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ACTIVE SYSTEMS IN THE VIBRATION CONTROL OF VEHICLES

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The optimization procedure for the vibrocontrol system of a vehicle is presented. The vehicle is represented by a two degrees of freedom model with discrete parameters. Input random kinematic excitation is assumed to be stationary and ergodic. The elektrohydraulic compensator is working parallely with the conventional suspension system. The structure and parameters of the active part, respectively, were obtained for the optimum criterion taken as a quadratic form. The relationships between values of the system parameters and the coefficients that describe fahr comfort and fahr safety are discussed.

1. Introduction

In the field of vehicle construction the tendencies to increase the fahr comfort and safety can be noticed. The problems of the vibroinsulating properties of the suspension system have been under intensive investigations for many years considering that the vibration of vehicle body is the important parameter of the fahr comfort. For the low frequency vibrations good result can be achieved introducing only an active vibration control system to the suspension system. This kind of the control of the dynamic systems is a complex mechatronic problem. If an active vibration control system is introduced to the vehicle suspension a compromise between fahr comfort and fahr safety is necessary. An acceptable design can be found for the vibroinsulating system in which passive and active elements are combined. This problem for an object with one degree of freedom was presented by Ballo (1989). In consideration of the restrictions imposed on the minimum value of the eigenfrequencies the passive vibroinsulating systems can be effective only for vibration the high frequencies. The vibrations occur in a vehicle due to the kinematic excitations when the vehicle moves along the road surface irregularities. The kinematic excitations have random character and by the stationary velocity is assumed to be the stationary, normal and ergodic process.

Essentially passive vibroinsulating systems of vehicles are considered to be nonlinear. The dynamic behaviour of vehicles was discussed by Mitschke (1984). To investigations of the vehicle in vibrations at low frequencies can be used a simple model of the vehicle in which the vibrations are isolated.

This paper presents the optimization procedure for the active vibro-control systems of vehicles. The vehicle is represented by a two degrees of freedom model in which an active compensator can be applied.

2. Synthesis of vibration control systems

The procedure of synthesis of the optimal vibration insulation systems based on the theory of filters was presented by Niziol, Chodacki and Michałowski (1990).

An optimal damper structure was proposed by Kolosov (1984).

For the complex system with many degrees of freedom the frequency method is more effective for finding the optimum vibrocontrol system.

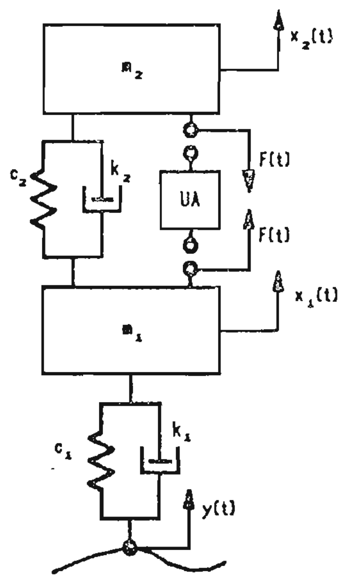

Fig. 1. Mechanical model of the vehicle

Fig.1 shows a one-dimensional model of a vehicle with wheels mass m_1 (degree of freedom $x_1(t)$) and reduced vehicle mass m_2 (degree of freedom $x_2(t)$), respectively. The tyre is represented by a spring (c_1) and a damping elements (k_1), respectively. The passive part of the suspension system consist of a spring

(c_2) and a damper (k_2). The force $F(t)$ is generated in an active compensator (UA). The kinematic excitation is denoted as $y(t)$.

The equations of motion for this system have the following form

$$m_1\frac{d^2x_1}{dt^2} = c_1(y-x_1)+k_1\frac{d}{dt}(y-x_1)+F(t)-c_2(x_1-x_2)-k_2\frac{d}{dt}(x_1-x_2) \tag{2.1}$$
$$m_2\frac{d^2x_2}{dt^2} = c_2(x_1-x_2)+k_2\frac{d}{dt}(x_1-x_2)-F(t)$$

The force $F(t)$ is the product of the active vibration control system counterbalanced by the dynamic forces acting through passive elements on the mass m_2. The value of the force $F(t)$ may depend on the relative displacement of the vehicle body and the wheel.

It can be described as

$$F(t) = W(j\omega)[x_2(t)-x_1(t)] \tag{2.2}$$

where $W(j\omega)$ – unknown transfer function of the active vibration control system.

The quality index of a system (UA) has the form

$$J = \int\limits_0^\infty F^2(t)dt + r_0^2\int\limits_0^\infty\Big[\frac{d^2x_2}{dt^2}\Big]^2dt \tag{2.3}$$

where r_0 – the weight coefficient.

Minimization of Eq (2.3) is used as a criterion of optimization of the active part. The system (UA) must be effective within the range of low frequencies. The kinematically excited model of a vehicle was investigated by Dimentberg (1980). The correlation functions of the road surface irregularities have been chosen in the following form

$$K_y(\tau) = \sigma_y^2 e^{-\alpha|\tau|} \tag{2.4}$$

where

σ_y^2 – variance of the road surface irregularities

α – coefficient of undulation.

If the velocity of a vehicle is constant, then the spectral density function can be calculated from Eq (2.4)

$$G_y(\omega) = G_0^2\frac{1}{\omega^2+\omega_0^2} \tag{2.5}$$

where

$$G_0^2 = 2\sigma_y^2\omega_0 \qquad\qquad \omega_0 = \alpha v$$

v – velocity of a vehicle.

If the following relationship is introduced

$$x_2(t) = H_2(j\omega)y(t) \tag{2.6}$$

and applied to Eq (2.3) (the Wiener-Hopf equation), from Eqs (2.1) and (2.2), taken into account Eq (2.5) is possible to obtain the transfer function $H_2(j\omega)$ for which the quality index reaches the minimum value.

The functions $H_2(j\omega)$ have the form

$$H_2(j\omega) = \frac{\sum\limits_{n=0}^{2} a_n(j\omega)^n}{\sum\limits_{n=0}^{4} b_n(j\omega)^n} \tag{2.7}$$

where the parameters a_n and b_n, depend on the following passive part parameters, frequency w_0 and weight coefficient r_0. Having in mind the transfer function $H_2(j\omega)$, the transfer function $W(j\omega)$ can be calculated

$$W(j\omega) = -m_2\frac{\sum\limits_{n=0}^{6} r_n(j\omega)^n}{\sum\limits_{n=0}^{5} \nu_n(j\omega)^n} \tag{2.8}$$

where r_n and ν_n, are functions of all the parameters.

3. Synthesis of the optimal system results

Investigations of the optimal system consist in obtaining the relationships between dynamic system characteristics and value of the weight coefficient r_0.

We introduce the following notations

$$\frac{c_1}{m_1} = \omega_1^2 \qquad \frac{c_2}{m_2} = \omega_2^2 \qquad \frac{k_1}{m_1} = 2\omega_1 h_1$$

$$\frac{k_2}{m_2} = 2\omega_2 h_2 \qquad \frac{m_2}{m_1} = \mu \qquad \alpha_0 = 4\omega_1\omega_2 h_1 h_2$$

$$\alpha_1 = 2\omega_1 h_1 + 2(1+\mu)\omega_2 h_2 \qquad r_0 = rm_2 \qquad \alpha_2 = \omega_1^2 + (1+\mu)\omega_2^2 + \alpha_0$$

$$\alpha_3 = 2(\omega_1\omega_2^2 h_1 + \omega_2\omega_1^2 h_2) \qquad \alpha_4 = \omega_1^2\omega_2^2$$

Parameters b_n in Eq (2.7) are calculated from the following system of nonlinear algebraic equations

$$2b_2 - b_3^2 = 2\alpha_2 - \alpha_1^2(1-2h_1^2)r^2$$

$$
\begin{aligned}
&b_2^2 + 2b_0 - 2b_1b_3 = \alpha_2^2 + 2\alpha_4 - 2\alpha_1\alpha_3 + \omega_1^4 r^2 \\
&2b_0b_2 - b_1^2 = 2\alpha_2\alpha_4 - \alpha_3^2 \\
&(1+r^2)b_4 = 1 \qquad\qquad b_0 = \alpha_4
\end{aligned}
\tag{3.1}
$$

while a_0, a_1, a_2 parameters can be obtained from the system of linear algebraic equation, described in the matrix form

$$
\mathbf{Aa} = \mathbf{q} \tag{3.2}
$$

where

$$
\mathbf{a} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_6 \end{bmatrix} \qquad\qquad \mathbf{q} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ \vdots \\ \vdots \\ q_6 \end{bmatrix} \tag{3.3}
$$

Matrix $\mathbf{A}$ has the form

$$
\mathbf{A} = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & -b_3 & 0 & 0 & 1 & e_2 \\
1 & -b_3 & b_2 & 0 & 1 & e_2 & e_1 \\
-b_3 & b_2 & -b_1 & 1 & e_2 & e_1 & e_0 \\
b_2 & -b_1 & b_0 & e_2 & e_1 & e_0 & 0 \\
-b_1 & b_0 & 0 & e_1 & e_0 & 0 & 0 \\
b_0 & 0 & 0 & e_0 & 0 & 0 & 0.
\end{bmatrix} \tag{3.4}
$$

where

$$
\begin{aligned}
&q_0 = \alpha_0 && q_1 = \alpha_3 - \alpha_0\alpha_1 \\
&q_2 = \alpha_0\alpha_2 + \alpha_4 - \alpha_1\alpha_3 && q_3 = \alpha_3(\alpha_2 - \alpha_0) - \alpha_1\alpha_4 \\
&q_4 = \alpha(\alpha_0 - \alpha_2) - \alpha_3^2 && q_5 = 0 \\
&q_6 = \alpha_4^2
\end{aligned}
$$

and

$$
e_0 = \omega_1^2\omega_0 \qquad\qquad e_1 = \omega_1^2 + 2\omega_1\omega_0 h_1 \qquad\qquad e_2 = 2\omega_1 h_1 + \omega_0
$$

Numerical calculations were carried out for the following values of parameters

- the passive part of the suspension system: $m_1 = 75$ kg, $m_2 = 375$ kg, $c_1 = 187.5$ kN/m, $c_2 = 54$ kN/m, $k_1 = 0.375$ kN·s/m, $k_2 = 1.8$ kN·s/m,
- the spectral density of excitation: $\sigma_y^2 = 3.33$ cm^2, $\alpha = 0.5$ m^{-1}, $v = 15$ m/s.

First, the values of factors were obtained as functions of r

$$K_c = \frac{\sigma_{x_2}}{\sigma_y} \tag{3.5}$$

and

$$K_w = \frac{\sigma_{x_1}}{\sigma_y} \tag{3.6}$$

where

- $\sigma_{x_1}, \sigma_{x_2}$ – standard deviations of the displacements of the vehicle body and the wheel, respectively
- σ_y – standard deviation of the road surface irregularities.

These relations are shown in Fig.2. The factor K_c is connected with the fahr comfort.

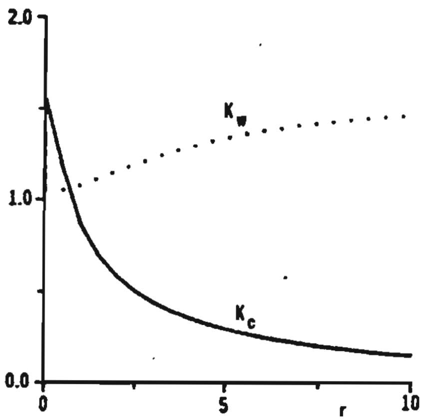

Fig. 2. K_c and K_w diagram versus r

Second, the values of factors were obtained

$$K_s = \frac{\sigma_w}{P} \qquad K_a = \frac{\sigma_F}{P} \tag{3.7}$$

where

- σ_w – standard deviation of the dynamic force in the tyre (for the considered model – in the spring c_1)
- σ_F – standard deviation of the compensation force (by two compensators for each axle)
- P – static force in the tyre.

Fig.3 shows these relationships as functions of r. The factor K_s is connected with the fahr safety.

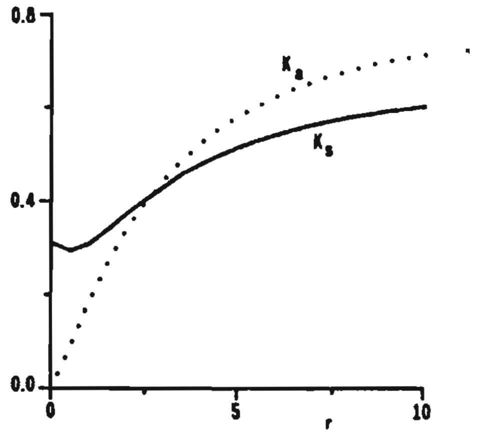

Fig. 3. K_s and K_a diagrams versus r

For $r = 0$ only the passive part of the vibroinsulating system is active. For the values of r changing with in the range $0 \div 5$, the active vibration control system displays high effectiveness. For this range of r the value of the factor K_c decreases. This is a very important property of the active vibration control system with regard to the relationships between driving comfort and the accelerations of the vehicle body.

On the other hand, if the value of r increases, it will bring about the amplification of wheel vibration and the dynamic load upon the tyre. For the optimal system certain characteristics in frequency domain can be obtained. The relative amplitude – frequency characteristic of the vehicle body is determinated by the formula based on Eq (2.7)

$$D_2 = \frac{|H_2(j\omega)|}{|H_{20}(j\omega)|} \tag{3.8}$$

where: $H_2(j\omega)$ and $H_{20}(j\omega)$ – moduli of the transfer functions of mass m_2 in the cases of active and passive systems, respectively.

In Fig.4 the relations D_2 versus ω are shown for two values of r. It can be seen, that the vibration reduction coefficient is extremaly high at the resonant frequency of the spring-mass system c_2, m_2 (about 2Hz).

For the wheel (mass m_1) one can write

$$D_1 = \frac{|H_1(j\omega)|}{|H_{10}(j\omega)|} \tag{3.9}$$

where: $H_1(j\omega)$ and $H_{10}(j\omega)$ – moduli of the transfer functions of the active and passive systems respectively.

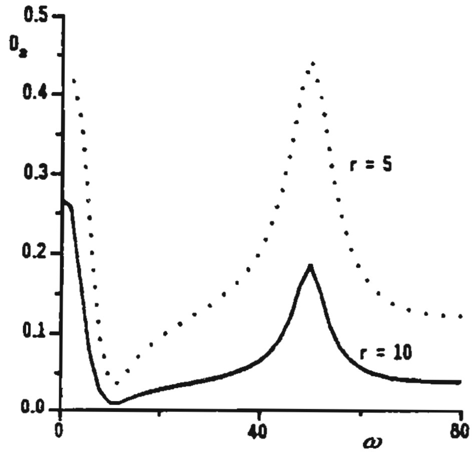

Fig. 4. Relative frequency response for the vehicle body

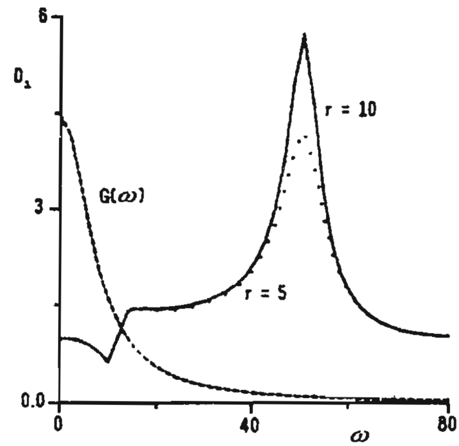

Fig. 5. Relative frequency response for the wheel

Fig.5 shows the relations D_1 versus ω for r as in Fig.4. At the frequencies within the region of the resonant frequency of the spring-mass system c_1, m_1 (about 8 Hz), in the active vibration control system wheel vibration increases in comparison with the passive system. An active vibration absorber of wheels resonant vibration was proposed by Luckel, Castiglioni, Jaker and Rutz (1992). The spectral density function is shown in Fig.5 according to Eq (2.5).

4. Structure of the compensator

The structure of compensator for the optimal vibroinsulating system is described by the transfer function $W(j\omega)$, according to Eq (2.8). Physically it can be realized by the application of a hydraulic actuator with a servovalve controlled by a regulator. Fig.6 shows the structure of a simple compensator, which consists of: 1 – double acting hydraulic actuator, 2 – servovalve, 3 – spring, 4 – damper, 5 – regulator, 6 – hydraulic supply system.

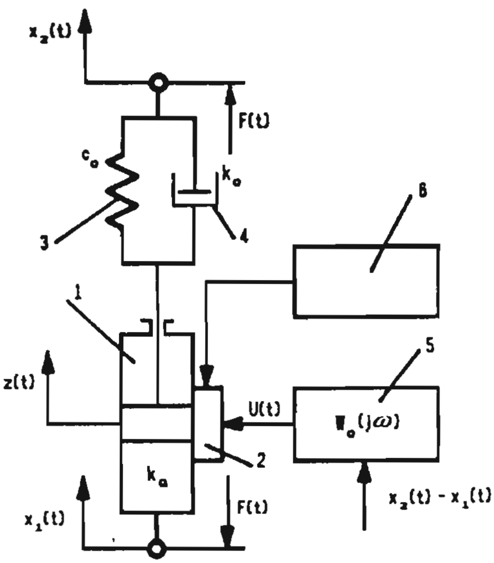

Fig. 6. Structure of the compensator

The voltage $U(t)$ is produced in the regulator as a function of the relative displacement of the vehicle body and the wheel. In the case, when a proportional servovalve is used, the equation of motion in the region of low frequencies has the following form

$$\frac{dz}{dt} = \frac{dx_1}{dt} - k_Q U(t) \tag{4.1}$$

where

$z(t)$ – displacement of the piston

k_Q – inflow coefficient (of oil into the actuator).

The compensation force can be obtained as

$$F(t) = k_0 \frac{d}{dt}[x_2(t) - z(t)] + c_0[x_2(t) - z(t)] \tag{4.2}$$

From Eq (2.2) and Eq (4.2) we have

$$U(t) = W_0(j\omega)[x_2(t) - x_1(t)] \tag{4.3}$$

where $W_0(j\omega)$ – transfer function of the regulator.

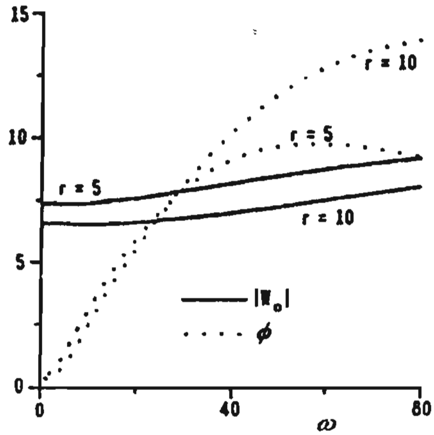

Fig. 7. Modulus and phase of the transfer function $W_0(j\omega)$

The transfer function $W_0(j\omega)$ of the optimal vibroinsulating system has a complex form. For example, Fig.7 shows the amplitude-frequency and the phase-frequency characteristics for two values of r in the case when the input signal to the regulator is the relative velocity of the vehicle body and the wheel. Within the region of low frequencies the following approximate relations can be obtained

$$|W_0(j\omega)| = \text{constant} \qquad \text{and} \qquad \phi = k\omega$$

where: k = constant.

In this way it is possible to simplify the structure of the regulator.

5. Conclusions

1. The application of the active vibration control system to vehicles improves the vibroinsulating properties of the suspension system. Driving comfort in a vehicle utilizing the active suspension system can be increased to maximum.
2. The results of the analysis have shown that the introduction of active elements into suspension systems of vehicles can cause problems, regarding keeping the dynamic load of the tyre at a same level as for a conventional system.

3. When taking into account the relations between the factors connected with fahr comfort and fahr safety and the weight coefficient, it is necessary to limit the value of the weight coefficient.

4. The presented method of finding the active vibration control system can be adopted to suspension systems of vehicles in which the passive elements have the physically nonlinear characteristics.

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Aktywne układy wibroizolacji pojazdów

Streszczenie

Praca dotyczy syntezy aktywnego układu wibroizolacji pojazdu reprezentowanego przez dyskretny układ o dwóch stopniach swobody, na który działa wymuszenie kinematyczne będące stacjonarnym i ergodycznym procesem losowym. Układ wykonawczy aktywnego wibroizolatora pracuje równolegle z pasywnym układem zawieszenia pojazdu. Określona została struktura i podane charakterystyki układu optymalnego dla funkcjonału jakości będącego formą kwadratową wartości przyspieszenia nadwozia i wartości siły kompensacyjnej. Zbadany został wpływ wybranych parametrów na wskaźniki określające komfort i bezpieczeństwo jazdy.

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