

THEORETICAL AND EXPERIMENTAL STRESS ANALYSIS IN A GEOMETRICALLY NONHOMOGENOUS ROTATING DISC

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The results of the numerical and experimental analysis of the stress in rotating part of an industrial centrifugal machine are presented.

1. Discrete model of structure

The considered centrifugal machine rotor (Fig.1) consisted of the drum (1) and the disc (2). The drum (Fig.1 and Fig.2) was treated as a thin-walled cylindrical shell. The disc (Fig.1 and Fig.3) was considered as a three-dimensional body loaded by centrifugal forces (Fig.3a) and forces caused by an interaction of the drum (Fig.3b and Fig.3c).

An IBM Computer and a corresponding program SAP IV were at the authors disposal.¹

Model was divided into 14 parts composed of the three-dimensional eight-nodal elements (Fig.4 and Fig.5).

¹The program was developed at the Faculty of Mechanical Engineering University of Beograd, by generation of points (three-dimensional and boundary element), generation of loadings caused by the centrifugal forces calculated automatically for each element, graphical control of input data.

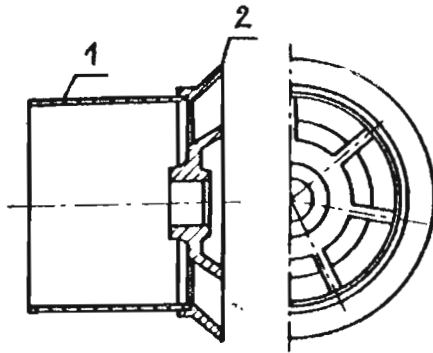


Fig. 1.

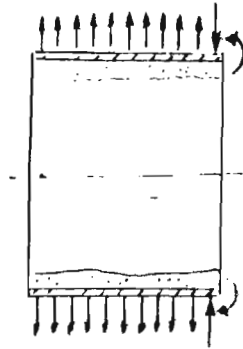


Fig. 2.

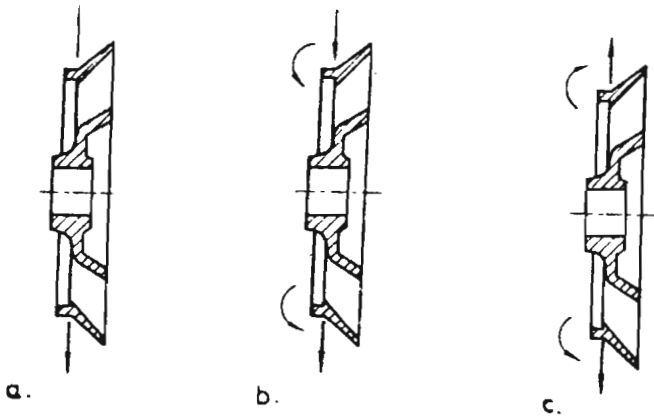


Fig. 3.

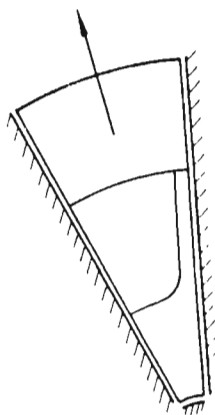


Fig. 4.

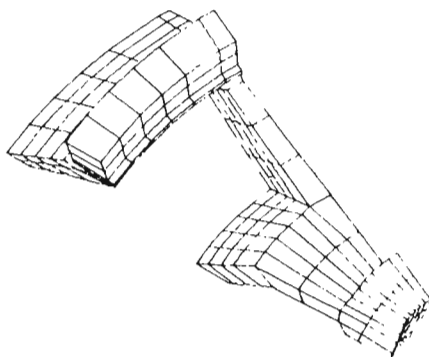


Fig. 5.

2. The experimental analysis

Among the experimental methods of the stress analysis of structures a particular attention is paid to the stress freezing photoelasticity – especially in the cases of three-dimensional stress analysis as well as of the elements subjected to the centrifugal forces caused by the rotation.

At first the model of the rotor (in 1:5 scale) was made of aluminium in order to obtain mould of silicon rubber (Fig.6) which was used for the manufacturing model of the epoxy resin (Fig.7).

The machined model was placed on a shaft with possibilities to apply programmable velocity of rotation and then, together with it, into the thermal chamber.

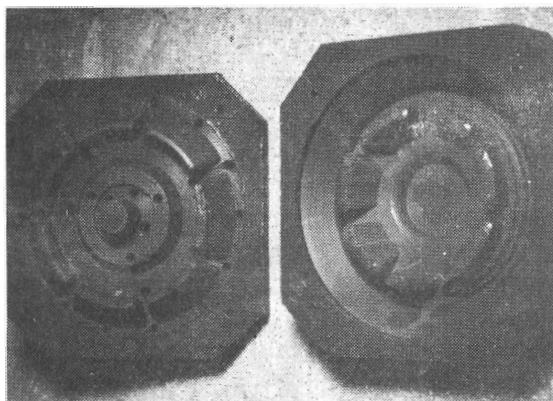


Fig. 6.

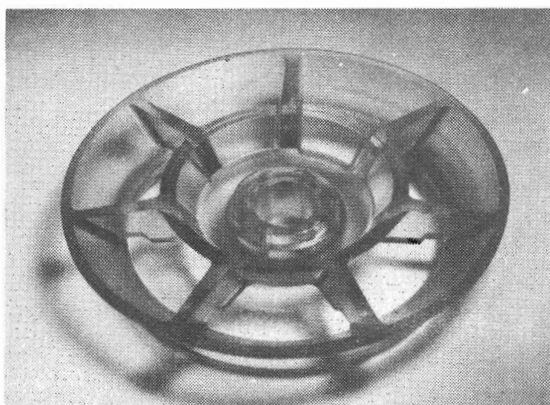


Fig. 7.

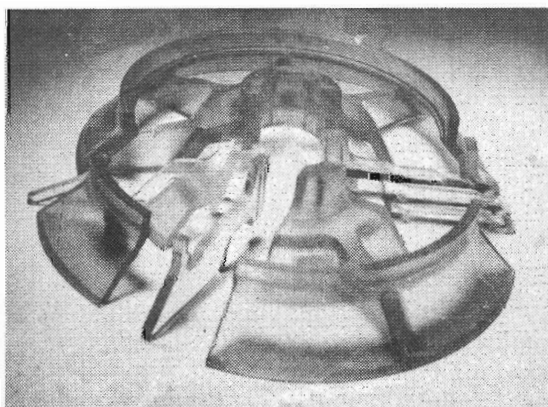


Fig. 8.

The temperature in the chamber was increased (1°C/h) up to 110°C, i.e. to the high elasticity state of the model material. After 4 hours of holding the temperature was decreased (0.5°C/h) automatically. The change of the temperature was controlled by the IBM personal computer.²

The model was then cut into thin slices (Fig.8) and analyzed. The high order isochromatic fringes corresponding to the high level of stress gradients are shown in Fig.9, 10, 11. The highest stress gradients appeared in the ribs at the places of their connections (Fig.11). Normal stresses at the boundary of the critical region are shown graphically (Fig.12 and 13).

It can be concluded (Fig.12) that the highest stress gradients appear between the points 12 and 18 and they are most probably caused by the great curvature of the boundary in that region. Between the points 23 and 25 the local concentration of isochromatic fringes caused by the geometry of the disc can be observed.

The structure can be improved by decreasing the high curvature of the boundary between the points 12 and 18.

The distribution of normal stresses at the boundary (Fig.13) suggests the change of the curvature of the boundary between points 9 and 12. According to the isochromatic fringe patterns shown in Fig.9 and 10, it is not necessary to change the geometry of other elements.

From the similarity criterion, according to the rules of the dimensional analysis, the following dimension matrix was obtained

	l	ν	ρ	n	σ	E
M	0	0	1	0	1	1
L	1	0	-3	0	-1	-1
T	0	0	0	-1	-2	-2

It results from the Buckingham's theorem that the number of dimensionless coefficients contained in the given set is equal to the number of introduced physical quantities decreased by the rank of the matrix determined for the basic units.

In this case the following dimensionless coefficients are obtained ($6 - 3 = 3$)

$$\Pi_1 = \frac{\sigma}{E}, \quad \Pi_2 = \nu, \quad \Pi_3 = \frac{\rho l^2 n^2}{E}$$

where

- E - Young modulus,
- ν - Poisson ratio,
- ρ - mass density,
- n - revolutions per time unit,
- l - length.

²The equipment was designed and made at the Aeronautical Institute of Technical University of Rzeszów, Poland.

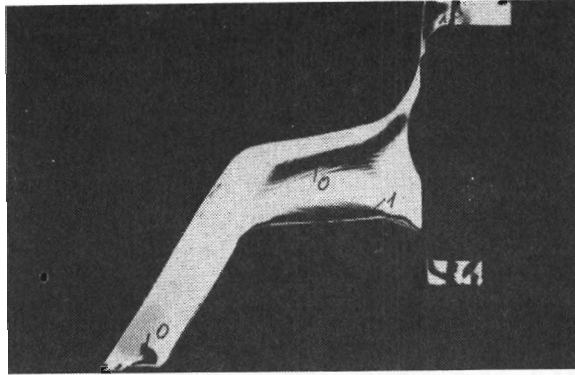


Fig. 9.

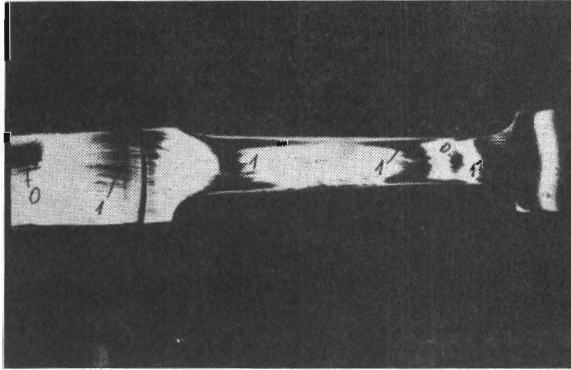


Fig. 10.

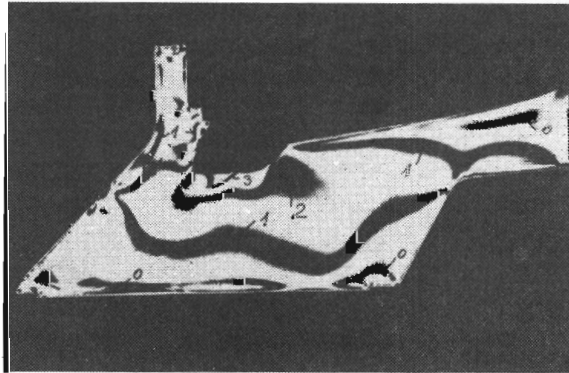


Fig. 11.

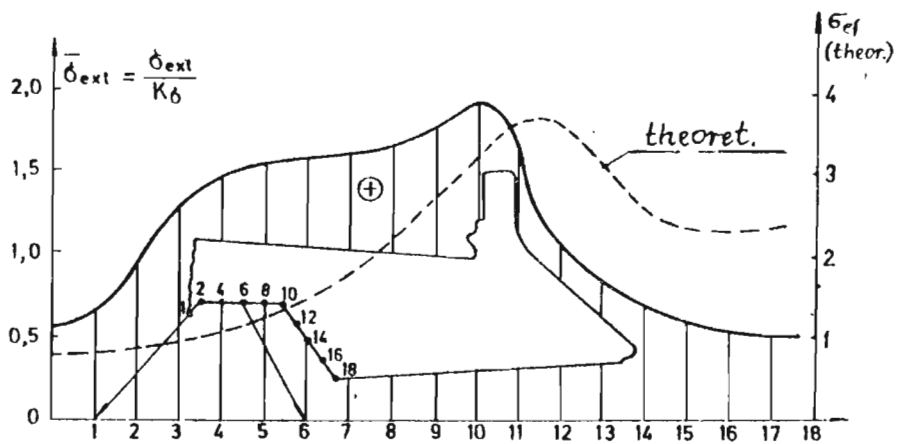


Fig. 12.

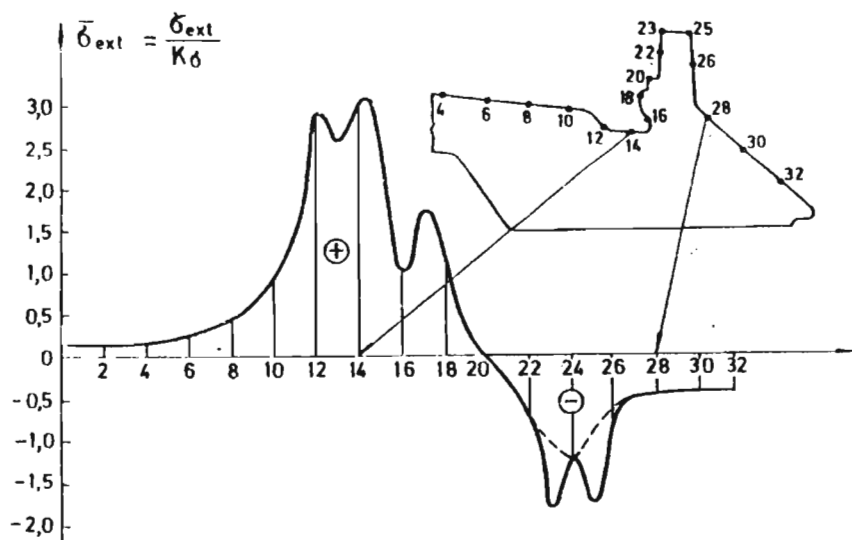


Fig. 13.

Hence we obtain the following relations

$$\frac{\sigma_m}{E_m} = \frac{\sigma_r}{E_r}, \quad \nu_m = \nu_r, \quad \frac{\rho_m l_m^2 n_m^2}{E_m} = \frac{\rho_r l_r^2 n_r^2}{E_r}.$$

The preceding relations permit the translation of the results from the model (m) to the real structure (r).

The most difficult condition to be satisfied is the condition of the equality of Poisson ratios.

$$\nu_m = \nu_r.$$

In the considered case $\nu_m = 0.50$ and $\nu_r = 0.30$ and it corresponds to the approximate model similarity. It has no influence on the stress distribution which is essential in the case of the identification of the regions stress concentration in the element.

The material and geometrical constants in the considered case assume the values

$$\frac{l_r}{l_m} = 5, \quad \frac{E_m}{E_r} = 1.51 \cdot 10^{-4}, \quad \frac{\rho_r}{\rho_m} = 7.36$$

whence

$$n_m = \frac{l_r}{l_m} \sqrt{\frac{\rho_r E_m}{\rho_m E_r}} n_r = 0.1667 n_r.$$

For $n_r = 1000 \text{ min}^{-1}$, $n_m = 166.7 \text{ min}^{-1}$. During the experiment $n_m = 500 \text{ min}^{-1}$ but even for such conditions the material of the model was linear and fulfilled the assumed similarity conditions of the model.

3. Conclusions

The numerical - as well as experimental methods described above are very useful from the practical point of view and can be used for designing of new structures, or for investigation of existing structures submitted to the influence of centrifugal forces.

The procedure permits the definition of a more general method for the shape optimization taking into account the minimization of the weight of the structure.

References

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Streszczenie

W pracy przedstawiono wyniki badań doświadczalnych stanu naprężenia w dysku przymysłowej maszyny wirnikowej o złożonej geometrii. Badania wykonano metodą elasto-optyczną zamrażania naprężeń. Poprzedzone one zostały analizą numeryczną metodą elementów skończonych, wyniki której wykorzystano do ustalenia kształtu modelu wirnika.

Otrzymane wyniki potwierdziły przydatność metody zamrażania naprężeń do analizy ich rozkładu w elementach wirujących.

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