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ORIGINAL AND STRAIN INDUCED PLASTIC ANISOTROPY IN METAL SURFACE LAYERS

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Metal articles, specially tools and machine elements should satisfy high requirements with respect to the fatigue and fracture toughness. For that reason, a great progress in development of metal surface layer technologies is observed recently. The surface layer after a heat, chemical, electrochemical or physical treatment is characterized by *an original plastic anisotropy* and a field of residual stresses. In a further mechanical treatment or in an exploitation process, the layer is squeezed what causes a change in the microstructure of material and redistribution of the residual stresses. The above leads to appearance and development of *a strain induced anisotropy*. In a design process of metal surface layers, one can apply the elastic-plastic analysis of metal elements. The analysis should take into account the both kinds of the plastic anisotropy. A description of selected surface layers obtained after various technological processes and a review of adequate mathematical models for elastic-plastic analysis are presented in the paper. The considered constitutive equations take into account: the original and strain induced plastic anisotropy, large plastic strains of the surface layer material, as well as, a polycrystalline microstructure of metals. Preliminary numerical examples, as an illustration of a surface layer behavior, are given.

1. Introduction

Physical, chemical and mechanical properties of a metal surface layer are different from the remaining part (the core) of a metal element. This is an effect of various surface treatments of metal units in a manufacturing process. Surface technologies enable us to obtain metal articles with better working parameters, such as smoothness, fatigue strength and corrosion resistance. It is particularly important in the case of machine elements and tools working under variable loading conditions.

From the mechanical point of view a surface layer is characterized by a strong plastic anisotropy and a concentration of residual stresses. The above properties appear when a metal element is subjected to one of the following surface technologies:

- heat treatment (quenching, tempering, annealing),
- chemical treatment (carbonizing, nitriding),
- electrochemical treatment (electro-plating),
- physical treatment (ions implantation).

The plastic anisotropy caused by the above technologies will be called *the original plastic anisotropy* of a metal surface layer.

To temper the stress concentration and to make the change of mechanical properties on the border between the surface layer and the core smooth, a mechanical treatment (burnishing, shot-peening) is commonly applied. A squeeze of the material during the mechanical treatment leads to *the strain induced plastic anisotropy* of the layer.

Combination of the previously described surface technologies with the plastic working gives the metal surface layer of desired technical parameters. Note, that the strain induced plastic anisotropy appears also in an exploitation process. Changes occurring in this process should be analyzed separately. Concluding, the final anisotropy and the stress concentration in the metal surface layer is caused by a conscious technological process and working conditions of the metal article.

2. Structure and mechanical properties of metal surface layers

According to the standard definition, the metal surface layer is a part of metal element laying under its external surface, which has mechanical, physical and sometimes even chemical properties different from those of the core of the element (Fig.1). Width of metal surface layers varies from 0.005 to 5 mm.

The most important characteristics describing a state of surface layer are the following:

- structure of the layer
- hardness field

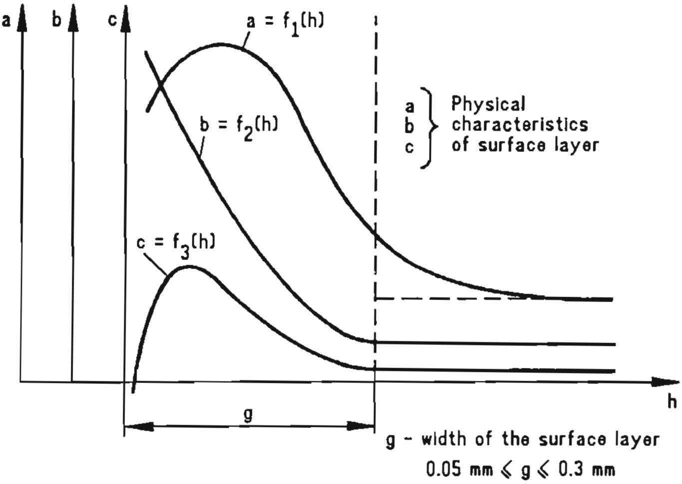

Fig. 1. Schematic description of the surface layer

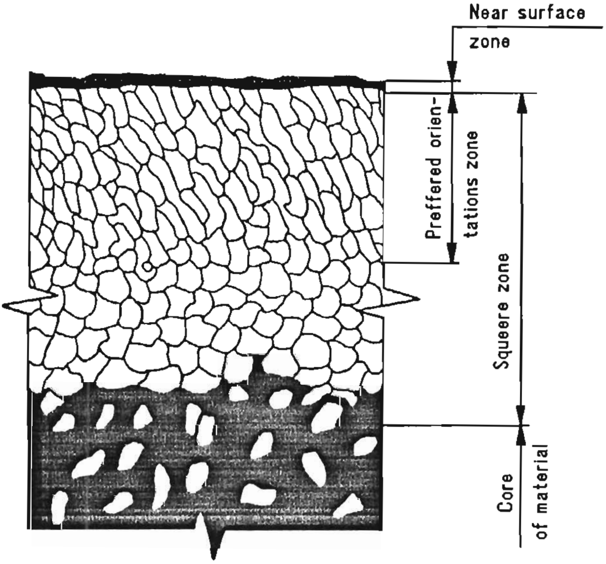

Fig. 2. Structure of metal surface layer according to the Polish Standards[1]

- residual stress field.

A simplified model of the structure of surface layer is shown in Fig.2. One can distinguish in the model: near-surface zone, preferred orientation zone and squeezed zone[1].

A more complex model of the surface layer structure distinguishes the following zones, Hebda and Janecki (1972):

- absorbed organic particles,
- adsorbed water particles,
- adsorbed gases,
- metal oxides,
- degraded crystallites,
- plastically deformed and textured grains,
- plastically deformed grains,
- elastically deformed grains.

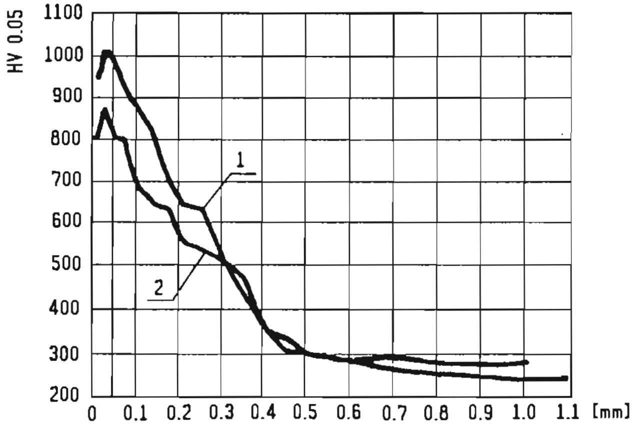

Fig. 3. Hardness distribution in the carbonized layer (1) and in the nitrided layer (2) for steel 18HGT

Examples of the hardness distribution in the carbonized and the nitrided layers, respectively, are shown in Fig.3. A residual stress distribution in a layer after carbonizing, as well as, after carbonizing and next shot-peening are presented in

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[1] *Surface layer. Terminology*, Polish Standards: PN 87/M-04250

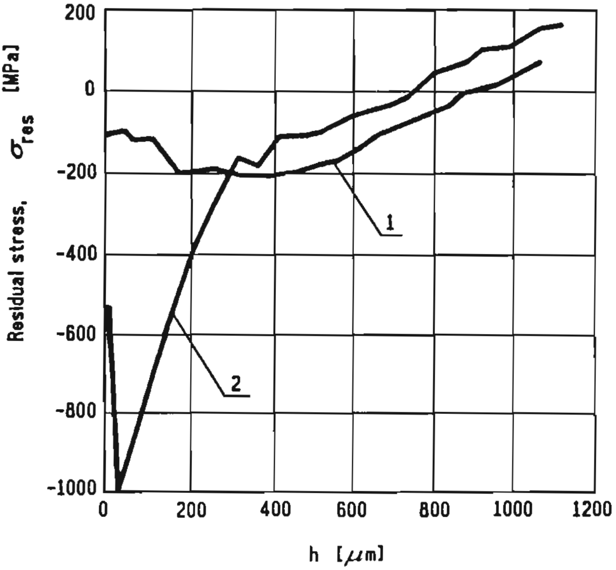

Fig. 4. Residual stress distribution after carbonizing (1) and after carbonizing and next shot-peening (2)

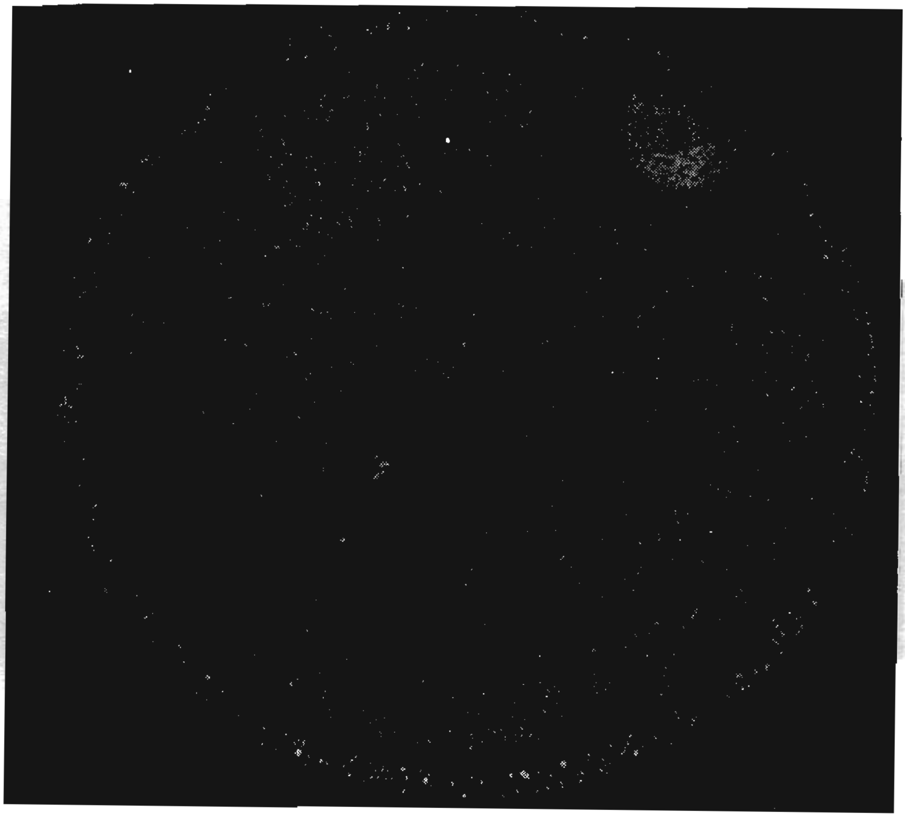

Fig. 5. Localization of a crack under a surface layer of metal element

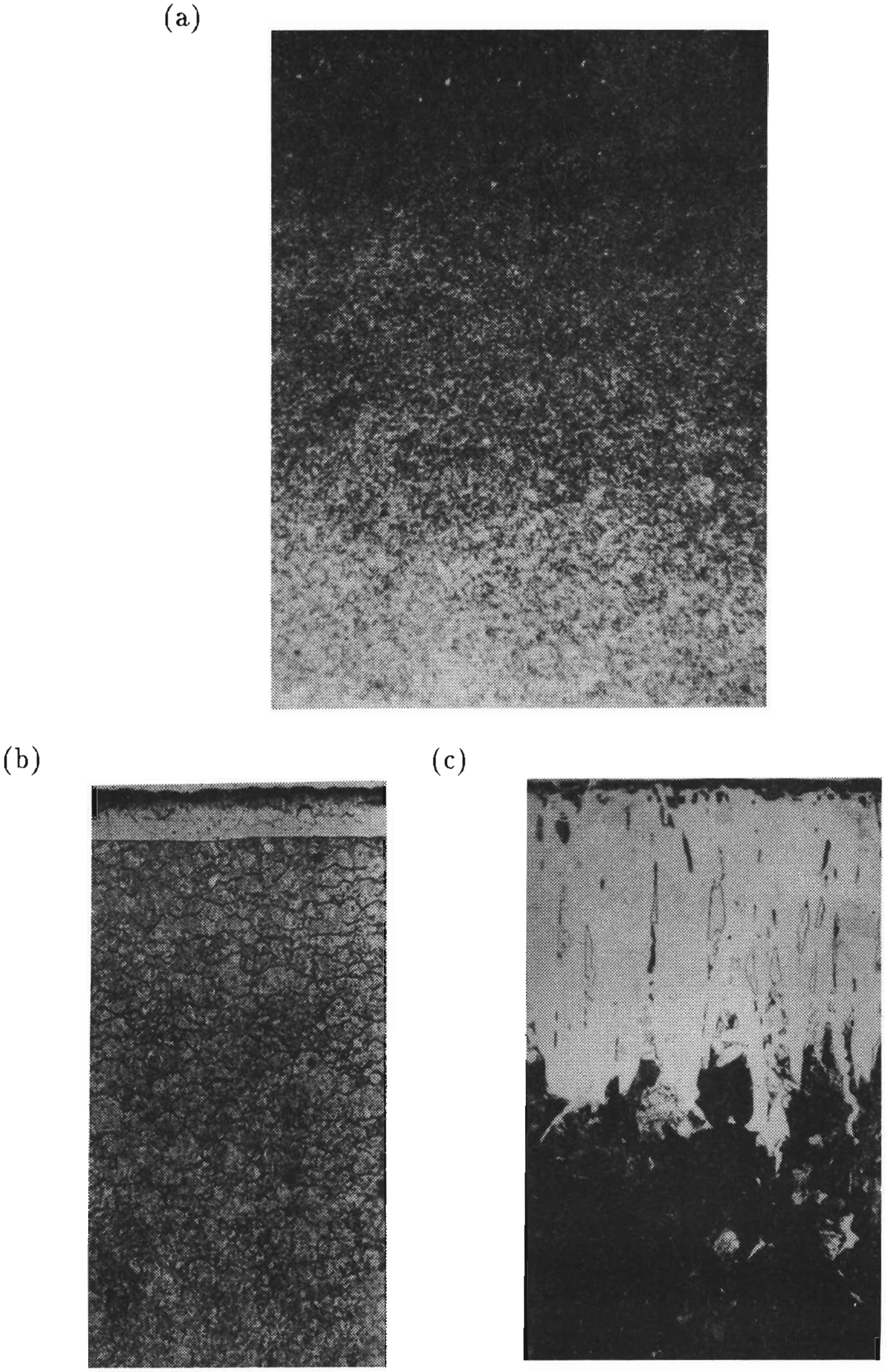

Fig. 6. Microstructure of (a) carbonized, (b) nitrided and (c) boroned layer

Fig.4. Note a positive influence of the mechanical treatment on a growth of residual stresses in the last case.

Hardening of metal surface layers has a strong influence on localization and initiation of the fatigue crack growth in metal elements. Due to the increment of the yield stress and the concentration of compressive residual stresses, a source of the fatigue crack is localized under a surface layer as it is shown in Fig.5. A number of loading cycles before the crack initiation increases considerably in this case.

3. Sources of plastic anisotropy in metal surface layers

As it was mentioned previously, *the original plastic anisotropy* in metal surface layers appears due to phenomena which occur in a manufacturing process. Microstructures of carbonized, nitrided or boroned surface layers (Fig.6) cause a local hardening and a stress concentration. Similar effects produce other surface technologies.

To improve the surface layer properties, metal elements are subjected to a plastic working. In Fig.7. are shown two examples of static burnishing process, and in Fig 8. – a dynamic shot-peening process.

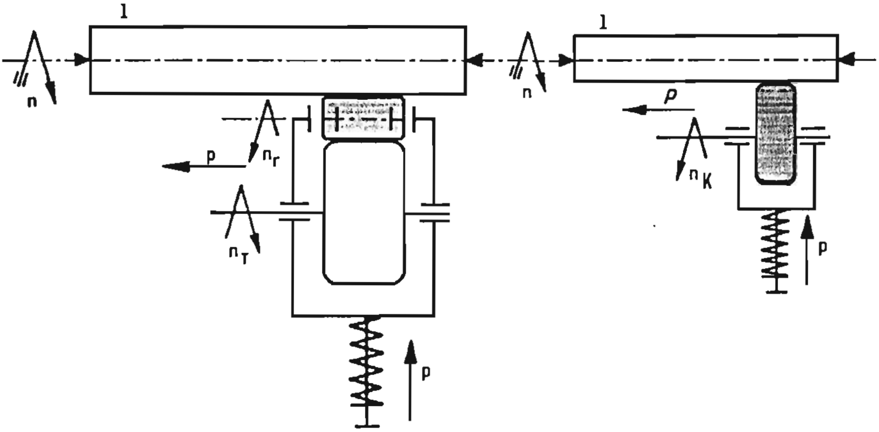

Fig. 7. Examples of a static burnishing process

The plastic working causes the following effects:

- growth of a fatigue strength in metal elements
- hardening of surface layers

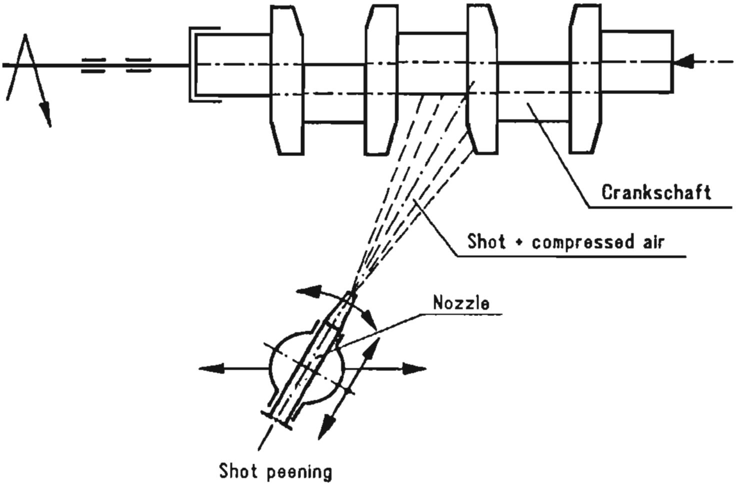

Fig. 8. Shot-peening process

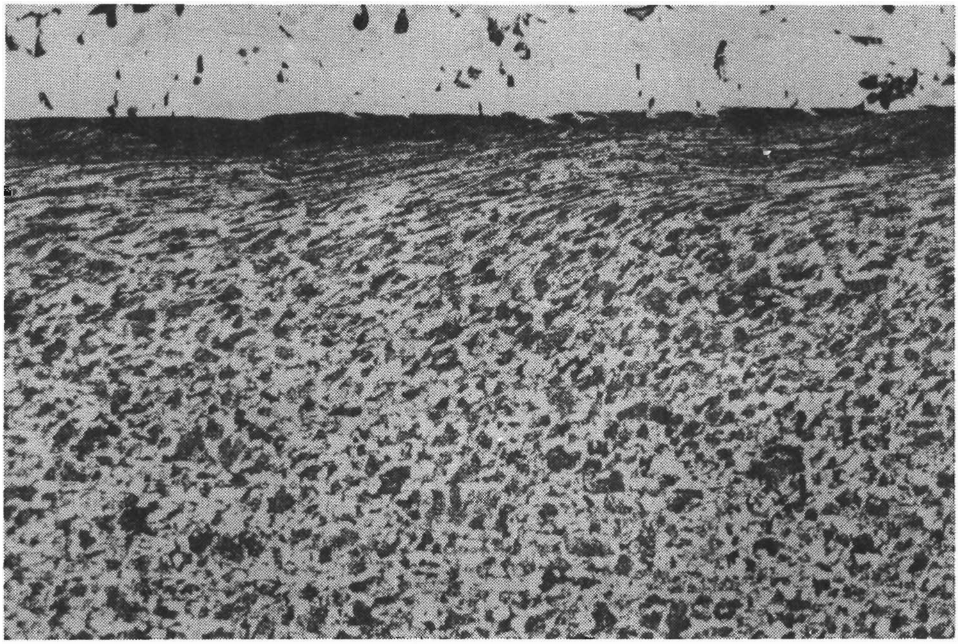

Fig. 9. Elongation of grains in the surface layer

- increment of a surface layer smoothness.

The first two of the above effects are due to an increment of compressive residual stresses and current yield stresses.

Plastic deformation of the surface layer cause an appearance and changes of *the strain induced plastic anisotropy.* Microscopic observations indicate two main reasons of these phenomena:

- elongation of metallic grains (morphological texture) (Fig.9)
- reorientation of crystalline lattices of the grains (crystallographical texture) (Fig.10).

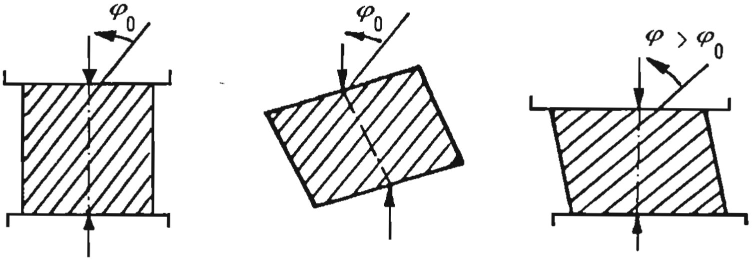

Fig. 10. Reorientation of a crystalline lattice

Because of a strong influence of the texture effects on macroscopic properties of metal surface layers, mathematical models of the layer behaviour should take into account both macroscopic and microscopic phenomena connected with the plastic deformations of metals.

4. Modelling of elastic-plastic behaviour of metal surface layers

Computer analysis of metal surface layers enables us to predict their elastic-plastic behaviour in both a forming and an exploitation process. It is assumed, that initial properties of the layers are prescribed by: elastic moduli, hardening moduli, a limit stress field and a residual stress field. As a result of the analysis, one obtains a final yield stress field and a change in the residual stress field after a given elastic-plastic deformation process.

There are two fundamental questions connected with the experimental input data for the computational analysis: how to measure the local limit stress and how to improve a measurement accuracy for residual stresses in thin surface layers? In many cases, it is necessary to estimate the limit stress on the basis of hardness measurements and make many different tests to separate residual stresses of the first order from those of the second order.

Other problems are connected with a formulation of physical and mathematical models for the computational analysis. A full description of the metal surface layer behaviour requires taking into account large plastic strains and changes in the plastic anisotropy of the material. Such a problem formulation needs a microstructural approach which leads to very expensive computations. For some materials and technological processes it is enough to consider small plastic strains of a surface layer and a core of a metal element. Sometimes, the normal anisotropy of the surface layer may be described by introduction of a system of sublayers with different plastic properties. The classical, macroscopic approach enables us to formulate simplified models: anisotropic plasticity at small strain and isotropic plasticity at large strain. Below, a short review of two groups of models of a surface layer behaviour is given. The macroscopic, as well as, the microscopic approach is presented.

4.1. Plasticity at small strain

4.1.1. Huber-Mises isotropic yield theory

Neglecting the anisotropy of metal surface layers, one can apply the classical yield theory, Hill (1950). In this case, a constitutive behaviour of an elastic-plastic material is described by:

– the Huber-Mises yield condition

$$\frac{1}{2}s_{ij}s_{ij} = k^2(\widehat{\varepsilon}^p) \tag{4.1}$$

– the flow rule (the Prandtl-Reuss equations):

$$\dot{\sigma}_{ij} = \frac{E}{1+\nu}\Big[\frac{\nu}{1-2\nu}\delta_{ij}\delta_{kl} + \frac{1}{2}\delta_{ik}\delta_{jl} + \frac{1}{2}\delta_{il}\delta_{jk} - \frac{s_{ij}s_{kl}}{\frac{2}{3}\widehat{\sigma}^2\Big(1+\frac{H}{3G}\Big)}\Big]\dot{\varepsilon}_{kl} \tag{4.2}$$

where

$$\begin{aligned} \widehat{\sigma}^2 &= \frac{3}{2}s_{ij}s_{ij} \qquad & H &= \frac{d\widehat{\sigma}}{d\widehat{\varepsilon}^p} \\ d\widehat{\varepsilon}^p &= \sqrt{\frac{2}{3}d\varepsilon^p_{ij}d\varepsilon^p_{ij}} \qquad & \dot{\varepsilon}^p_{ij} &= \frac{4}{9}\frac{s_{ij}s_{kl}}{\widehat{\sigma}^2 H}\dot{\sigma}_{kl} \end{aligned} \tag{4.3}$$

s_{ij} – a deviator of the Cauchy stress σ_{ij}
k – yield shear stress
E, G, ν – isotropic elastic moduli
$\dot{\sigma}_{ij}, \dot{\varepsilon}_{ij}, \dot{\varepsilon}^{p}_{ij}$ – stress, total strain and plastic strain increment, respectively.

4.1.2. Hill anisotropic plasticity

More general description gives the anisotropic theory of plasticity proposed by Hill (1950). The model allows an initial orthotropy of plastic properties which does not change in a deformation process. For the case of rigid-plastic materials, we have at our disposal:

– the yield condition

$$\begin{aligned} 2f(\sigma_{ij}) \equiv F(\sigma_{22}-\sigma_{33}) + G(\sigma_{33}-\sigma_{11}) + H(\sigma_{11}-\sigma_{22}) + \\ +2L\tau_{23}^2 + 2M\tau_{31}^2 + 2N\tau_{12}^2 - 1 = 0 \end{aligned} \tag{4.4}$$

– the flow rule

$$\dot{\varepsilon}^{p}_{ij} = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}} \tag{4.5}$$

where: F, G, H, L, M, N are parameters characterizing the anisotropy of plastic yield, and $\dot{\lambda}$ is a scalar positive function which can be determined from the yield criterion (4.4).

4.1.3. Baltov-Sawczuk model of plastic anisotropy development

A proposition of a model which describes changes of plastic anisotropy due to deformation process has been given by Baltov and Sawczuk (cf Lee, 1969). In this model, moduli of plastic anisotropy depend on plastic strain. The yield condition, for the case of rigid-plastic materials without the Bauschinger effect is the following

$$2f(\sigma_{ij},\varepsilon^{p}_{ij}) \equiv \Big[\mu\Big(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\Big) + A\varepsilon^{p}_{ij}\varepsilon^{p}_{kl}\Big]s_{ij}s_{kl} - 1 = 0 \tag{4.6}$$

where ε^{p}_{ij} is the plastic strain, and λ, A are material constants. The flow rule is given by the relation (4.5).

4.2. Plasticity at large strain

4.2.1. Macroscopic theory for isotropic materials

In the models at small strain, the plastic strain was defined as that which remains after elastic unloading of a specimen. One can adapt the above definition for the case of large plastic strain under condition of a uniform deformation of the specimen. In general case, we cannot split the total strain on its elastic and plastic part because of residual distortions which remain in the specimen after unloading. To remove these distortions, one can cut the unloaded specimen into small pieces. The final strains of the pieces may be regarded as the plastic ones. Lee (1969) has proposed to assume that it is possible to cut mentally the unloaded specimen on a finite number of unstressed pieces for any elastic-plastic deformation field. The above assumption is known in the literature as the hypothesis of intermediate (or unstressed) configuration. Note, that we cannot determine the final orientation of the unstressed configuration. For that reason the hypothesis is valid only in the case when mechanical response of the material is independent on its orientation. It is the case of an isotropic material. The hypothesis of intermediate configuration leads to the multiplicative decomposition of the total deformation gradient F_{ij}, i.e. separation into its elastic and plastic part

$$F_{ij} = F^e_{ik} F^p_{kj} \tag{4.7}$$

On the base of the above decomposition McMeeking and Rice have proposed the following generalization of the Prandtl-Reuss equations (4.2) on the case of large plastic strains (McMeeking and Rice, 1975)

$$\tau^{\nabla}_{ij} = \frac{E}{1+\nu}\Big[\frac{\nu}{1-2\nu}\delta_{ij}\delta_{kl} + \frac{1}{2}\delta_{ik}\delta_{jl} + \frac{1}{2}\delta_{il}\delta_{jk} - \frac{s_{ij}s_{kl}}{\frac{2}{3}\widehat{\sigma}^2\Big(1 + \frac{H}{3G}\Big)}\Big]d_{kl} \tag{4.8}$$

where

d_{ij} – strain rate tensor

$\tau_{ij} = (\rho/\rho_0)\sigma_{ij}$ – Kirchhoff stress

τ^{∇}_{ij} – Zaremba-Jaumann derivative of τ_{ij}

ρ – current density

ρ_0 – reference density.

The remain denotations are given under Eqs (4.3).

4.2.2. *Microstructural theories for textured materials*

Microstructural theories are applied in the case of large plastic strains which occur in a metal forming processes. They allow us to describe appearance and development of plastic anisotropy caused by plastic deformations. If a polycrystalline structure of metals is taken into account, the plastic yield appears as a result of glides on slip systems of each of the grains composing the considered metal specimen. Because the glides may occur on certain crystallographical planes and in certain crystallographical directions only, a single grain is plastically anisotropic. During a plastic deformation the slip planes and directions rotate with respect to the specimen frame taking certain privileged orientations. Existence of such orientations in a material is called *a crystallographical texture.* Change of the texture during a deformation process leads to an evolution of plastic anisotropy of the polycrystalline material.

To describe the elastic-plastic behaviour of polycrystals it is necessary to have a model of single crystals behaviour, at first. Next, it is necessary to assume a model of polycrystal which satisfies the continuity and equilibrium conditions for the crystalline aggregate.

In the case of large plastic strains, the Taylor's model (1938) of the grain aggregate is usually applied. It assumes, that the local strain fields, in every grain, are the same like the global, macroscopic strain field. This assumption satisfies the continuity condition, but the forces on grain boundaries are not in equilibrium.

The first, complete formulation of a single crystal plasticity has been proposed by Hill and Rice (1975). The model bases on the Schmid law: a crystal yields when a shear stress on a certain slip system reaches its critical value. The Schmid law plays the role of a plasticity condition which is described by a three-parametric family of yield surfaces. Flow rule for the crystal is associated with the above plasticity condition through the orthogonality principle. Such a formulation leads to an ambiguity in the choice of active slip systems when the end of stress vector is situated in a plastic corner of a yield surface. To overcome this problem a nonlinear, viscous approximation of the plastic crystal behaviour is applied in some papers, Asaro and Needleman (1985). The approximation gives good results if a parameter describing the material viscosity is large enough. The above model is commonly used in a computational analysis of polycrystals. However, because of viscous effects its application is limited to a certain class of deformation processes. Recently, it has been proposed a model which avoids the problem of nonuniqueness in the choice of active lip system and describes solely the elastic-plastic behaviour of crystals (Gambin, 1992). The system of equations of the model is very similar to those proposed by McMeeking and Rice for isotropic, elastic-plastic continuum at large strain (cf Gambin, 1992)

$$\tau_{ij}^{\nabla} = \Big(\mathcal{L}_{ijkl} - \frac{\mathcal{F}_{ij}\mathcal{F}_{kl}}{\mathcal{F}_{mn}\mathcal{G}_{mn} + h_0}\Big) d_{kl} \tag{4.9}$$

where

$\mathcal{L}_{ijkl}$ – elastic moduli tensor
h_0 – hardening parameter
$\mathcal{F}_{ij}$, $\mathcal{G}_{ij}$ – functions of stress, elastic moduli and parameters describing a crystal geometry.

Details are given by Gambin (1992).

The last model of a single crystal behaviour together with the Taylor's model of polycrystal allows us to predict texture development due to large plastic strain.

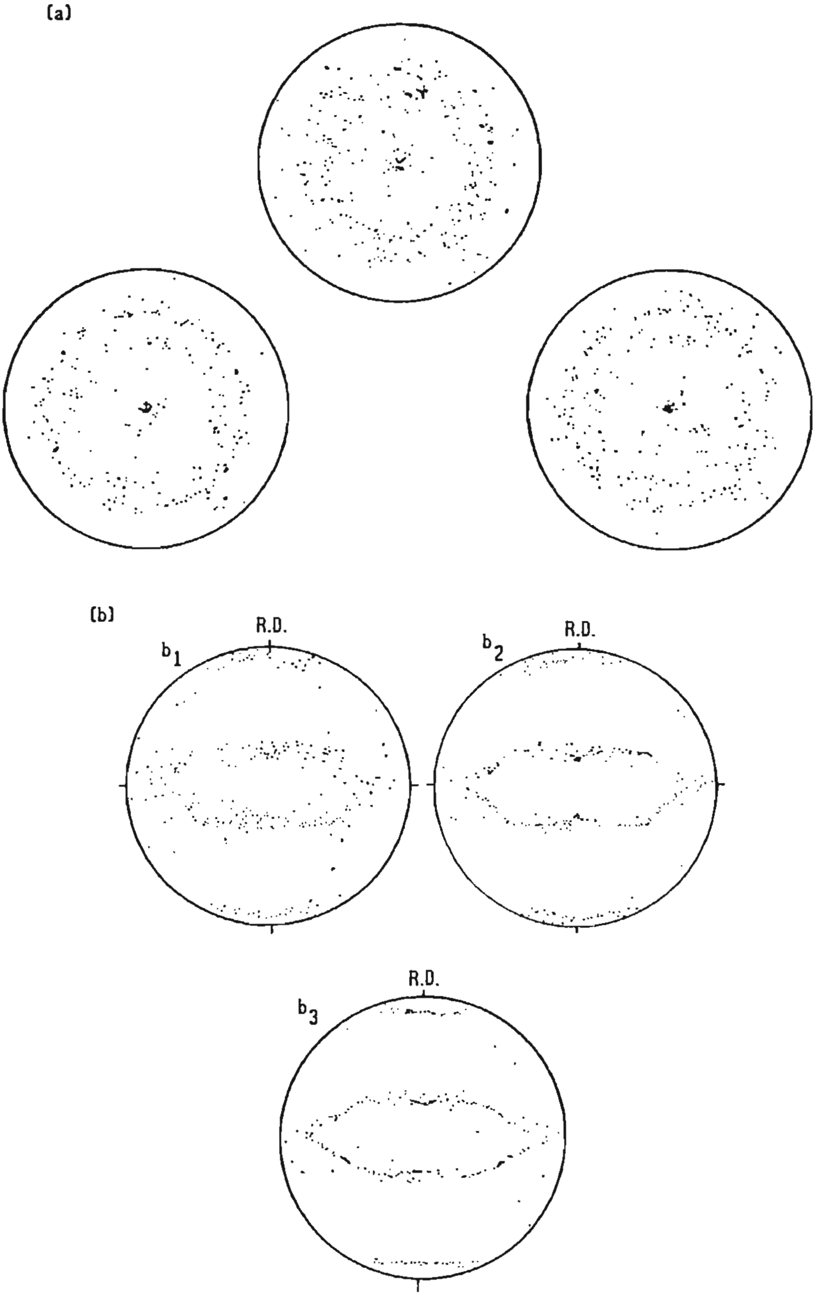

Fig. 11. (111) pole figures for (a) drawing and (b) rolling, after 50%, 75% and 88% reduction

In Fig.11, the (111) pole figures show the texture changes in drawing and rolling processes.

5. Preliminary numerical results

The presented models give many possibilities for computational analysis. The proper choice of a model depends on a structure of surface layer and a deformation process. Below, preliminary results of numerical calculations for the model of isotropic continuum at large plastic strain are given. Consider a metal specimen with a surface layer under plane strain conditions. The specimen is locally squeezed (Fig.12 and 13). Two cases are considered: (a) the surface layer is softer than the core (Fig.12), (b) the surface layer is harder than the core (Fig.13). In the first case, the elastic limit R_e is 300 MPa and 500 MPa for the surface layer and the core, respectively. In the second case, the corresponding values of R_e are equal 700 MPa and 325 MPa, respectively.

6. Conclusions

The present paper indicates a possibility of a computer analysis in the process of designing of metal surface layers. Such an analysis, based on the modern theories of elastic-plastic deformations, enables us to predict a metal surface layer behaviour during forming and exploitation processes. Efficiency of the analysis depend on a solution of two kinds of problems. The first is connected with its experimental basis: to find a hardness/yield stress relation and to unify residual stress measurements in thin surface layers. The second group of problems is connected with a constitutive modelling of surface layers: to give an effective description of the anisotropic plastic flow for materials with microstructure and to describe the fracture of such materials under monotonic and variable loading.

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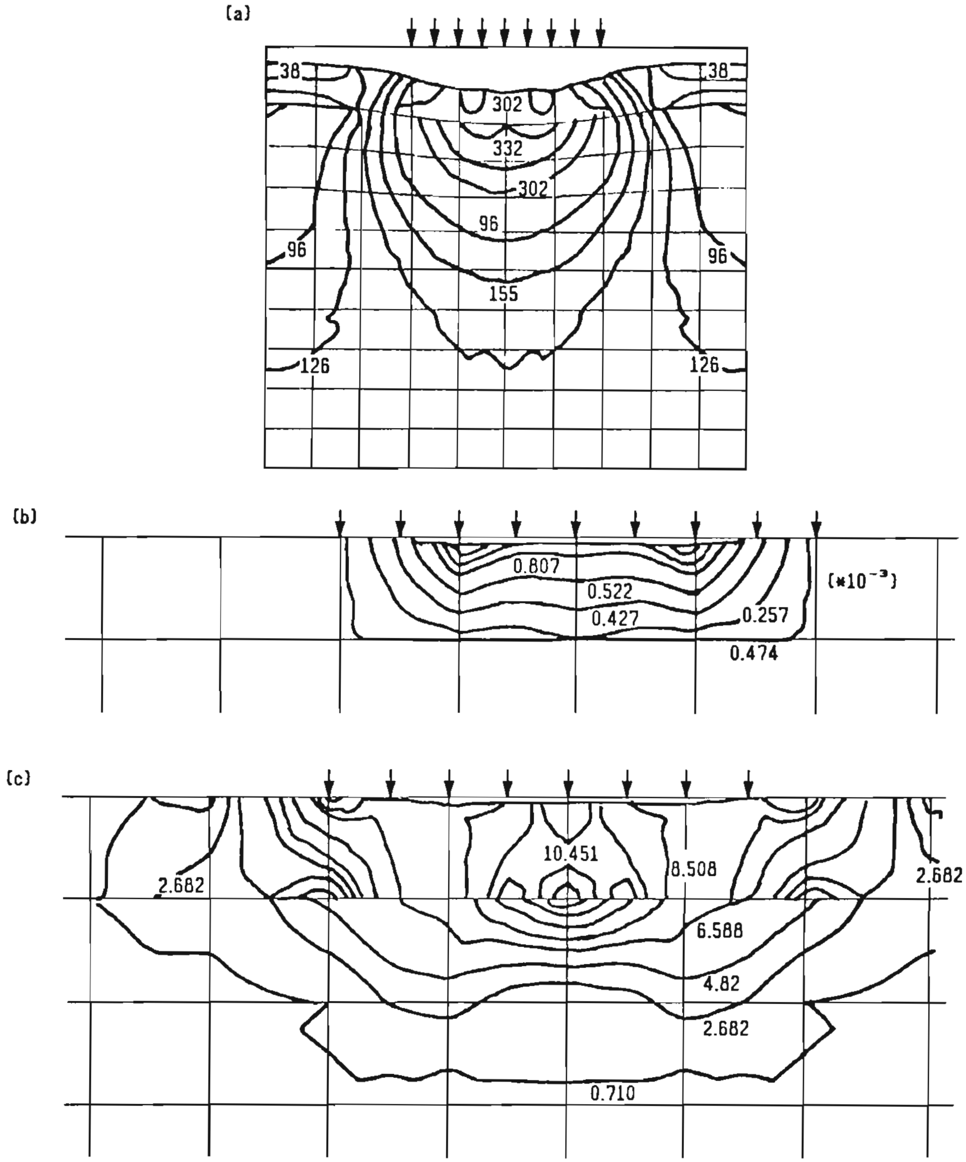

Fig. 12. Burnishing of the soft surface layer, (a) effective stress [MPa], (b) plastic strain [–], (c) residual stress [MPa]

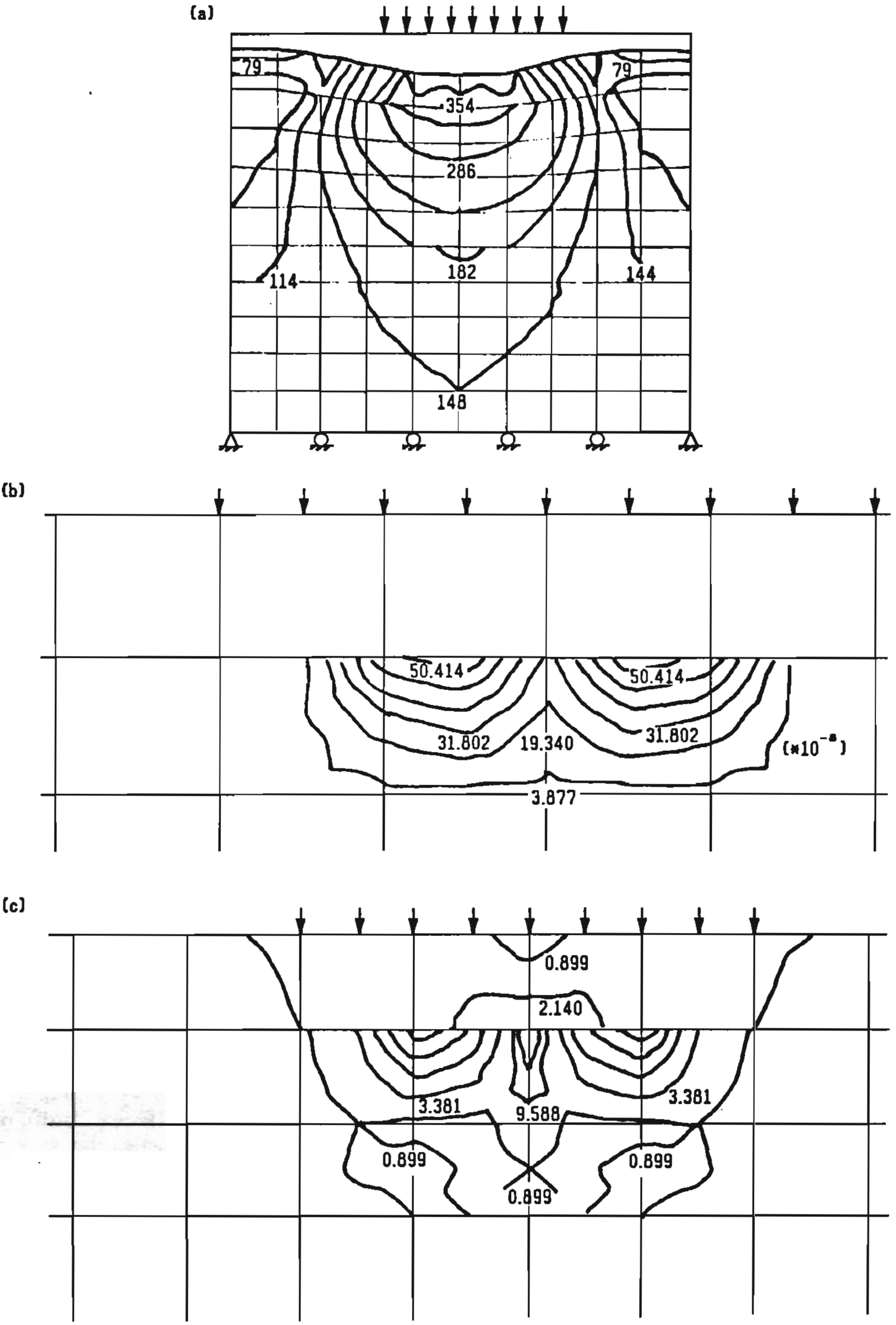

Fig. 13. Burnishing of the hard surface layer, (a) effective stress [MPa], (b) plastic strain [–], (c) residual stress [MPa]

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Początkowa i indukowana anizotropia plastyczna w warstwach wierzchnich metali

Streszczenie

Szeregu wyrobom, specjalistycznym narzędziom oraz częściom maszyn stawiane są wysokie wymagania z uwagi na wytrzymałość zmęczeniową oraz odporność na kruche pękanie. Z tego też powodu obserwuje się obecnie intensywny rozwój technologii warstw wierzchnich. Warstwy wierzchnie metali po obróbkach cieplno-chemicznych czy elektrofizycznych charakteryzują się zazwyczaj początkową anizotropią plastyczną. W kolejnych fazach różnych procesów obróbek mechanicznych bądź podczas eksploatacji warstwy wierzchnie części maszyn ulegają zgniotowi, który wywołuje zmiany w mikrostrukturze materiału oraz redystrybucję naprężeń własnych. Powyższe zjawiska prowadzą następnie do wystąpienia indukowanej anizotropii odkształceniowej. Podczas projektowania stanu warstwy wierzchniej do opisu tych zjawisk wykorzystuje się teorię sprężysto-plastycznych deformacji metali. Analiza taka winna jednak uwzględniać oba rodzaje anizotropii. W pracy zaprezentowano opis wybranych modeli warstw wierzchnich otrzymanych po różnych procesach technologicznych oraz przegląd odpowiednich modeli matematycznych sprężysto-plastycznych deformacji. Rozpatrywane równania konstytutywne uwzględniają początkową i indukowaną plastyczną anizotropię oraz duże odkształcenia materiału warstwy wierzchniej a także polikrystaliczną mikrostrukturę metali. Przedstawiono także wstępne wyniki obliczeń numerycznych będące ilustracją sygnalizowanych zmian w warstwie wierzchniej.

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