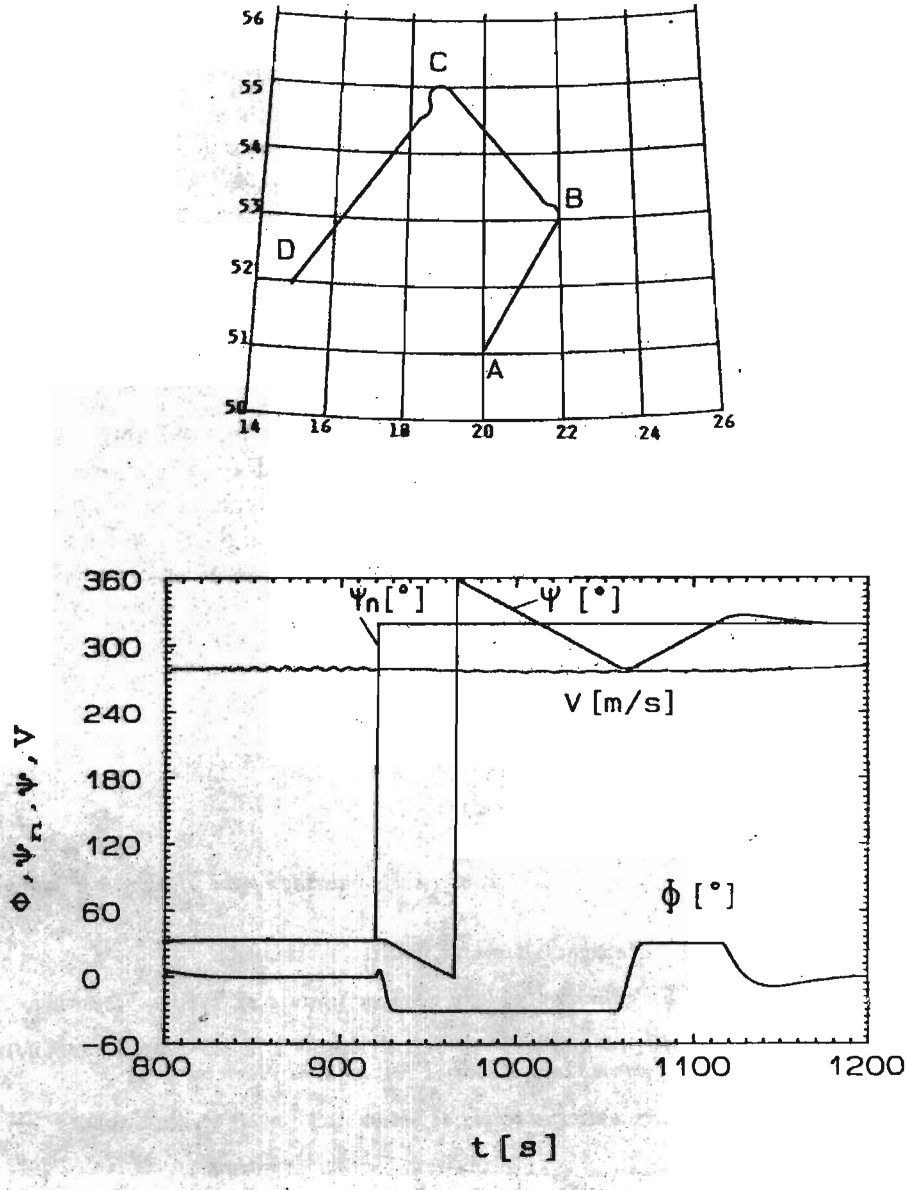

80
60
40
20
0
φ [°]
Warszawa
A
B
C
Władywostok
0 20 40 60 80 100 120 140
λ [°]
360
270
180
90
0
ψ [°]
A
B
C
D
0E+00 1E+04 2E+04
s [km]

Fig. 4.



56
55
54
53
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50
14
16
18
20
22
24
26
A
B
C
D
360
300
240
180
120
60
0
-60
800
900
1000
1100
1200
Φ, Ψn, Ψ, V
t [s]
Ψn [°]
Ψ [°]
V [m/s]
Φ [°]

Fig. 5.

a supersonic strike aircraft. Together with elevator control principles they are to be found elsewhere (cf [4]).

Fig.4 illustrates flight path ABC on a map of Merkator projection $\varphi = f(\lambda)$. At the bottom of Fig.4 one can easily observe a change of the true course (Ψ) plotted against the route covered.

Fig.5 shows the aircraft flight trajectory $ABCD$ on a map of conical projection. The time functions of some selected flight characteristics: speed V, aircraft roll Φ, true course Ψ, and flight heading as required $\Psi_n = \Psi_D$ (Fig.3) have been inserted, as well. The results refer to the final route stage AB and to the initial one $-BC$, respectively.

5. Final remarks

The equations, algorithms, and computer programms presented above render analysis of the controlled aircraft flight characteristics in the navigational coordinate systems possible. The applied assumptions correspond with real aircraft navigation and aircraft control (in terms of the automatic flight control systems) principles. The results being obtained can be applied, to solve real problems of aircraft navigation, to develop control principles of the automatic control systems, etc. Characteristics of the foregoing algorithms show their numerical identity and stability.

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Model dynamiki lotu samolotu w nawigacyjnych układach wspó rzędnych

Streszczenie

Przedstawiono równania, algorytm i program komputerowy dynamiki lotu sterowanego samolotu w nawigacyjnych układach współrzędnych. Wyznaczone równania umożliwiają wykorzystanie klasycznych równań ruchu samolotu do opracowania ww. modelu. Zamieszczono wyniki przykładowych obliczeń.

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