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# METHOD OF SUPPORTING FUNCTIONALS IN THE ESTIMATION OF ALARM CONDITIONS IN SYSTEMS OF VIBRATIONAL MONITORING

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The paper presents methodological assumptions for the search for time prognoses of possible perturbations in the operation of a monitored bearing node. A way of determining the departure time of the monitored diagnostic signals from permissible areas is proposed. Its realization is determined by a solution originated from the method of "functionals supporting the availability area". An idea of proposed estimation construction is presented on the example of a two-dimensional isothermal model of a hydrodynamic bearing.

## 1. Introduction

Early detection of perturbations in machine is supported by the systems that monitor its condition. With regard to high-speed machines with slide bearings these are most often systems that monitor vibrations of the shaft neck in the bearing bushing. Machine condition is estimated by the continuous measurement of shaft neck vibration in two mutually perpendicular directions, related to assumed criteria values. Monitoring of vibrations realized in such a way, i.e. determination of the trajectory of shaft neck motion and its position relative to areas of stable equilibrium is a useful method of recognizing symptoms of destruction phenomena that occur at such nodes. Since the trajectory changes are related to exploitational disturbances (changes in lubricant viscosity, external loading, rotational speed) or changes in bearing parameters (geometrical dimensions, clearance, etc.). Those changes can bring about the disturbance of the equilibrium of external forces and the hydraulic force, which in consequence, can lead to the self-excited vibration of high amplitude causing the carrying film of oil to be broken and the bearing to be destroyed.

Two of the most essential problems of the design of such systems when monitoring the condition of bearing nodes are the following:

- Problem of filtration of the disturbances occurring change trends recognition
- The task of selecting algorithms for values of the monitored vibration symptoms which permit the time of their departure from the admissible areas to be estimated; the estimation determines the prognosis of the time up to the occurrence of the failure conditions.

Introducing those solutions into monitoring systems results in the increase of effectiveness of their functioning.

The direction of searching for proper algorithms was determined so far by the solutions to filtration (cf Cempel (1989)) and prediction problems (cf Batko (1984); Batko and Kaźmierczak (1985)). Their major drawback is a relatively low generality despite the fact that the results are interesting from the practical applications point of view. The lack of generally applicable results from their incomplete verifiability, due to the assumptions made.

The present work will present a new approach to the estimation problem of the time till to the departure of the monitored signals from the admissible areas, similarly as with regard to filtration problems (cf Batko and Banek (1993a,b)).

## 2. Problem formulation

The question of forecasting the machine failure conditions is consists in the estimation of the time up to departure from the admissible areas controlled by the system that monitors diagnostic signals. In the solutions suggested so far, that problem is presented as a question of estimation and extrapolation of the change trends in the values of the monitored trajectories basing on various calculation procedures (cf Batko (1984); Batko and Kaźmierczak (1985)). The paper presents a theoretical analysis of the possibility to formulate this problem on the ground of control theory, which results in a greater universality. The approach allows a more complete analysis of the phenomena monitored.

As the basis of our considerations we shall adopt the observation equation of perturbed movements of a shaft neck in the bearing bushing of the slide bearing monitored. The equation corresponds to the model and to the simplifying assumptions made by Kurnik and Starczewski (1985). Concerning the conditions of determining the operation area of the oil film and hydrodynamic forces.

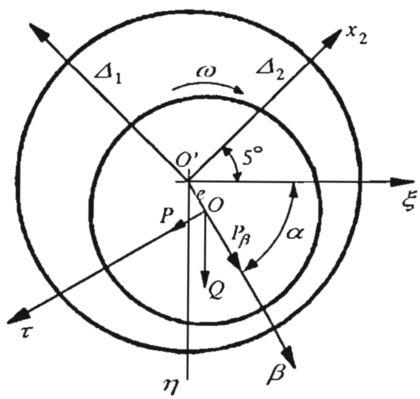


Fig. 1. Considered model of the bearing node system

In the formulation given by Kurnik and Starczewski (1985), the forces that act on the shaft neck have a distribution as in Fig.1, while the equation of neck motion in the bearing bush in the $\xi,\eta$ system of coordinates is

$$\begin{aligned} m\ddot{\xi} &= P_\beta \cos\alpha - P_\tau \sin\alpha \\ m\ddot{\eta} &= P_\beta \sin\alpha + P_\tau \cos\alpha + Q \end{aligned} \tag{2.1}$$

where

$\ddot{\xi},\ddot{\eta}$ – acceleration of the neck centre in the directions $\xi$ and $\eta$ respectively

$P_\beta, P_\tau$ – components of the buoyancy (uplift) forces of the oil wedge

$Q$ – external loading

$m$ – mass of the shaft neck

$\alpha$ – angle between the $OO'$ straight line that connects the centres of neck and bush, respectively, and the positive direction of $\xi$ axis.

The lubricant buoyancy (uplift) forces

$$P_\beta = P_\beta(\omega, \beta, \dot{\beta}, \dot{\alpha}) \qquad\qquad P_\tau = P_\tau(\omega, \beta, \dot{\beta}, \dot{\alpha})$$

are nonlinear functions of the relative eccentricity $\beta = e/\varepsilon$ (i.e. the ratio of neck eccentricity $e = OO'$ and absolute clearance $\varepsilon = R_0 - R$), as well as of the neck rotational speed $\omega$, the radial velocity $\dot{\beta}$ and circumferential velocity $\dot{\alpha}$.

In the model formulation (2.1), the neck motion relative the coordinate system $\beta, \alpha$ (which is more convenient for analysis) is given by

$$\begin{aligned} \ddot{\beta} &= f_\beta(\alpha, \beta, \dot{\alpha}, \dot{\beta}) \\ \ddot{\alpha} &= f_\alpha(\beta, \dot{\beta}, \alpha, \dot{\alpha}) \end{aligned} \tag{2.2}$$

For stationary operation conditions, of load $Q$ and the rotational speed $\omega$, the neck reaches the position $\beta_0, \alpha_0$ in the circle $\beta \leq 1$. That position is determined by the conditions $\alpha = \beta = \dot{\alpha} = \dot{\beta} = 0$, that determine the geometric locus of the equilibrium positions of the neck centre for various pairs of $[\omega, Q]$ that depend on the design parameters of the bearing node. Monitoring of the machine condition is reduced to an analysis of the neck motion around the equilibrium position. It can be described by the variables

$$\beta - \beta_0 \qquad\qquad \alpha - \alpha_0 \tag{2.3}$$

The linearized equation of the neck motion (that bears out the simplifying assumptions adopted) around the equilibrium position $(\beta_0, \alpha_0, 0, 0)$ in matrix formulation takes the form

$$\dot{\boldsymbol{X}} = \mathbf{A}\boldsymbol{X} \qquad\qquad \boldsymbol{X}(0) = \boldsymbol{x} \tag{2.4}$$

The matrix $\mathbf{A}$ appearing in Eqs (2.4) is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ k_{11} & c_{11} & k_{12} & c_{12} \\ 0 & 0 & 0 & 1 \\ k_{21} & c_{21} & k_{22} & c_{22} \end{bmatrix} \tag{2.5}$$

where: $k_{ij}$, $c_{ij}$ are coefficients of elasticity and attenuation (damping), respectively, and the vector $\boldsymbol{X}$ is

$$\boldsymbol{X} \equiv [x_1, x_2, x_3, x_4]^\top = [\beta - \beta_0, \dot{\beta}, \alpha - \alpha_0, \dot{\alpha}]^\top$$

The values of coefficients of elasticity $k_{ij}$ and of damping $c_{ij}$, respectively, for arbitrary bearing design can be determined by means of the perturbation method or its generalization (cf Kiciński (1993)). The analysis of changes in the trends monitored, can be related to the equation being perturbed by a "small noise"

$$\dot{\boldsymbol{X}} = \mathsf{A}\boldsymbol{X} + \varepsilon \mathsf{B}\dot{\boldsymbol{w}} \qquad\qquad \boldsymbol{X}(0) = \boldsymbol{x} \tag{2.6}$$

where $\boldsymbol{w} = [w_1, w_2]^\top$ is the Wiener process, and the matrix $\mathsf{B}$ has the form

$$\mathsf{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.7}$$

The perturbing term $\varepsilon \mathsf{B}\dot{\boldsymbol{w}}$ appearing in Eq (2.6) can be related to small external perturbations that act upon the system in the process of its exploitation, or to the interactions due to nonlinearities which were neglected when the linearized description (2.4) was introduced. Their presence can be reduced to changes in the system parameters, and, in consequence, to the departure of trajectory from the stable positions. As a result, the self-excited vibrations can be generated, with high amplitudes that cause the carrying oil film to be broken and the bearing to be destroyed.

Let the solution of Eqs (2.4) and (2.6) be denoted by $X^x, X^{x,\varepsilon}$. Let $D \subset \mathcal{R}^4$ be an open set including zero. The set $D$ consists of the stable neck positions that correspond to the correct bearing operation. The time of departure from the area of admissible changes $x \in D$ (being of interest in the monitoring process) is defined by the relationship

$$\tau^{x,\varepsilon} \equiv \inf\{t \geq 0 : \quad X^{x,\varepsilon}(t) \in \partial D\} \qquad\qquad x \in D$$

Proper interpretation of the time $\tau^{x,\varepsilon}$ depends not only on the set $D$ form, which determines permissible shaft neck vibrations, but also on the location of the point at which the trajectory $X^{x,\varepsilon}$ reaches the boundary $\partial D$ of the set $D$.

If the set $D$ is of the form

$$D \equiv \{x \in \mathcal{R}^4 : \quad |x_k| < r_k, \;\; r_k > 0, \;\; k = 1,2,3,4\} \tag{2.8}$$

then, for the corresponding $r_k$, and $t > \tau^{x,\varepsilon}$ the linearized equation of motion (2.4) may no more be a sufficiently correct description of the shaft neck behaviour in the bearing bushing.

If, on the other hand, we have

$$|X_I^{x,\varepsilon}(\tau^{x,\varepsilon})| \geq r_I$$

where $r_I$ is the criterion value in the monitoring system, that determines the permissible amplitude of the shaft neck vibration, then $\tau^{x,\varepsilon}$ is the time of signalling by the monitoring system the possibility of a failure, or conditioning machine shut-down.

The aim of the present work is an analysis of the possibility to estimate the time $\tau^{x,\varepsilon}$ of the departure from the areas of neck stable positions, as a result of perturbations that occur in such systems (see Eq (2.6)).

## 3. Application of the results to estimation of departure time of the monitored trends from admissible areas

The task of estimating the instant when the monitored trajectory of the shaft rotor neck exceeds the permissible boundaries for the first time, can be related to the problem put forward originally by A.N.Kolmogorov pertaining the departure time of the trajectory of a given dynamic system from given areas. There exists a comprehensive literature on theoretical mathematical considerations on the subject. However, only few publications and results are suitable for the applications given in the present work due to the assumption of "uniform ellipticity" that nearly always appears in mathematical works concerning diffusion processes. Because of the properties of the matrix $\mathbf{B}$ that appears in the problem analyzed, this assumption is not valid.

The solutions given by Zabczyk (1985a,b) do not have the limitation mentioned above. Thus, they constitute a good basis for searching for solutions to the problem under investigation.

A synthesis of these results constitutes the Theorems 1 and 2 presented below. They pertain to the following problem.

Let $y^{x,u}$ stand for solution (for $u \in L^2([0,\infty),\mathcal{R})$) to the differential equation

$$\dot{y} = \mathbf{A}y + \mathbf{B}u \qquad y(0) = x \tag{3.1}$$

defined for $t \geq 0$. For arbitrary $x \in \mathcal{R}^4$ and for the number $\eta > 0$ we define the following subset $\gamma^x(\eta) \subset \mathcal{R}^4$

$$\gamma^x(\eta) \equiv \mathrm{cl}\Big\{y \in \mathcal{R}^4 : \quad y = y^{x,u}(t),$$

$$\text{for a certain } t \geq 0 \text{ and control } u \text{ such that } \frac{1}{2}\int_0^t \|u(s)\|^2\, ds \leq \eta\Big\}$$

where the abbreviation "cl" denotes the closure of a set, that is $\mathrm{cl}A = \overline{A}$. $\gamma^x(\eta)$ denotes the closure of the set of end points of the trajectories $y^{x,u}(t)$, when the control energy

$$\frac{1}{2}\int_0^t \|u(s)\|^2\, ds$$

does not exceed $\eta$. Let $K(x,r)$ denote a sphere with the centre at $x$ and the radius $r$; $\rho(x,A)$ denotes the Euclidean distance of the point $x$ and the set $A$; the symbol $E$ denotes the mathematical expectation with respect to the probability $P$ from the space $(\Omega, F, P)$, where the Wiener process is defined. Let (Fig.2.)

$$D_0 \equiv \Big\{x \in D : \quad y^{x,0}(t) \in D \ \ \forall t \geq 0\Big\}$$

where $D_0$ denotes the set of initial conditions for Eq (3.1), such that all free trajectories starting from $D_0$ stay in $D$ for all $t > 0$.

$$\overline{\eta} \equiv \sup\Big\{\eta : \quad \gamma^0(\eta) \subset \overline{D}\Big\}$$

where $\overline{\eta}$ denotes the maximum of control energy, such that all trajectories of (3.1) starting from $x = 0$ stay in $\overline{D}$ all the time

$$F \equiv \overline{D} \setminus K(0, r_0)$$
$$G \equiv \partial F \cap \gamma^0(\overline{\eta})$$

where the symbols $\cap$, $\setminus$ denote the intersection and the difference of sets, respectively.

With this notation we have the following theorems, that result directly from Zabczyk (1985a,b).

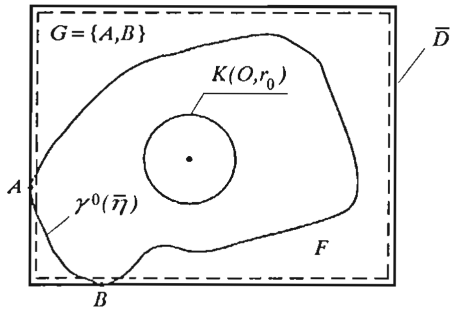


Fig. 2. Geometrical interpretation of the analyzed dynamical neck-bushing system conditions

- **Theorem 1**

If matrix **A** is stable, then for all $x \in D$

$$\overline{\lim_{\varepsilon \downarrow 0}} \varepsilon^2 \ln E(\tau^{x,\varepsilon}) \leq \overline{\eta} \quad \square$$

Theorem 1 says that the expected value of the exit time, $E(\tau^{x,\varepsilon})$, does not exceed $\exp(\overline{\eta}/\varepsilon^2)$, where the constant $\overline{\eta}$ depends only on the deterministic system (3.1). Hence, the Theorem 1 expresses some probabilistic properties of Eq (2.6) in terms of the classical control theory.

- **Theorem 2**

If matrix **A** is stable, then, for any $x \in D_0$ and $\delta > 0$ we have

$$\lim_{\varepsilon \downarrow 0} P\Big\{\rho(X^{x,\varepsilon}(\tau^{x,\varepsilon}, G\Big) > \delta\Big\} = 0 \quad \square$$

Theorem 2 says that, with probability close to one, trajectories of Eq (2.6) may leave $D$ only through the neighbourhoods of the points $A$ or $B$ (see figures).

As can be seen from the above theorems, the set $\gamma^0(\overline{\eta})$ and its common points with $\partial D$ are of basic importance. Below we assume that the set $D$ has the form of Eq (2.7). Let $\mathbf{\Phi} = (\phi_{ij})$, $i, j = 1, ..., 4$, be the fundamental matrix of Eq (2.6), and let $\boldsymbol{b} \in \mathcal{R}^4$ be an arbitrary vector.

Let us denote

$$\lambda_t(\boldsymbol{b}, \eta) \equiv \sup \left\{ \Big\langle \boldsymbol{b}, \int\limits_0^t \mathbf{\Phi}(t-s)\mathsf{B}u(s)\, ds \Big\rangle : \quad \frac{1}{2} \int\limits_0^t \|u(s)\|^2 \, ds \leq \eta \right\} \tag{3.2}$$

and

$$\lambda_t(b,\eta) \equiv \sup\left\{\lambda_t(b,\eta): \quad t \geq 0\right\}$$

$$R_t(\eta) \equiv \left\{\int_0^t \Phi(t-s)\mathsf{B}u(s)\, ds : \quad \frac{1}{2}\int_0^t \|u(s)\|^2\, ds \leq \eta\right\}$$

The set $R_t(\eta)$ is called (in the control theory) the area of availability (cf Górecki and Turowicz (1970)). It can be proved that $R_t(\eta)$ is a closed and bounded domain, symmetrical with respect to the point $x = 0$, continuous for the argument $t \geq 0$; in addition $\mathbf{A}$ is a convex body (cf Górecki and Turowicz (1970)). It can also be proved that the boundary $\partial R_t(\eta)$ of the set $R_t(\eta)$ has no corner points, i.e. that at any point of the boundary $\partial R_t(\eta)$ there are unique external normal and tangent (supporting) hyperplanes. It can easily be seen that

$$\lambda_t(b,\eta) = \langle b, d\rangle$$

where $d$ is a certain point of $\partial R_t(\eta)$, and that the equation

$$\langle x, b\rangle = \lambda_t(b,\eta)$$

is represents the hyperplane supporting the set $R_t(\eta)$ at the point $d$. It can also be easily seen that

$$\gamma^0(\eta) = \mathrm{cl}\left\{\bigcup_{t\geq 0} R_t(\eta)\right\}$$

where $\bigcup$ is the union of sets, and that the equation

$$\langle x, b\rangle = \lambda(b,\eta) = \langle b, \Delta\rangle$$

where $\Delta$ is the point of the boundary $\partial\gamma^0(\eta)$ of the set $\gamma^0(\eta)$, represents the hyperplane that supports the set $\gamma^0(\eta)$ at $\Delta$.

We shall now determine the functions $\lambda_t(b,\eta)$ and $\lambda(b,\eta)$. Let

$$\Psi_b(t) \equiv I_0\Phi(t)b$$

where $I_0\mathrm{diag}(0,1,0,1)$.

- **Lemma 1**

$$\lambda_t(b,\eta) = \sqrt{2\eta\int_0^t \|\Psi_b(s)\|^2\, ds}$$

**Proof**

From Eq (3.2) it follows that

$$\lambda_t(\boldsymbol{b},\eta) = \sup\left\{ \int_0^t \langle I_0\Phi(t-s)\boldsymbol{b}, \widetilde{\boldsymbol{u}}(s)\rangle\, ds : \right. \qquad (3.3)$$

$$\left. \frac{1}{2}\int_0^t \|\widetilde{u}(s)\|^2\, ds \le \eta,\quad \widetilde{\boldsymbol{u}} = [0, u_1, 0, u_2]^{\top} \right\}$$

The term under the symbol "supremum" is the scalar product in the space $L^2([0,t];\mathcal{R}^4)$ of the vectors $\boldsymbol{\Psi}_b(t-\cdot)$ and $\widetilde{\boldsymbol{u}}(\cdot)$. Thus, for a fixed $t \ge 0$, the supremum is reached on collinear vectors. Hence, the best $\widetilde{\boldsymbol{u}} = \widetilde{\boldsymbol{u}}_0$ is given by the formula

$$\widetilde{\boldsymbol{u}}_0(s) = \frac{1}{\chi_t}\boldsymbol{\Psi}_b(t-s) \qquad s \in [0,t] \qquad (3.4)$$

where

$$\chi_t \equiv \sqrt{\frac{1}{2\eta}\int_0^t \|\boldsymbol{\Psi}_b(t-s)\|^2\, ds}$$

Substitution for $\widetilde{\boldsymbol{u}} = \widetilde{\boldsymbol{u}}_0$ into Eq (3.3) and change of the integration variable $s \to t-s$ yields the proof of the lemma.

**Remark 1**

It can easily be verified that, for any $t \ge 0$

$$\frac{1}{2}\int_0^t \|\widetilde{u}_0(s)\|^2\, ds = \eta$$

- **Lemma 2**

$$\lambda(\boldsymbol{b},\eta) = \lim_{t\uparrow\infty} \lambda_t(\boldsymbol{b},\eta)$$

From Lemma 1 it follows that the function $t \mapsto \lambda_t(\boldsymbol{b},\eta)$ is smooth. From the general results of the theory of linear differential equations with constant coefficients it follows that the elements $\phi_{ij}$ of the fundamental matrix $\boldsymbol{\Phi}$ are of the form $p_k(t)\exp(\sigma_k t)$, where $\sigma_k$ are the roots of characteristic equation, and $p_k$ are polynomials of the order depending on the multiplicity of the corresponding root. For a stable matrix $\mathbf{A}$, all $\sigma_k$ have negative real parts, thus $\phi_{ij} \in L^2([0,\infty),\mathcal{R})$ and the following estimates hold true

$$|\lambda_t(b,\eta)| \leq \sqrt{2\eta\Big[b\cdot\Big(\int_0^t \|\Phi^*(t-s)\Phi(t-s)\|ds\Big)b\Big]} \leq M\sqrt{2\eta}\|b\|$$

for a certain $M > 0$ (depending only on the matrix $\mathbf{A}$, and independent of $t$), and for all $t \geq 0$.

Let us employ the matrix norm defined as the maximum of eigenvalues of the fundamental matrix. It is of the form

$$p(t)\mathrm{e}^{-\sigma t}$$

where $p(\cdot)$ is a polynomial of degree at most $n$, and $\sigma > 0$. Since

$$\int_0^\infty t^k \mathrm{e}^{-\sigma t}\, dt = \frac{k!}{\sigma^{k+1}}$$

then the constant $M$ does not depend on $t$; it does depend on $p(\cdot)$, and consequently, on $\mathbf{A}$. As the function $t \mapsto \lambda_t(b,\eta)$ is an integral of a nonnegative function, thus it is a non-decreasing and bounded function. This implies the existence of a limit $\lambda(b,\eta)$, towards which the function tends asymptotically.

Let $e_k \in \mathcal{R}^4$, $k = 1,2,3,4$, such that $< e_i, e_j >= \delta_{ij}$, where $\delta_{ij}$ is the Kronecker delta.

Because of the special form of set $D$, we take the vector $b$ of the form $b = \{e_k\}$; $k = 1,2,3,4$.

The equality

$$\lambda(e_k,\eta) = \sqrt{2\eta\int_0^\infty \|I_0\mathbf{\Phi}(s)e_k\|ds}$$

results immediately from the Lemmas 1 and 2. As the expression under the symbol of integration does not depend on $\eta$, we conclude that

$$\overline{\eta} \equiv \sup\Big\{\eta;\ \gamma^0(\eta) \subset \overline{D}\Big\}$$

is equal to the number $\eta$ for which

$$\lambda^2(e_k,\eta) = r_k^2$$

for the first time for some $k \in \{1,2,3,4\}$.

Thus

$$\overline{\eta} = \min \left\{ \eta_k; \ \eta_k = \frac{r_k^2}{2 \int\limits_0^\infty \|I_0 \Phi(s) e_k\|^2 ds}, \ k = 1, ..., 4 \right\}$$

At the same time, the number $k$, for which $\eta_k = \overline{\eta}$, determines the coordinate of the border of the rectangle $D$ that touches the set $\gamma^0(\overline{\eta})$. We can summarize the above conditions in the following form:

- **Theorem 3**

If the matrix $\mathbf{A}$ is stable and the set $D$ has in the form of Eq (2.7), then for the time $\tau^{x,\varepsilon}$ of the system given by Eq (2.6), we have

$$\overline{\lim_{\varepsilon \downarrow 0}} \varepsilon^2 \ln E(\tau^{x,\varepsilon}) \leq \min_{k=1,...,4} \left\{ \frac{\frac{1}{2} r_k^2}{\int\limits_0^\infty \|I_0 \Phi(s) e_k\|^2 ds} \right\}$$

and for any $x \in D$ and $\delta > 0$

$$\lim_{\varepsilon \downarrow 0} P\Big\{ \rho\Big(X^{x,\varepsilon}(\tau^{x,\varepsilon}, G\Big) > \delta \Big\} = 0$$

where $G$ is the subset consisting of these points of the boundary $\partial D$ of the set $D$, for which $\overline{\eta} = r_k$. □

Theorem 3 says that $E(\tau^{x,\varepsilon})$ does not exceed $\exp(\overline{\eta}/\varepsilon^2)$, where $\overline{\eta}$ is defined above. The second part of theorem says that a trajectory of Eq (2.6) leaves $D$ through a point at which $k$-coordinate

$$r_k = \lambda(e_k, \overline{\eta})$$

Hence, it specifies the critical coordinates.

The above results determine, for the monitoring system, the estimation conditions of the time at which the alarm signals can be generated by the monitoring system. This requires only the analyzed state vector to be transformed to a system of coordinates that are determined by the location of the axis of measuring sensors of a given monitoring system. They can enforce the necessity for a closer analysis of the behaviour of the bearing node in the exploitation process, or shutting the monitored machine down. This will result from the problem algorithmization elaborated on the basis of its formalization corresponding to the constraints determining the form of the set $D$.

## 4. Conclusions

The methodological suggestion presented in the paper, and the results obtained are not of a final nature. Basing on a simplified two-dimensional isothermal model of a hydrodynamic bearing they present new possibilities of realization of monitoring tasks within the scope of the search for time prognoses of possible perturbations in the operation of the monitored bearing node. A new research field related to possible estimation of failure conditions formulated in such a way is presented.

An essential advantage of the suggestion formulated is its universality. It results from the possibility of considering the dynamic properties of arbitrary bearing designs when simulating the estimated phenomena.

The solution presented in the paper (after its proper algorithmization, suitable for the adopted model of the system "neck-bushing-external support") can be used in looking for prognoses of possible perturbations in the correct operation of the bearing node of a machine.

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**Metoda funkcjonałów podpierających w procesie estymacji stanów alarmowych w systemach drganiowego monitoringu**

Streszczenie

W artykule przedstawiono założenia metodologiczne dla poszukiwań prognoz czasowych możliwych zaburzeń w pracy monitorowanego węzła łożyskowego. Zaproponowano pewien sposób wyznaczenia czasu wyjścia kontrolowanych przez system monitorujący sygnałów diagnostycznych z obszarów dopuszczalnych. Jego realizację określa rozwiązanie wywodzące się z metody "funkcjonałów podpierających obszar osiągalności". Ideę proponowanej konstrukcji estymacyjnej przedstawiono na przykładzie dwuwymiarowego izotermicznego modelu łożyska hydrodynamicznego.

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