

REMARKS ON STABILITY OF DISCRETE-CONTINUOUS STRUCTURE UNDER CIRCULATORY LOAD

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The paper deals with a problem of stability of a discrete-continuous structure under circulatory load. An example of the elastic column with localized loss of stiffness modeled as a flexible diminishing beam element is discussed in detail. Stabilization or destabilization of the system is observed, depending on localization and flexibility of a diminishing element. Continuous and discontinuous variations of the critical force are observed and thoroughly explained as a consequence of the mutual interaction of two kinds of characteristic curves, which were distinguished on the force-frequency plane. An analytical solution to the problem is found by the use of the transfer matrix method.

1. Introduction

The design of structures subjected to nonconservative loads very often occurs as a very complex question from the point of view of stability and optimization. Particularly a study of behaviour of systems under follower forces is important in engineering practice. A comprehensive review of the literature on this problem is given by Bogacz and Janiszewski (1985). The presented paper considers in general the problem of stability of a structure with localized loss of stiffness. The Beck column, being a linear elastic column subjected to the compressive tangent load of constant magnitude acting at its free end, is discussed in detail.

Investigations concerning the stability of the Beck column with localized loss of stiffness, were made by Anifantis and Dimarogonas (1983). The elastic

(cf Bogacz and Mahrenholtz (1980), (1986); Bogacz and Imielowski (1986); Bogacz and Niespodziana (1987); Bogacz et al. (1991); Imielowski et al. (1992)) and viscoelastic (cf Bogacz and Imielowski (1986); Bogacz and Niespodziana (1987)) models of localized weakness of the column were then developed by Bogacz, Mahrenholtz et al. Stabilization or destabilization of such a system is observed depending on the value of its parameters. A considerable decrease in the critical force value is observed when the localized loss of shear stiffness is placed near the fixed end of the column, while above fourfold increase of the critical force is possible when the localized loss of bending stiffness is situated in the centre of the structure. A very disadvantageous position of the weakness occurs above the centre of the column, for which the discontinuous changes of the critical force with a destabilization of the structure can follow a variation of the design parameter.

An optimization and application of the active control is possible when the phenomenon of stability of the considered structure is precisely determined. Particularly the mutual interaction of two kinds of characteristic curves which were distinguished by Bogacz et al. (1991) are substantial in understanding of this phenomenon. Optimization research began with a model of the stepped column, then Bogacz and Mahrenholtz (1980) took the column with a single segment of vanishing dimensions and stiffness, located at the centre of the structure as an initial shape for the maximum load optimization. One of the best results, compared with findings of Tada et al. (1989), was obtained using the gradient projection method in a multi-modal analysis (cf Mahrenholtz and Imielowski (1991)). The critical force is very sensitive to even a small variation in the acting force direction (cf Imielowski (1993)).

In the present paper a phenomenon of the continuous and discontinuous changes of critical force is thoroughly explained as a consequence of quantitative or qualitative variation of the characteristic curves on the force-frequency plane. Some generalizations of the results obtained previously by Bogacz et al. (1991), Imielowski et al. (1992) were made and special attention will be paid to two kinds of characteristic curves. New findings are discussed.

2. Formulation of the problem

The structure under consideration, i.e. the linear elastic column with the localized weakness placed in the cross-section $x_1 = l_1/L$, is shown in Fig.1. In the following, the column will be treated as a system consisting of two beam segments connected by a flexible element as an idealization of a crack or any

flexible element of stiffness and dimensions considerable smaller than these of the structure.

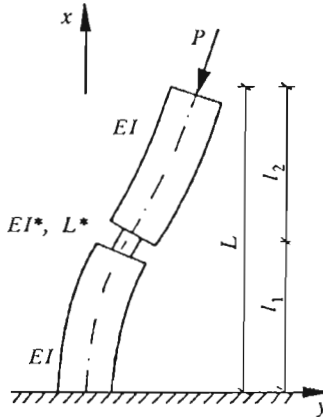


Fig. 1. Beck column with localized weakness

The equation of motion for the beam segment i , is given by the following expression

$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 y_i}{\partial x^2} \right) + P \frac{\partial^2 y_i}{\partial x^2} + \rho A \frac{\partial^2 y_i}{\partial t^2} = 0 \quad (2.1)$$

where

- EI - flexural rigidity
- P - axial tangential load
- ρA - mass per unit length.

The exact solution for the segment of constant mass and stiffness distribution is assumed in the form

$$y_i(x, t) = w_i(x) \exp(i\omega t) \quad (2.2)$$

where

$$w_i(x) = A_1 \sinh \lambda_1 x + A_2 \cosh \lambda_1 x + A_3 \sin \lambda_2 x + A_4 \cos \lambda_2 x \quad (2.3)$$

$$\lambda_{1/2} = \sqrt{\mp \frac{P}{2EI} + \sqrt{\left(\frac{P}{2EI} \right)^2 + \frac{\rho A \omega^2}{EI}}} \quad (2.4)$$

and ω is the frequency of vibration.

Boundary conditions for the column (the Beck problem) are given by

$$w_1(0) = w_1'(0) = w_2''(L) = w_2'''(L) = 0 \quad (2.5)$$

and the conditions of equilibrium of forces and continuity of displacements in the cross-section x_1 are given by

$$\begin{aligned} w_1''(x_1) &= w_2''(x_1) \\ w_1'''(x_1) &= w_2'''(x_1) \\ w_1(x_1) - w_2(x_1) &= \gamma_S^* w_1'''(x_1) \\ w_1'(x_1) - w_2'(x_1) &= \gamma_R^* w_1''(x_1) \end{aligned} \quad (2.6)$$

where $\gamma_R^* = \gamma_R EI/L$ and $\gamma_S^* = \gamma_S EI/L^3$ are nondimensional parameters of the rotational flexibility and the flexibility in a direction of the shear force, respectively. In an analysis sometimes the stiffness parameters, defined as $\kappa_R^* = 1/\gamma_R^*$ and $\kappa_S^* = 1/\gamma_S^*$, are used for convenience.

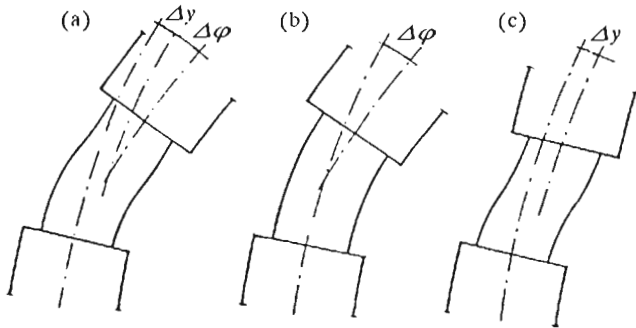


Fig. 2. (a) Complete deformation of the additional segment; (b) deformation with the first eigenform (rotation); (c) deformation with the second eigenform (transverse displacement)

Conditions (2.6) accept a general case of the local loss of stiffness, i.e. loss of the rotary stiffness and loss of stiffness in the direction of shear force. For a model of the local weakness taken as an additional beam element of dimensions tending to zero, the rotation is an idealization of the deformation with the first natural form and is shown in Fig.2b, whereas the transverse displacement represents the deformation with the second eigenform as shown in Fig.2c. A rotationally-slidable joint, presented in Fig.3, satisfies the complete deformation of such an additional segment, rotation and transverse displacement simultaneously. Such idealization is a generalization of the usually developed model of the elastic hinge-joint (cf Bogacz and Mahrenholtz (1986); Bogacz and Niespodziana (1987)) and the model of the transverse-slidable joint (cf Bogacz and Imielowski (1986); Bogacz and Niespodziana (1987)).

An analytical solution to the problem is possible by the use of the transfer matrix technique. The variables y , φ , M , Q , have a similar constitutive form

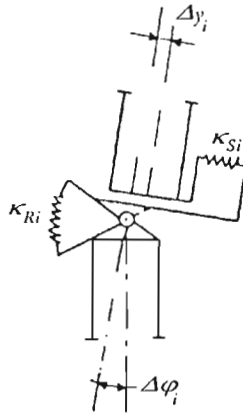


Fig. 3. Model of rotationally-slidable joint

and in the method there are closed in the state vector \mathbf{G} defined as

$$\mathbf{G} = [y, \varphi, M, Q]^T = [w, w', -EIw'', -EIw''']^T \quad (2.7)$$

For a generalized segment (beam or joint) the transfer matrix \mathbf{T}_i enables a relation between the states vectors for its both boundaries

$$\mathbf{G}_{i+1}^0 = \mathbf{T}_i \mathbf{G}_i^0 \quad (2.8)$$

where $\mathbf{G}_i^0 = \mathbf{G}(x_i = 0)$ and $\mathbf{G}_{i+1}^0 = \mathbf{G}(x_{i+1} = 0)$.

The successive multiplying results in the global transfer matrix \mathbf{T} . For the complete structure we get

$$\mathbf{G}_{n+1}^0 = \mathbf{T}_n \dots \mathbf{T}_2 \mathbf{T}_1 \mathbf{G}_1^0 = \mathbf{T} \mathbf{G}_1^0 \quad (2.9)$$

The partial transfer matrix \mathbf{T}_i for the segment can be found using the solution of Eq (2.3) and is given by Bogacz et al. (1991). However, for the rotationally-slidable joint \mathbf{T}_i is obtained from Eqs (2.6). The nonzero elements of the transfer matrix for the joint presented in Fig.2 are

$$t_{ii} = 1 \quad t_{14} = \gamma_S^* \quad t_{23} = \gamma_R^* \quad (2.10)$$

The boundary conditions (2.5) lead to the characteristic equation which expresses the relation between force and frequency

$$\begin{vmatrix} t_{33} & t_{34} \\ t_{43} & t_{44} \end{vmatrix} = \Phi(P, \omega) = 0 \quad (2.11)$$

In the following, an analysis of the characteristic curves (eigencurves) on the force-frequency plane, determined by the successive real roots of Eq (2.10), is taken into account. The critical state is defined by the critical force. The structure loses its stability by flutter with P_{cr} determined as a lowest double root of Eq (2.10). For $P > P_{cr}$ the respective eigenfrequencies became conjugate couple $\omega_{ij} = \omega_0 \pm ip$, an amplitude of vibration increases, due to the exponential term of Eq (2.12)

$$y_i(x, t) = w_i(x) \exp(i\omega_0 \pm p)t = w_i(x) \exp(i\omega_0) \exp(\pm pt) \quad (2.12)$$

On the force-frequency plane the flutter critical force is determined as maximum, minimum or a point of intersection of curves on the $P_{cr}^* = P_{cr}^*(\omega^*)$ plane appearing with $\omega^* > 0$. The structure loses its stability by divergence with P_{cr}^* occurring with zero frequency. The calculations are done for nondimensional values P^* and ω^* , where $P^* = PL^2/EI$ and $\omega_{cr}^{*2} = \omega_{cr}^2 L^4 \rho/EI$.

3. Influence of system parameters on a shape of the eigencurves

A configuration of eigencurves on the load-frequency plane typical for the Beck column is given in Fig.4f. It is obvious that the localization of the discontinuity on the free end of column does not influence its stability. The condition of disappearance of the bending moment is satisfied as an assumption. Also the condition of disappearance of the shear force is fulfilled, because the circulatory force acts tangentially to the free end of the column by definition.

Appearance of the localized loss of stiffness in any other position results in variation of a shape of the characteristic curves. Quantitative and qualitative variations cause the continuous and discontinuous changes of the critical load. It was observed that the thorough stability analysis of the system is possible when the simultaneous rotation and transverse displacement of joint is taken into account. Only for a such model, two kinds of the characteristic curves can be easily distinguished on the force-frequency plane. Any variation of the shape of the eigencurves may be then treated as a result of interaction between curves of this two kinds.

Let us begin the consideration with the structure characterized by a perfectly flexible joint, i.e. $\kappa_R^* = \kappa_S^* = 0.0$. Configurations of characteristic curves for $x_1 = 0.0, 0.1, 0.3, 0.5, 0.7, 1.0$, are shown in Fig.4. It is seen that for two selected localizations $x_1 = 0.0$ and $x_1 = 1.0$ only one type of eigencurves is observed. In the former case, successive double roots of Eq (2.10) tend to zero,

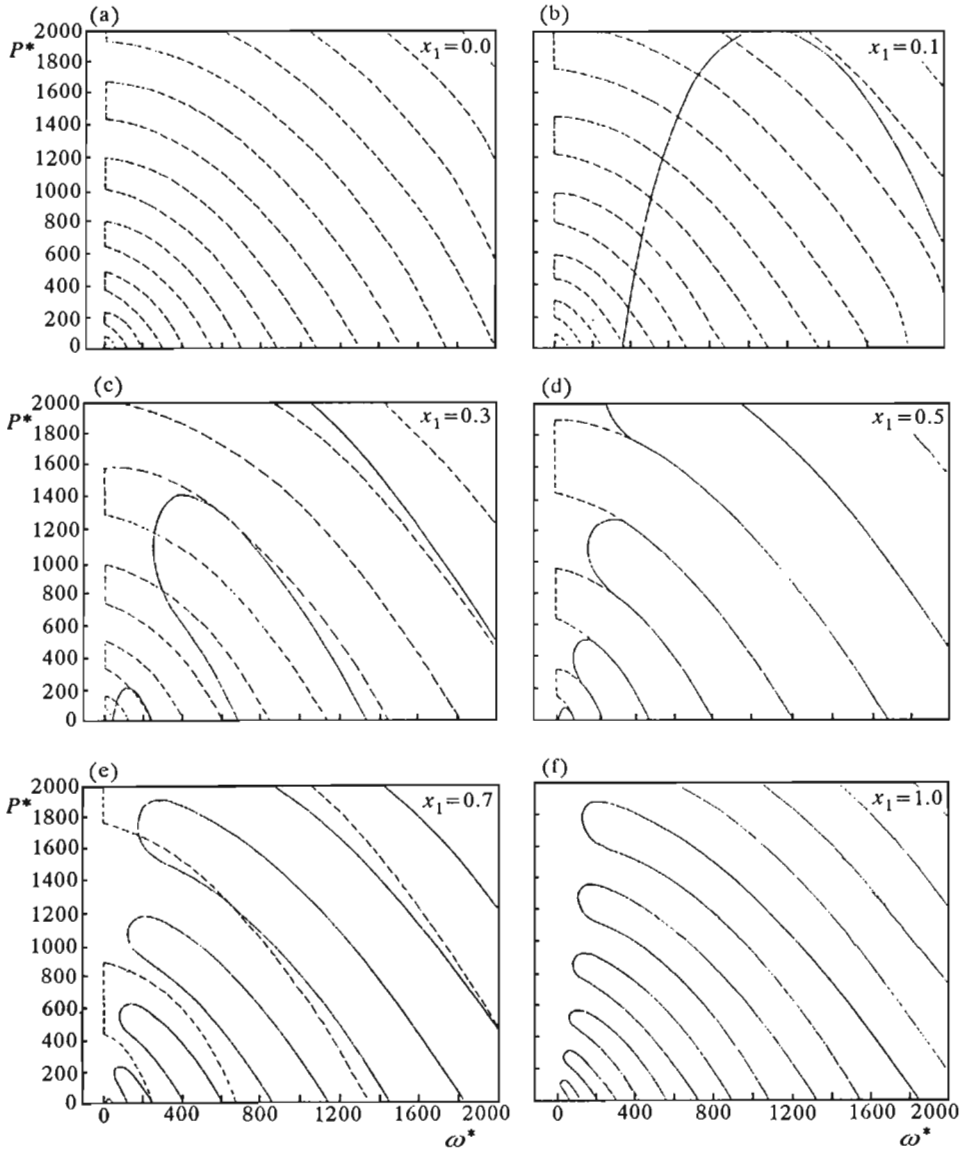


Fig. 4. Configuration of characteristic curves for the column with a perfectly flexible joint, $\kappa_S^* = \kappa_R^* = 0.0$, located in different cross-sections

in calculation $\omega_{cr}^* = 0.00006$. The column loses its stability by divergence. However in the latter one, the configuration of eigencurves is typical for the Beck column. Successive double roots occur at greater frequency. The column loses its stability by flutter.

Let us consider a localization of the joint at a small distance from the fixed end of the column. It is seen that a few curves of the "flutter" configuration is added to the "divergence" one. In Fig.4b for $x_1 = 0.1$, two additional curves of the second configuration are observed within the considered range of frequencies. When the joint takes a position closer to the centre of the column, the "flutter" curves bring near the origin of coordinates, whereas the "divergence" curves displace from it. It is seen from Fig.4d that for $x_1 = 0.5$ the curves coincide in pairs which, for $P^* = 0.0$, become double roots of the characteristic equation. The critical force diminishes to zero.

A graph of $P_{cr}^* = P_{cr}^*(x_1)$ for $\kappa_R^* = \kappa_S^* = 0.0$ is depicted in Fig.5a. The thick line is determined from an analysis of the first four frequencies. It is seen that for $x_1 < 0.622$, instability of the divergent type ($\omega_{cr}^* = 0.00006$) occurs with the first and the second natural form. However for $x_1 > 0.622$ the column loses its stability by flutter with the third and the fourth mode. It is seen that for $x_1 = 0.622$, the column loses its stability oscillating at first four frequencies. Variation of localization of the joint about this position results in a switch of the critical frequency from $\omega_{cr}^* = 0.00006$ to $\omega_{cr}^* = 28.56$. On the graph for some values of x_1 there is marked a drop and then a steep increase of P_{cr}^* . The successively marked areas appear a consequence of meeting and then of crossing-over of the successive 7th, 6th, 5th, 4th and 3rd curves of the divergence type by the first flutter type curve. The first of them, denoted by VII takes place for the joint situated near $x_1 = 0.1$, as shown in Fig.4b. However, the last one, denoted by III, takes place for the joint positioned near $x_1 = 0.3$, and is explained in Fig.4c. Because an infinite number of such areas follows the crossing of both configuration of curves, we conclude instability of the structure with a perfectly flexible joint.

Let us consider a column with a joint for which the stiffness of transverse or of rotational degree of freedom tends to but does not reach the zero value. It is seen from Fig.6, that now the point of intersection of the eigencurves is always placed above the horizontal axis and determines the value of critical force always bigger than zero. High sensitivity of the critical force to the joint stiffness is typical for this set of system parameters. However, the first eigenfrequency remains approximately equal to zero. The critical force is of the divergent type with the critical frequency tending to zero and appears as the point of instability of the first and the second eigenfrequencies.

An increase of the first eigenfrequency from zero value, is caused by the

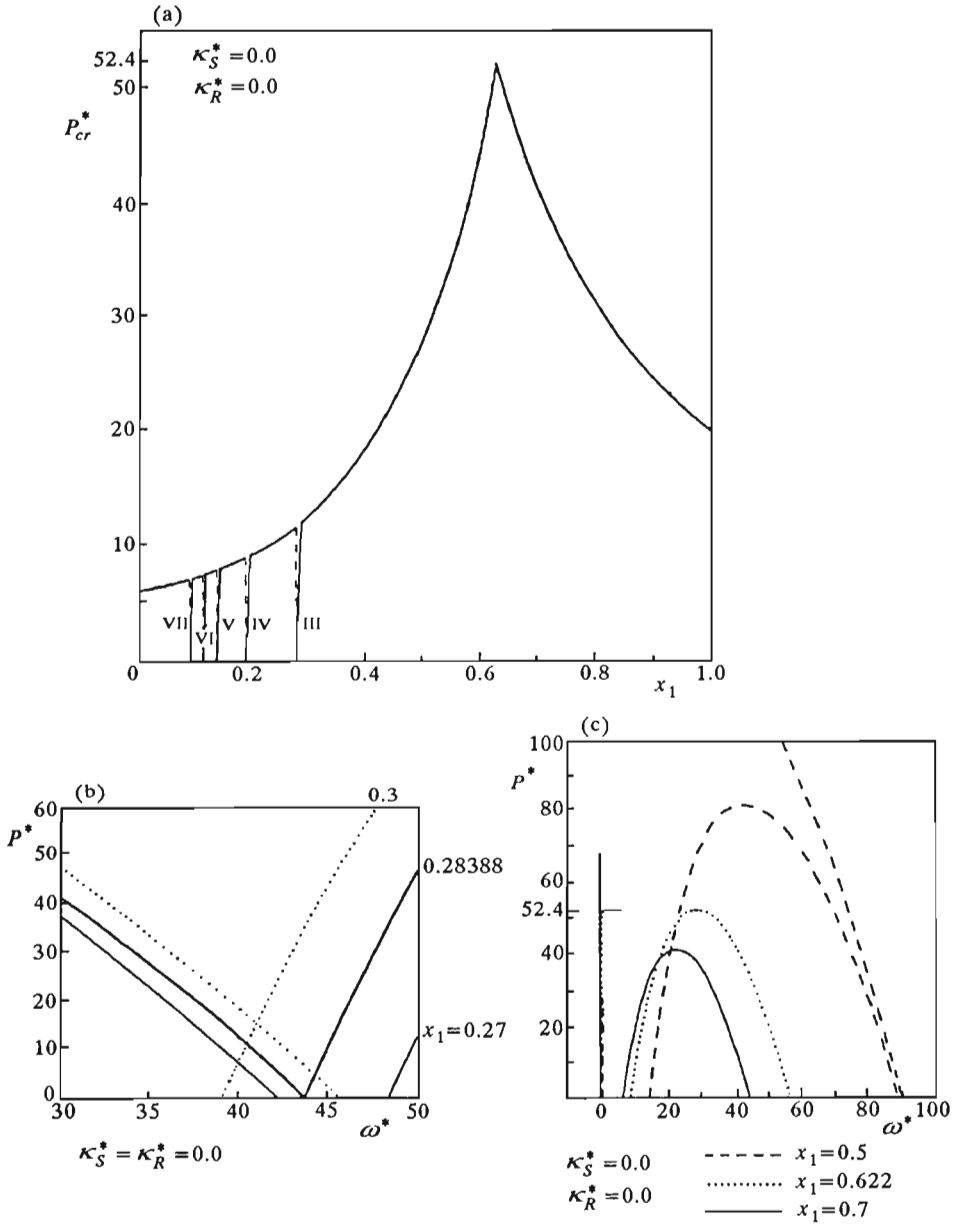


Fig. 5. (a) Critical force versus joint location for the column with perfectly flexible joint $\kappa_S^* = \kappa_R^* = 0.0$; (b) shape of eigencurves for the unstable structure; (c) "Jump" of critical value of frequency

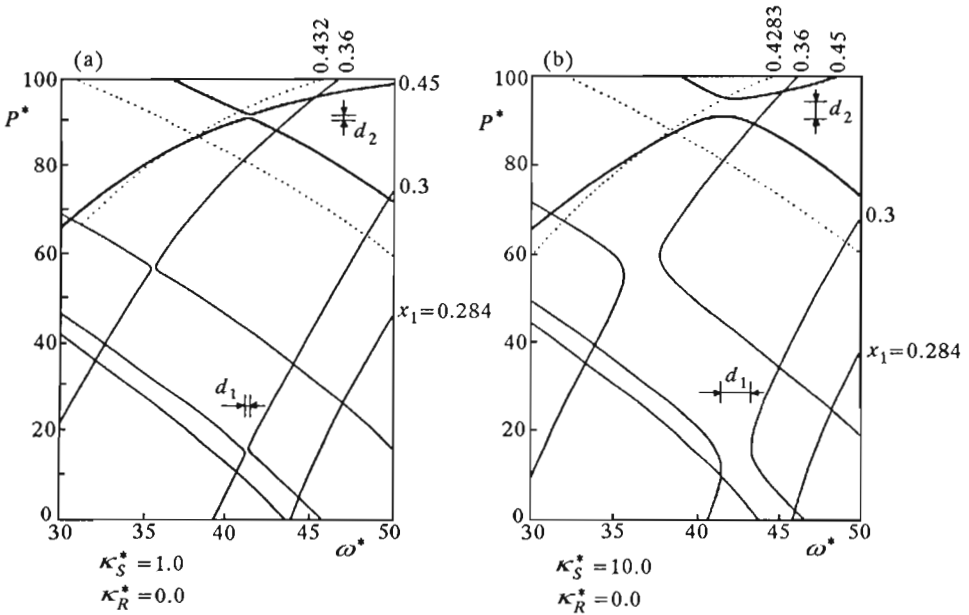


Fig. 6. Influence of the joint position on a shape of characteristic curves for the joint with equal to zero stiffness of the rotary degree of freedom

stiffening of the second degree of freedom of the joint ($\kappa_S^* > 0.0$ and $\kappa_R^* > 0.0$). On the force-frequency plane a shift of the first eigencurve from the vertical axis is observed. The column loses its stability by flutter. Let us turn to the effect of the joint position. The graphs of $P^* = P^*(\omega^*)$ for different x_1 are presented in Fig.7. The "movement" of two kinds of eigencurves with respect to the origin of coordinates is seen. On the graph the first curve of a "flutter" configuration changes its position and crosses the 3rd, 4th and 5th curves of the "divergence" one. The curves intersect each other for a so called "critical" positions of the joint, e.g. $x_1 = 0.186$, $x_1 = 0.23796$ and $x_1 = 0.3277$, as depicted in Fig.7. For the successive "critical" positions x_1 the respective critical forces determined as a point of intersection of curves are bigger than the previous ones and do not influence the P_{cr}^* appearing at the first and the second natural frequency. Such behaviour is typical for each localization below the centre of column.

It is an important observation that a variation of the joint position does not influence minimal distances d_1 and d_2 between the curves. Beyond the "critical" position, and positions differing slightly from the critical ones, the distances d_1, d_2 depend proportionally directly on the joint stiffness value.

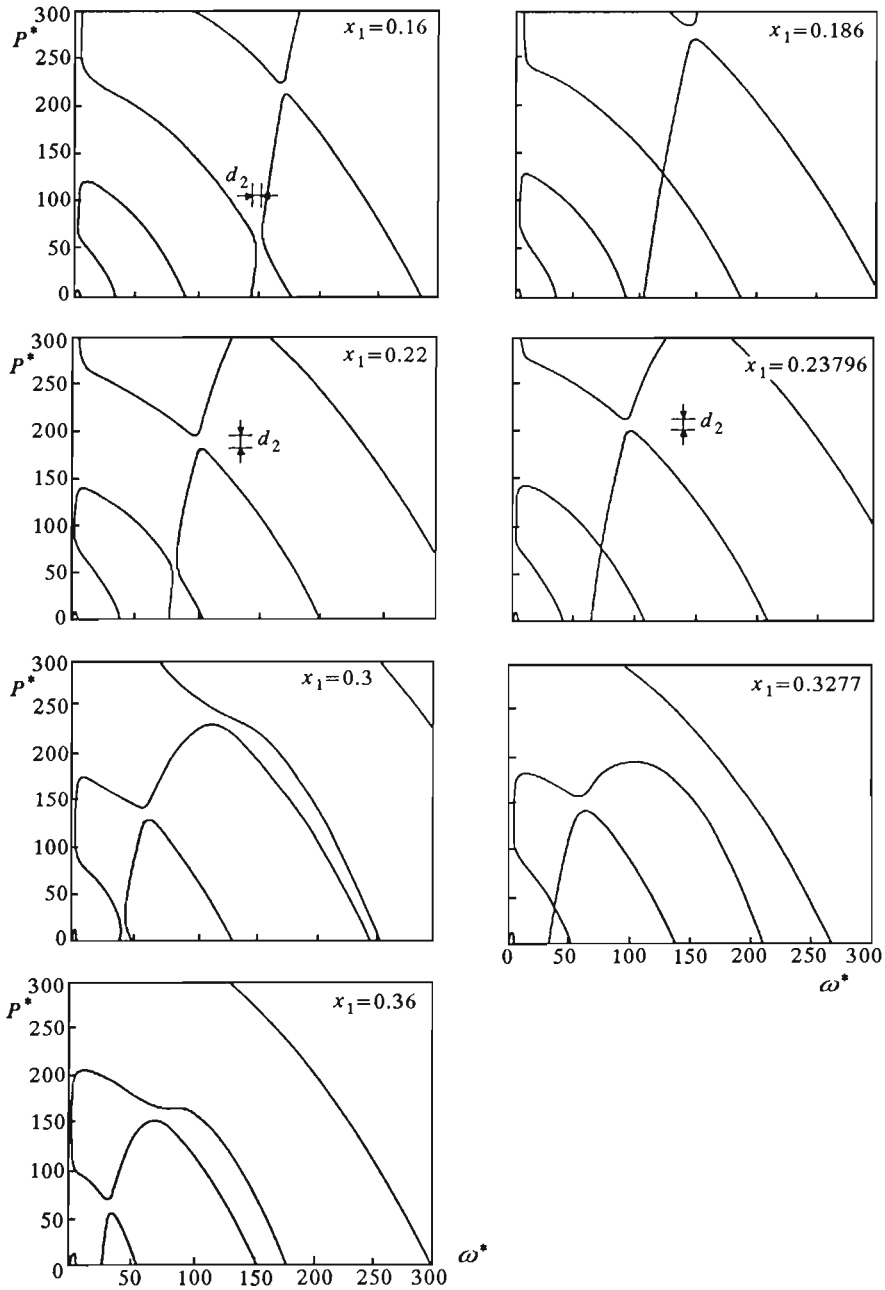


Fig. 7. Influence of the joint position $\kappa_S^* > 0.0, \kappa_R^* > 0.0$ on a shape of characteristic curves

It is depicted in Fig.6 for (κ_R^*, κ_S^*) taking values equal to $(0.0, 1.0)$ and $(0.0, 10.0)$, respectively.

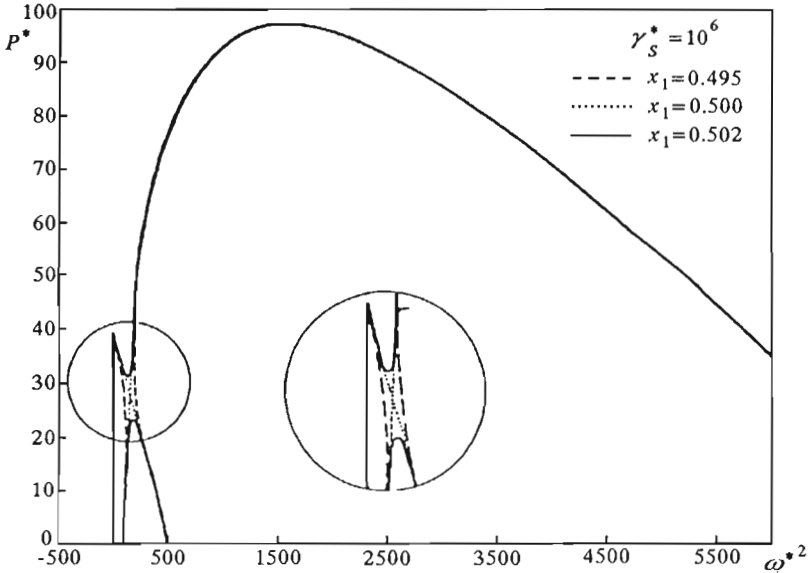


Fig. 8. Switch-over of eigencurves for the case of interaction between eigencurves of both kinds

Let us discuss the joint positioned above the centre of the column. In such localization interactions between the two first eigencurves of each configuration take place. The qualitative changes of the characteristic curves result in discontinuous variation of the critical force. This phenomenon is explained in Fig.8. Bogacz and Mahrenholtz (1986) and Bogacz and Niespodziana (1987) also described it. For the set of parameters $(\gamma_R^* \rightarrow 0.0, \gamma_S^* \rightarrow \infty, x_1 = 0.495)$ the column loses its stability oscillating with the first and the second modes while for $(\gamma_R^* \rightarrow 0.0, \gamma_S^* \rightarrow \infty, x_1 = 0.75)$ instability occurs with the second and the third natural frequencies. The discontinuous change in the critical force value is connected with a discontinuous variation of the critical frequency.

For the localization of the joint above the centre of the structure a skip of the critical force may occur also at high eigenfrequencies. It is obvious that an analysis of the first four frequencies may occur insufficient in determining the critical force value.

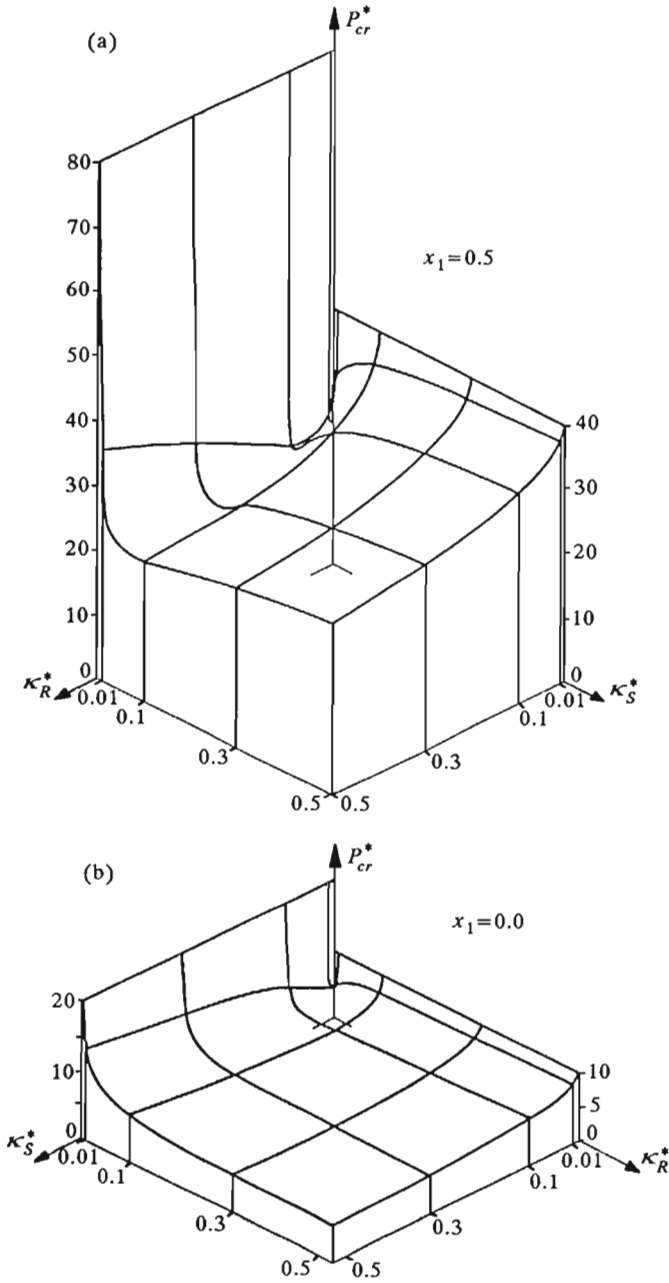


Fig. 9. Critical force versus joint stiffness for $x_1 = 0.0$ and $x_1 = 0.6$

4. Influence of system parameters on the critical force value

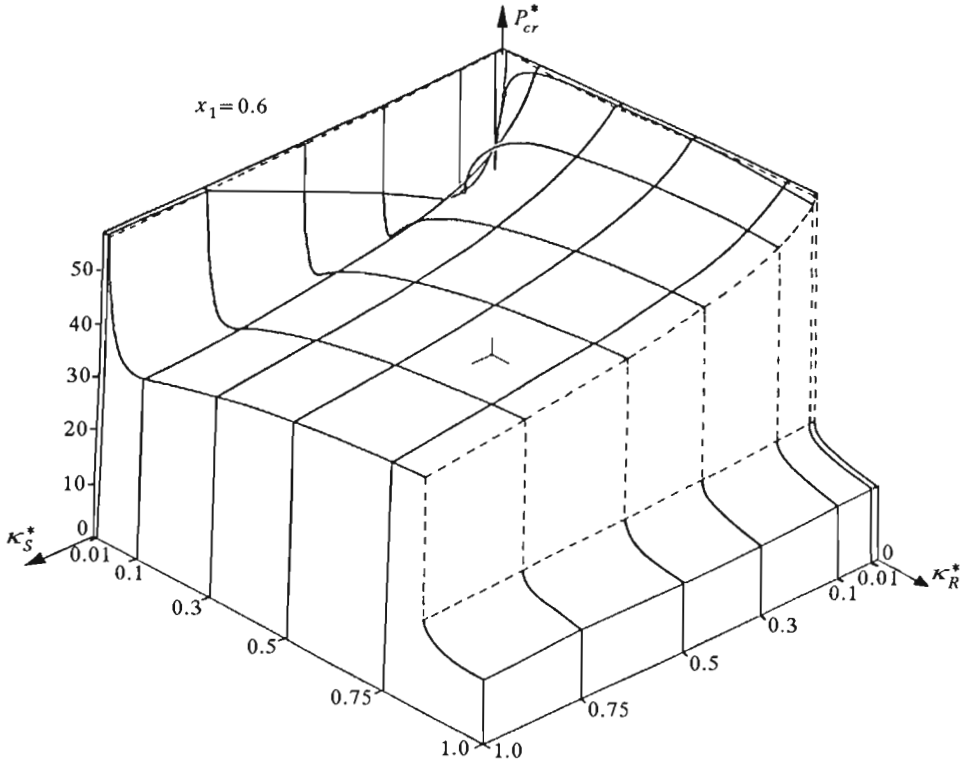


Fig. 10. Discontinuous changes of critical force for the joint located above centre of column $x_1 = 0.6$

In consideration the joint stiffness is taken from the range $\kappa_S^*, \kappa_R^* \in (0, \infty)$. The typical graphs of $P_{cr}^* = P_{cr}^*(\kappa_S^*, \kappa_R^*)$ are presented in Fig.9 and Fig.10 for the localized weakness placed at $x_1 = 0.0$, $x_1 = 0.5$ and $x_1 = 0.6$. It is seen that for localizations of the joint below the centre of the structure only continuous variation of the critical force takes place. A considerable decrease of the critical force follows the location of the joint near the base of the column. When the joint flexibility does not change, any higher position (but not exceeding the centre of the column) results in an increase of the critical force. Particularly advantageous from the point of view of critical force maximization, is a column with the joint stiffness characterized by $\kappa_R^* \rightarrow 0.0$ and $\kappa_S^* \neq 0.0$ or $\kappa_S^* \rightarrow 0.0$ and $\kappa_R^* \neq 0.0$. This high value of the critical force is kept independently of the stiffness value for the second degree of freedom of

the joint. The highest value of P_{cr} equal to $80.8EI/l^2$ (about fourfold increase in a comparison with the uniform column) is observed for the joint of stiffnesses $\kappa_R^* \rightarrow 0.0$ and $\kappa_S^* \neq 0.0$, localized in the cross-section $x_1 = 0.5$ ¹. The same value was obtained by Bogacz and Mahrenholtz (1986) for a structure with an elastic hinge-joint. For the column with the joint of tending to zero value of the shear stiffness only double increase of the critical force, i.e. up to $39.9EI/l^2$, is possible for this localization.

It is to notice that for a localization of the joint below the centre of the structure the obtained surfaces $P_{cr}^* = P_{cr}^*(\kappa_S^*, \kappa_R^*)$ are concave. The minimal value of the critical force occurs for rotary stiffness about two times smaller than the shear rigidity. However, when the joint is located above the position $x_1 = 0.5$ the critical force represents a point on the discontinuous surface in the space of system parameters. The lines of discontinuity indicate a skip of the critical force and take place for the critical values of structure parameters, when qualitative variations of the shape of the characteristic curves take place. Points of the surface which are placed on the both sides of the line of discontinuity are represented by a qualitatively different configuration of the eigencurves. An example of the discontinuous surface is shown in Fig.10 for the flexible element placed in the cross-section $x_1 = 0.6$.

5. Conclusions

In the paper an influence of localized weakness on stability of the Beck column is discussed. In general the local loss of stiffness is modeled by an additional element, i.e. joint, with a possibility of simultaneous rotational and transverse displacement. A location of such an element can stabilize or destabilize the system, depending on its position and flexibility. It was shown, that for a local weakness placed below a centre of the structure, the instability occurs with the first and the second eigenfrequency. For other localizations the column may lose its stability oscillating at higher frequencies and an analysis of the only first four eigenfrequencies may appear to be insufficient to determine a value of the critical force.

The graphs of $P_{cr}^* = P_{cr}^*(\kappa_S^*, \kappa_R^*)$, for various joint positions are presented. For the model of the column with a generalized flexible element, realizing simultaneous shear and rotational displacement, the critical force is never

¹The phenomenon of a considerable increase of the critical force when the diminishing segment is located in the centre of the column has an application to the shape optimization of this structure (cf Imielowski and Mahrenholtz (1994))

higher than the one obtained for the column with the localized loss of bending stiffness or shear stiffness (cf Bogacz and Mahrenholtz (1986); Bogacz and Imielowski (1986)). There is noticed a near fourfold increase of the critical load $P_{cr} = 80.8EI/l^3$, when the ideal flexible hinge-joint is located in the centre. It can be seen that for transverse-slidable joint only double increase of P_{cr} is possible for this position. It was shown that when stiffness of at least one degree of freedom of the joint tends to zero, the critical force takes its maximal value independently of the stiffness value of the second degree of freedom. From the point of view of maximization of the critical force the model of the structure with the single elastic hinge-joint located in the centre of the column is suitable for considerations as an initial for optimization.

When the joint is located above the central position, the critical force represents a point on the discontinuous surface in the space of system parameters. Such positions of the flexible element occur very disadvantageous because a small variation of the system parameters can result in a drop of the critical force with a destabilization of the structure.

It was shown that the discontinuous changes of the critical force follow the qualitative variation in configuration of the characteristic curves, whereas the continuous changes are caused by quantitative variation in this configuration.

Due to the fact that the linear approach does not always describe well a nature of the phenomenon, the next investigations will be devoted to the nonlinear generalization of the above studies brought for discussion with the findings presented by Kounadis (1991a) and (1991b).

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Uwagi o stateczności układów dyskretno-ciągłych poddanych obciążeniu cyrkulacyjnemu

Streszczenie

W pracy rozważane są zagadnienia stateczności układów dyskretno-ciągłych poddanych obciążeniom niekonserwatywnym. Na przykładzie kolumny ze zlokalizowaną nieciągłością sztywności, modelowaną węzłem obrotowo-przesuwным, badany jest wpływ parametrów układu na stateczność. Układ jest obciążony siłą cyrkulacyjną. Badany jest przypadek siły stycznej do swobodnego końca kolumny. Omówiono zjawiska stabilizacji i destabilizacji konstrukcji oraz ciągłych i skokowych zmian siły krytycznej będące rezultatem wzajemnego oddziaływania dwóch rodzajów krzywych charakterystycznych na płaszczyźnie obciążenie-częstość. Do rozwiązania zagadnienia wykorzystano metodę macierzy przeniesienia.