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STABILIZATION OF BEAM PARAMETRIC VIBRATIONS

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A theoretical investigation of dynamic stability for linear elastic beams due to time dependent harmonic axial forces is presented. The concept of intelligent structure is used to insure the active damping. In the present paper the applicability of active vibration control is extended to linear continuous systems with parametric harmonic excitations. The study is based on the application of distributed sensors, actuators, and an appropriate feedback and is adopted for stability problems of system consisting of beam with control part governed by uniform partial differential equations with time dependent coefficients. To estimate deviations of solutions from the equilibrium state (the distance between a solution with nontrivial initial conditions and the trivial solution) a scalar measure of distance equal to the square root of the functional is introduced. The Lyapunov method is used to derive a velocity feedback implying nonincreasing of the functional along an arbitrary beam motion and in consequence to balance the supplied energy by the parametric excitation and the dissipated energy by the inner and control damping. In order to calculate the energetic norm of disturbed solution as a function of time the partial differential equation is solved numerically. The numerical tests performed for the simply supported beam with surface bonded actuators and sensors show the influence of the fedback constant on the vibration decrease.

1. Introduction

Piezoelectrics show a great promise as elements of **intelligent structures**, i.e. structures with highly distributed actuators, sensors and processor networks. Such a system allows the use of software adjustment to modify and tune the closed-loop behaviour via distributed sensor and actuators. Spatially-varying piezoelectric actuator distributions has been applied to control all vibrational modes of flexible beams with a wide class of boundary conditions (cf Burke and Hubbard, 1988; Newman, 1991). Control strategies were derived using Lyapunov's direct method. The flexural vibration of an elastic beam having a piezoelectric actuator bonded to one face was described by a partial differential equation with the inhomogeneous term representing space and time variations of control distributions. As the control input is modelled by distributed moments generated by an electric field, intrinsic features of mechanical coupling between the beam and piezoelectric layers were neglected.

Crawley and de Luis (1987) presented a comprehensive static model for a piezoelectric actuator, bonding layer and structurally coupled beam. In their model the effective formulas describing static structural strains both in the structure and in the actuator as well as shear stresses in the bonding layer were derived and discussed. Jie Pan, Hansen and Snyder (1991) analysed a dynamic model of a simply supported beam response to excitation by actuators made using piezoelectric ceramics glued to the beam surface. The dynamic extensional strains on the beam surface were calculated neglecting the presence of a finite bonding layer and assuming a perfect bonding actuator. Wang and Rogers (1991) alternatively modelled the dynamics of the beam – actuator system using a strain – energy approach.

The present paper is devoted to formulating control laws without the necessity of modelling the beam in terms of its vibrational modes. The beam is supposed to be axially compressed by a harmonic force, which can excite parametric vibrations and destabilize the system. The previous assumptions are relaxed and the finite bonding layer is analysed.

In the second section the dynamic equations governing a mechanical coupling are considered. Under the assumption that the bonding layers are very thin the control strategies are derived via the direct Lyapunov method in the third section. In the fourth section a full dynamic model taking into account the finite bonding actuators and sensors is developed. Control laws are derived using the Lyapunov approach with a functional in the form of total mechanical energy. Collocated sensor/actuator system (Dosch, Inman and Garcia, 1992) is discussed and compared to the classical solution, where the actuator and the sensor are mounted on opposite sides of the beam.

In the sixth section of this paper a behaviour of system energy is numerically simulated.

2. Equations of an axially loaded beam with perfectly bonded layers – model 1

We start our consideration from a Bernoulli-Euler beam axially loaded by a time-dependent force $S(t)$ with identical piezoelectric layers mounted on each of two opposite sides of the beam. The layers are perfectly bonded onto the beam, which allows the assumption of strain continuity at the bonding interface. The beam is divided into three sections and the dynamics of each section should

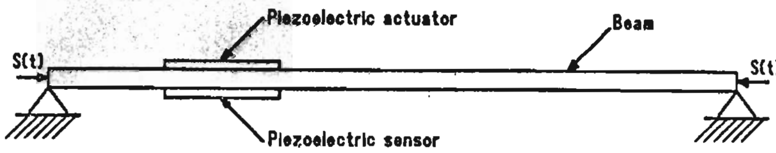

Fig. 1. Beam bonded with piezoelectric layers

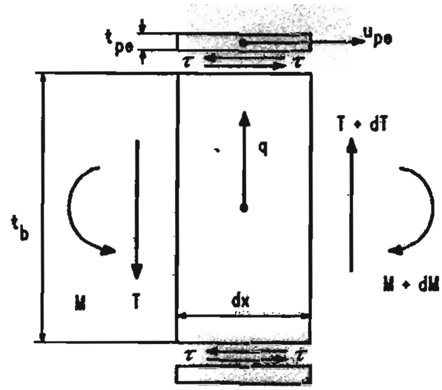

Fig. 2. Geometry of beam element with perfectly bonded piezoelectric layers

be described separately (see Fig.1). The piezoelectric layers are located between $x = x_1$ and $x = x_2$. The element of length dx in the second section $x_1 < x < x_2$ is shown in Fig.2. The thickness t_{pe} of piezoelectric layers is assumed to be small as compared with the beam thickness t_b and therefore the longitudinal stress in the element is assumed to be uniform in the transverse direction. Static equations of the element are given as follows

$$T_{,x} + q = 0 \tag{2.1}$$

$$M_{,x} - T + \tau b t_b = 0 \tag{2.2}$$

where τ is the shear stress on the interface surface, t_b is the beam thickness, and b is the beam width. In order to derive the dynamic equations we substitute q for the inertia force in the transverse motion. A comma denotes a partial derivative of the main symbol with respect to the index. Assuming the pure onedimensional shear in the bonding layer and pure extensional strain in the piezoelectric and beam and using the strain displacement relationships (Crawley and de Luis, 1987) the governing equations have the form

$$E_{pe} t_{pe} \varepsilon_{pe,xx} - \tau_{,x} = \rho_{pe} \varepsilon_{pe,tt} \tag{2.3}$$

$$E_b J_b \varepsilon_{b,xx} + t_b b \tau_{,x} = \rho_b t_b b w_{,tt} \tag{2.4}$$

where ε_{pe} and ε_b are the strains in the piezoelectric element and in the surface of the beam, respectively.

The geometrical moment of the beam cross-section is defined by $J_b = bt_b^3/12$. The beam transverse displacement is related to the beam surface strain by

$$w_{,xx} = -\frac{2\varepsilon_b}{t_b} \tag{2.5}$$

Expressing the transverse load acting on the beam by the inertia force and a component due to the axial load S in the form

$$q = -\rho_b t_b b w_{,tt} - S(t)w_{,xx} \tag{2.6}$$

eliminating the shear τ and using the perfect bonding condition on the interface between the piezoelectric element and the beam

$$\varepsilon_{pe} = \varepsilon_b \qquad\qquad x \in (x_1, x_2) \tag{2.7}$$

dynamic equations of motion in displacements can be written down as
— for $x_1 \le x \le x_2$

$$(E_bJ_b + E_{pe}J_{pe})w_{,xxxx} - \rho_{pe}J_{pe}w_{,xxtt} + \rho_b t_b w,tt + S(t)w,xx = 0 \tag{2.8}$$

— for $0 \le x \le x_1$ and $x_2 \le x \le \ell$,

$$E_bJ_bw,xxxx + \rho_b t_b w,xxtt + S(t)w,xx = 0 \tag{2.9}$$

where $J_{pe} = bt_{pe}t_b^2/2$.

Introducing x dependent coefficients $E_pJ_p(x)$ and $E_{pe}J_{pe}(x)$

$$E_bJ_b(x) = \begin{cases} E_bJ_b & \text{if} \quad 0 \le x \le x_1 \\ E_bJ_b + E_{pe}J_{pe} & \text{if} \quad x_1 \le x \le x_2 \\ E_bJ_b & \text{if} \quad x_2 \le x \le l \end{cases}$$

$$\rho_{pe}J_{pe}(x) = \begin{cases} 0 & \text{if} \quad 0 \le x \le x_1 \\ \rho_{pe}J_{pe} & \text{if} \quad x_1 \le x \le x_2 \\ 0 & \text{if} \quad x_2 \le x \le l \end{cases}$$

we can rewrite Eqs (2.8) and (2.9) in the form

$$E_bJ_b(x)w_{,xxxx} - \rho_{pe}J_{pe}(x)w_{,xxtt} + \rho_b t_b w_{,tt} + S(t)w,xx = 0 \qquad \text{for} \quad x \in (0,l) \tag{2.10}$$

The component $\rho J_{pe}w_{,xxtt}$ in Eq (2.10) represents the rotary inertia term of the piezoelectric elements and therefore Eq (2.10) is the same as in the Timoshenko beam theory with omitting a shear effect.

Boundary conditions corresponding to simply supported ends for $x = 0, l$ have the form

$$w(0,t) = 0 \qquad w(l,t) = 0 \tag{2.11}$$

$$w_{,xx}(0,t) = 0 \qquad w_{,xx}(l,t) = 0 \tag{2.12}$$

At the joints between sections 1 and 2 $x = x_1$ and between sections 2 and 3 $x = x_2$ we have the continuity in beam transverse displacement, slope, curvature and transverse forces

$$w(x_1^-) = w(x_1^+) \tag{2.13}$$

$$w_{,x}(x_1^-) = w_{,x}(x_1^+) \tag{2.14}$$

$$w_{,xx}(x_1^-) = w_{,xx}(x_1^+) \tag{2.15}$$

$$E_b J_b w_{,xxx}(x_1^-) = (E_b J_b + E_{pe} J_{pe}) w_{,xxx}(x_1^+) - \rho_{pe} J_{pe} w_{,xtt}(x_1^+) \tag{2.16}$$

$$w(x_2^-) = w(x_2^+) \tag{2.17}$$

$$w_{,x}(x_2^-) = w_{,x}(x_2^+) \tag{2.18}$$

$$w_{,xx}(x_2^-) = w_{,xx}(x_2^+) \tag{2.19}$$

$$E_b J_b w_{,xxx}(x_2^+) = (E_b J_b + E_{pe} J_{pe}) w_{,xxx}(x_2^-) - \rho_{pe} J_{pe} w_{,xtt}(x_2^-) \tag{2.20}$$

The stress free conditions for the piezoelectric layers have the form $\sigma_{pe}(x_1) = \sigma_{pe}(x_2) = 0$. Using the constitutive equations of a piezoelectric material, Lee (1990), they can be written down as

$$w_{,xx}(x_1) = -\frac{2}{t_b}\varepsilon_{pe}(x_1) = -\frac{2}{t_b}\Lambda \tag{2.21}$$

$$w_{,xx}(x_2) = -\frac{2}{t_b}\varepsilon_{pe}(x_2) = -\frac{2}{t_b}\Lambda \tag{2.22}$$

$$\Lambda = \frac{d_{31}\nu}{t_{pe}} \tag{2.23}$$

where Λ denotes the piezoelectric strain, d_{31} is the piezoelectric constant, and ν is the voltage applied across the piezoelectric.

3. Deriving the control strategy for model 1

The partial differential equation (2.10) with the nonuniform boundary conditions, Eqs (2.17) and (2.18) does not have the trivial solution $w = w_{,t} = 0$ and we are going to derive the stabilizing control strategy using the direct Lyapunov method. The crucial point of the method is a construction of a suitable Lyapunov functional, which is positive for any motion of analysed system. Due to the term $\rho J_{pe} w_{,xxtt}$ in Eq (2.10) it is necessary to introduce a term representing the rotary kinetic energy (Tylikowski, 1986 and 1991) to the Lyapunov functional

$$V = \frac{1}{2}\int\limits_0^l \Big[\rho_b b t_b w_{,t}^2 + E_b J_b(x) w_{,xx}^2 + \rho_{pe} J_{pe}(x) w_{,xt}^2\Big]\,dx \tag{3.1}$$

Time-derivative of functional V with respect to time is equal to

$$\frac{dV}{dt} = \int\limits_0^l \big[\rho_b b t_b w_{,t} w_{,tt} + E_b J_b(x) w_{,xx} w_{,xxt} + \rho_{pe} J_{pe}(x) w_{,xt} w_{xtt}\big]dx \tag{3.2}$$

Integrating by parts and using zero boundary conditions we have

$$\int\limits_0^l \rho_{pe} J_{pe}(x) w_{,xt} w_{,xtt}\,dx = -\int\limits_0^l \rho_{pe} J_{pe}(x) w_{,t} w_{,xxtt}\,dx + \rho_{pe} J_{pe} w_{,t} w_{,xtt}\Big|_{x_1}^{x_2} \tag{3.3}$$

Substituting into the time-derivative yields

$$\begin{aligned} \frac{dV}{dt} &= \int\limits_0^l \Big\{w_{,t}[\rho_b b t_b w_{,tt} - E_{pe} J_{pe}(x) w_{,xxtt}] - E_b J_b(x) w_{,xx} w_{,xxt}\Big\}dx + \\ &+ \rho_{pe} J_{pe} w_{,t} w_{,xtt}\Big|_{x_1}^{x_2} \end{aligned} \tag{3.4}$$

Eliminating the first integrand term by means of dynamic equations

$$\begin{aligned} \frac{dV}{dt} &= \int\limits_0^l \Big[-E_b J_b(x) w_{,t} w_{,xxxx} + E_b J_b(x) w_{,xx} w_{,xxt} - S(t) w_{,t} w_{,xx}\Big]dx + \\ &+ \rho_{pe} J_{pe} w_{,t} w_{,xtt}\Big|_{x_1}^{x_2} \end{aligned} \tag{3.5}$$

Integrating by parts separately in intervals: $(0, x_1)$, (x_1, x_2) and (x_2, l) yields

$$
\begin{aligned}
\frac{dV}{dt} = &-\int\limits_0^l S(t)w_{,t}w_{,xx}\,dx + \\
&+w_{,t}(x_1)\Big[(E_{pe}J_{pe}+E_bJ_b)w_{,xxx}(x_1^+) - \rho_{pe}J_{pe}w_{,xtt}(x_1^+) - E_bJ_bw_{,xxx}(x_1^-)\Big] - \\
&-w_{,t}(x_2)\Big[(E_{pe}J_{pe}+E_bJ_b)w_{,xxx}(x_2^-) - \rho_{pe}J_{pe}w_{,xtt}(x_2^-) - E_bJ_bw_{,xxx}(x_2^+)\Big] + \\
&+w_{,xt}(x_1)\Big[E_bJ_bw_{,xx}(x_1^-) - (E_bJ_b+E_{pe}J_{pe})w_{,xx}(x_1^+)\Big] + \\
&+w_{,xt}(x_2)\Big[(E_bJ_b+E_{pe}J_{pe})w_{,xx}(x_2^-) - E_bJ_bw_{,xx}(x_2^+)\Big]
\end{aligned}
\tag{3.6}
$$

Due to the joint boundary conditions (equivalent to the equality of transverse force at $x = x_1$ and $x = x_2$) the second and the third component in Eq (3.6) are equal to zero. Similarly, the continuity of beam curvature at x_1 and x_2 simplifies the last two components in Eq (3.2).

$$
\frac{dV}{dt} = -\int\limits_0^l S(t)w_{,t}w_{,xx}\,dx - E_{pe}J_{pe}\frac{2\Lambda}{t_b}\Big[w_{,xt}(x_2) - w_{,xt}(x_1)\Big]
\tag{3.7}
$$

where Λ is the strain induced by the piezoelectric actuaturs, which can be calculated from Eqs (2.17) and (2.18).

Assuming a velocity feedback in the form

$$
\Lambda = K_s\int\limits_{x_1}^{x_2} w_{,xxt}\,dx
\tag{3.8}
$$

where K_s is a sensor constant, Newman (1991)

$$
K_s = \frac{d_{31}E(t_s+t_{pe})}{2C}
$$

the time derivative of functional can be written in the form

$$
\frac{dV}{dt} = -\int\limits_0^l S(t)w_{,t}w_{,xx}\,dx - E_{pe}J_{pe}\frac{2}{t_b}\Big[\int\limits_{x_1}^{x_2} w_{,xxt}\,dx\Big]^2
\tag{3.9}
$$

or

$$
\frac{dV}{dt} = -\int\limits_0^l S(t)w_{,t}w_{,xx}\,dx - E_{pe}J_{pe}\frac{2}{t_b}\Big[w_{,xt}(x_2) - w_{,xt}(x_1)\Big]^2
\tag{3.10}
$$

The second quadratic component in Eq (3.10) represents the rate at which the distributed controller extracts energy from the beam.

4. Equations of an axially loaded beam with finite bonding layers and piezoelectric layers – model 2

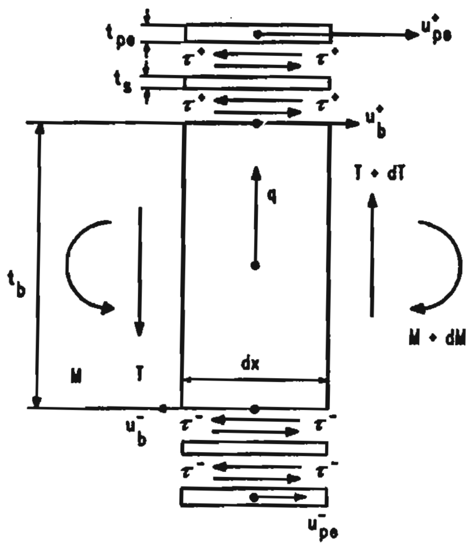

Fig. 3. Geometry of beam element with finite bonding layers and piezoelectric layers

Let us consider two piezoelectric elements bonded by a finite-thickness bonding layer to an elastic beam. In order to derive the dynamic equations the equilibrium of the element shown in Fig.3 is examined. The equations have the form

$$\rho_{pe}Au^+_{pe,tt} = N^+_{pe,x} - \tau^+ b \tag{4.1}$$

$$\rho_{pe}Au^-_{pe,tt} = N^-_{pe,x} - \tau^+ b \tag{4.2}$$

$$\rho_b t_b b w_{,tt} = T_{,x} + \big(S(t)w_{,x}\big)_{,x} \tag{4.3}$$

where $A = t_{pe}b$ is a piezoelectric layer cross-section.

Axial forces in piezoelectrics: N^+_{pe}, N^-_{pe}, interlayer shear stresses: τ^+, τ^-, and beam transverse force T and bending moment M can be found from classical formulae

$$N^+_{pe} = AE_{pe}u^+_{pe,x} \tag{4.4}$$

$$N^-_{pe} = AE_{pe}u^-_{pe,x} \tag{4.5}$$

$$\tau^{+} = \frac{G}{t_s}\Big(u^{+}_{pe} + \frac{t_b}{2}w_{,x}\Big) \tag{4.6}$$

$$\tau^{-} = \frac{G}{t_s}\Big(u^{-}_{pe} - \frac{t_b}{2}w_{,x}\Big) \tag{4.7}$$

$$M = -E_b J_b w_{,xx} \tag{4.8}$$

$$T = -E_b J_b w_{,xxx} + \frac{t_b b(\tau^{+} - \tau^{-})}{2} \tag{4.9}$$

Using Eqs (4.1) ÷ (4.9) the dynamic equations of system motion can be written in the form

— $x \in (x_1, x_2)$

$$\rho_{pe} A u^{+}_{pe,tt} - E_{pe} A u^{+}_{pe,xx} + \frac{Gb}{t_s}\Big(u^{+}_{pe} + \frac{t_b}{2}w_{,x}\Big) = 0 \tag{4.10}$$

$$\rho_{pe} A u^{-}_{pe,tt} - E_{pe} A u^{-}_{pe,xx} + \frac{Gb}{t_s}\Big(u^{-}_{pe} - \frac{t_b}{2}w_{,x}\Big) = 0 \tag{4.11}$$

$$\rho_b t_b b w_{,tt} + E_b J_b w_{,xxxx} - \frac{Gbt_b}{2t_s}\Big(u^{-}_{pe} - u^{-}_{pe} + \frac{t_b}{2}w_{,x}\Big)_{,x} + S(t) w_{,xx} = 0 \tag{4.12}$$

— $x \in (0, x_1) \cup (x_2, l)$

$$\rho_b t_b b w_{,tt} + E_b J_b w_{,xxxx} + S(t) w_{,xx} = 0 \tag{4.13}$$

We assume simply supported boundary conditions imposed on the solution of Eq (4.13) at $x = 0$ and $x = l$, continuity of deflection, slope, curvature and transverse force for $x = x_1$ and $x = x_2$.

Remembering Eq (4.9) the conditions corresponding to the continuity of transverse forces can be written down as

$$E_b J_b w_{,xxx}(x_1^{-}) = E_b J_b w_{,xxx}(x_1^{+}) - \frac{t_b b}{2}\Big(\tau^{+}(x_1^{+}) - \tau^{-}(x_1^{+})\Big) \tag{4.14}$$

$$E_b J_b w_{,xxx}(x_2^{+}) = E_b J_b w_{,xxx}(x_2^{-}) - \frac{t_b b}{2}\Big(\tau^{+}(x_2^{-}) - \tau^{-}(x_2^{-})\Big) \tag{4.15}$$

Solutions of Eqs (4.10) and (4.11) should satisfy free edge conditions, which can be written in the form

$$u^{+}_{pe,x} = \Lambda^{+} \qquad \text{at} \qquad x = x_1\,, \quad x = x_2 \tag{4.16}$$

$$u^{-}_{pe,x} = \Lambda^{-} \qquad \text{at} \qquad x = x_1\,, \quad x = x_2 \tag{4.17}$$

5. Deriving the control strategy for model 2

We choose the Lyapunov functional as a total mechanical energy in the form

$$
\begin{aligned}
V &= \frac{1}{2}\int\limits_0^l \Big(\rho_b t_b b w_{,t}^2 + E_b J_b w_{,xx}^2\Big)\,dx + \\
&+ \frac{1}{2}\int\limits_{x_1}^{x_2} \Big(\rho_{pe} A u_{pe_t}^{+\,2} + E_{pe} A u_{pe,x}^{+\,2}\Big)\,dx + \frac{1}{2}\int\limits_{x_1}^{x_2} \Big(\rho_{pe} A u_{pe_t}^{-\,2} + E_{pe} A u_{pe,x}^{-\,2}\Big)\,dx + \\
&+ \frac{1}{2}\int\limits_{x_1}^{x_2} \frac{Gb}{t_s}\Big(u_{pe}^{+} + \frac{t_b}{2} w_{,x}\Big)^2\,dx + \frac{1}{2}\int\limits_{x_1}^{x_2} \frac{Gb}{t_s}\Big(u_{pe}^{-} - \frac{t_b}{2} w_{,x}\Big)^2\,dx
\end{aligned}
\tag{5.1}
$$

Proceeding similarly as in section 3 after integrating by parts and using boundary conditions the time-derivative of functional is given by

$$
\begin{aligned}
\frac{dV}{dt} &= -\int\limits_0^l S(t) w_{,t} w_{,xx}\,dx + \\
&+ E_{pe} A \Lambda\Big(u_{pe,t}^{+}(x_2) - u_{pe,t}^{+}(x_1) + u_{pe,t}^{-}(x_2) - u_{pe,t}^{-}(x_1)\Big)
\end{aligned}
\tag{5.2}
$$

If the feedback has the form

$$
\Lambda = K_s \int\limits_{x_1}^{x_2} u_{pe,x}^{-}\,dx
\tag{5.3}
$$

the time-derivative of functional is equal to

$$
\begin{aligned}
\frac{dV}{dt} &= -\int\limits_0^l S(t) w_{,t} w_{,xx}\,dx + \\
&+ E_{pe} A K_s \Big(u_{pe,t}^{+}(x_2) - u_{pe,t}^{+}(x_1)\Big)\Big(u_{pe,t}^{-}(x_2) - u_{pe,t}^{-}(x_1)\Big)
\end{aligned}
\tag{5.4}
$$

Therefore the second term does not have a definite sign. If a collocated sensor-actuator is used (Dosch, Inman and Garcia, 1992) then the feedback is given by

$$
\Lambda = -K_s \int\limits_{x_1}^{x_2} u_{pe,x}^{+}\,dx
\tag{5.5}
$$

and the system extracts energy as

$$\frac{dV}{dt} = -\int_0^l S(t)w_{,t}w_{,xx}\,dx - E_{pe}AK_s\Big(u^+_{pe,t}(x_2) - u^+_{pe,t}(x_1)\Big)^2 \tag{5.6}$$

6. Numerical simulation and results

In order to simulate the dynamic behaviour of the beam we start with Eq (2.4), where the shear distribution τ is taken from Crawley and de Luis (1987)

$$\rho t_b b w_{,tt} + 2\beta\rho t_b b w_{,t} + EJw_{,xxxx} - bt_b\tau_{,x} + S(t)w,xx = 0 \tag{6.1}$$

$$\tau = -\Lambda\frac{Gm}{2t_s\Gamma}\frac{\sinh\Gamma\frac{x-m}{a}}{\cosh\Gamma}\Big(H(x-x_1) - H(x-x_2)\Big) \tag{6.2}$$

where

$$\Gamma^2 = \frac{E_b t_b + 6E_{pe}t_{pe}}{E_b t_b}\frac{m^2 G}{t_s t_p e E_{pe}} \tag{6.3}$$

and $H(x)$ is a Heaviside function.

The axial parametric excitation is given as follows

$$S(t) = S_0 + A_0\sin(\omega t)$$

The dimensions and properties of materials are the same as used by Jie Pan, Hansen and Snyder (1991)

$l = 0.038\text{m}$	$b = 0.04\text{m}$	$t_b = 0.002\text{m}$
$x_1 = 0.078\text{m}$	$x_2 = 0.118\text{m}$	$\rho_b = 7800\text{kg/m}^3$
$E_b = 21.6\times10^{10}\text{N/m}^2$	$\rho_{pe} = 7250\text{kg/m}^3$	$E_{pe} = 6.3\times10^{10}\text{N/m}^2$
$d_{31} = 1.9\times10^{-10}\text{m/V}$	$G = 10^9\text{N/m}^2$	$t_s = 0.0001\text{m}$

The first eigenfrequency of beam is equal to $\omega_1 = 207.38$ 1/sec. A passive viscous model of external damping with a constant proportionality coefficient is assumed to describe a dissipation of the beam energy. The reduced damping coefficient β in Eq (5.3) is equal to $\beta = 0.01$. Calculations were performed for the main parametric resonance, where the excitation frequency ω is equal to the half of the first eigen-frequency of the beam.

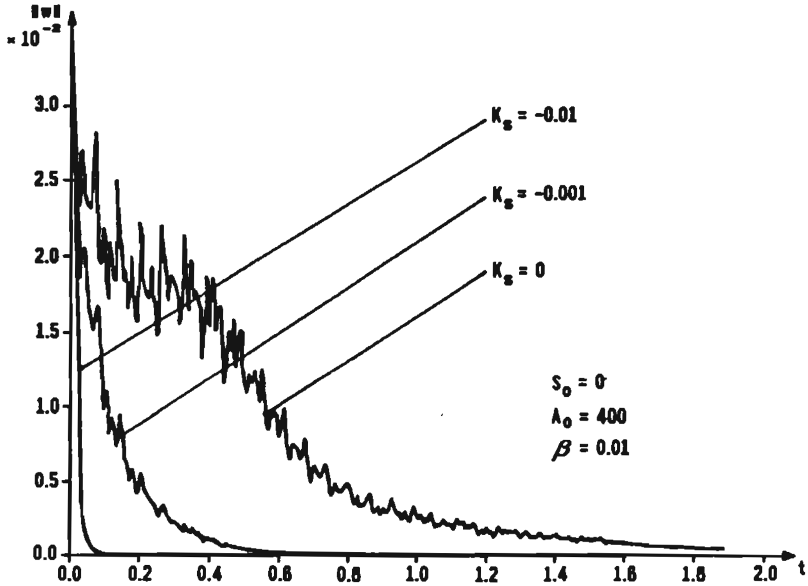

Fig. 4. Influence of the feedback gain on vibrations in the stable region of parametric excitation

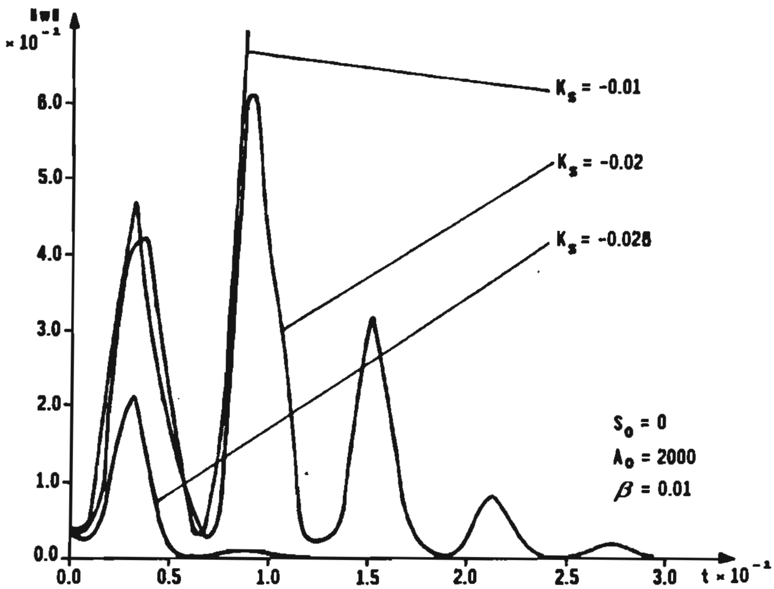

Fig. 5. Influence of the feedback gain on vibrations in the unstable region of parametric excitation

The square root of mechanical energy denoted by $\|w\|$ represents the distance of a disturbed solution from the stright equilibrium $w = 0$

$$\|w\| = \sqrt{\frac{1}{2}\int\limits_0^1 (w_{,t}^2 + w_{,xx}^2)dx}$$

Fig.4 shows an influence of the feedback gain K_s on the vibration behaviour for the amplitude A_0 near to the unstable value. In Fig.5 the similar situation is shown, when the axial force destabilizes the beam motion without the feedback $K_s = 0$. It can be observed that the velocity feedback dramatically stabilizes the beam motion in both cases.

Acknowledgement

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Stabilizacja drgań parametrycznych belki

Streszczenie

W pracy przedstawiono teoretyczną analizę dynamicznej stateczności liniowej sprężystej belki ściskanej siłą harmonicznie zmienną. Posłużono się pojęciem **konstrukcji inteligentnej** w celu wprowadzenia aktywnego tlumienia. W analizie skorzystano z zastosowania rozłożonych czujników, elementów wykonawczych i odpowiedniego sprzężenia zwrotnego do zbadania stateczności układu złożonego z belki opisanej równaniem jednorodnym o pochodnych cząstkowych ze współczynnikami jawnie zależnymi od czasu. W celu oceny odchylenia rozwiązania od prostoliniowego stanu równowagi wprowadzono miarę odległości równą pierwiastkowi z energetycznego funkcjonału. Zastosowano bezpośrednią metodę Lapunowa w celu pokazania, że prędkościowe sprzężenie zwrotne zmniejsza energię mechaniczną belki mogąc w końcowym efekcie zbilansować energię dostarczaną przez wymuszenie parametryczne i energię rozpraszaną przez tlumienie pasywne i aktywne.

Dokonano symulację numeryczną normy energetycznej. Testy numeryczne przeprowadzone dla belki przegubowo podpartej na obu końcach z przyklejonymi po obu stronach płytkami czujnika i elementu wykonawczego pokazują wplyw sprzężenia zwrotnego na zanik drgań.

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