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APPLICATION OF THE FOURIER TRANSFORMATION TO FLUTTER TESTS

FRANCISZEK LENORT

Institute of Aviation, Warsaw

Hammond and Doggett (1976) present the method for determination of a damping coefficient of vibrations in flutter tests in terms of the Fourier transformation. The invented formula for the damping coefficient is complex and demands for many simplifications to be accepted when producing it. It has been proved in the present contribution that the plain and precise formula for the damping coefficient can be invented without any simplifications when employing the Fourier transform. This method has been tested both on the model and the experimental data. It is currently being used in the analysis of flutter characteristics of an aircraft in flight.

1. Introduction

The term "flutter" means the self-excited aeroelastic vibrations of an aircraft in flight. In accordance with this definition these vibrations do not originate either from the operating power unit or from the separation of an air flow on various aircraft parts, e.g. in flights at high angles of incidence.

Typical examples of flutter vibrations are bending-torsional vibrations of a wing. When the bending frequency close to the frequency of torsion it can be found in the resulting configuration of angles of incidence that a wing starts to absorb energy from the incoming air flow and converts it into the energy of rapidly extending wing vibrations. The similar increase of the bending vibrations of a wing can appear in the phase-delayed relative motion of the aileron system.

The flight testing of flutter tendency consists in the precise analysis of aircraft vibrations, which appear in flight with stepwise increasing speeds. When the natural vibrations due to the turbulence of ambient air are small and difficult to analyze the excitation is being applied by means of the impulse or harmonic method. These forced vibrations (see Fig.1) are then analyzed with respect to vibration mode, frequency, amplitude and damping coefficients of particular vibration components, respectively.

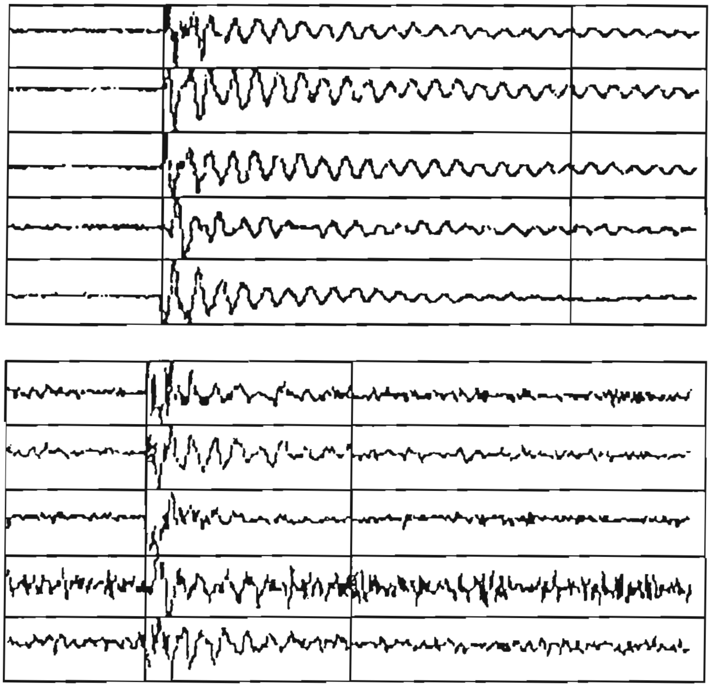

Fig. 1. Vibrations of an aircraft with underwing stores after the impulse excitation, mesured by means of vibration sensors. The upper part shows the results of test on the ground at $V = 0$ speed and the lower part – results of flight tests at the speed $V = 870$ kph

The damping coefficient value for vibrations of the aircraft structure has been taken as the basic one.

The A.C.23.629-1 Advisory Circular to the US Airworthiness Regulations FAR-23 demands for the damping coefficient g value to be not less than 0.03. It means that the relative vibration damping coefficient ζ should not be less than 0.015 within the whole range of permissible flight speeds.

The freely decaying signal for the vibrating one-degree-of-freedom system in the form

$$y(t) = A\exp(-\zeta\omega_0 t)\sin(\omega_n t + \varphi) \tag{1.1}$$

has been analyzed by Hammond and Dogget (1976).

The frequency ω_0 for a system without damping may be written as

$$\omega_0 = \sqrt{\frac{k}{m}}$$

In mechanical systems k is a stiffness constant and m is a generalized mass of vibrating system.

The natural frequency of a system can be written as

$$\omega_n = \omega_0\sqrt{1-\zeta^2}$$

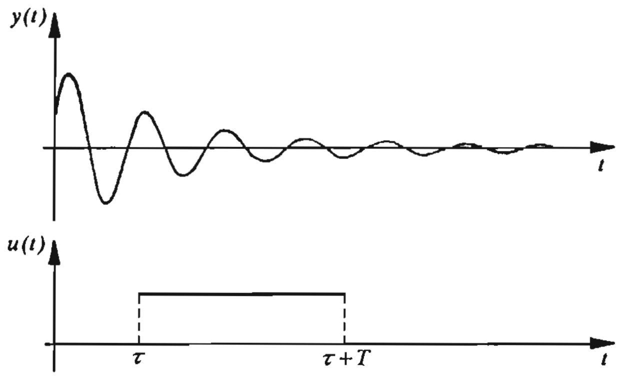

Fig. 2. Theoretical form of the freely diminishing signal $y(t)$ for the linear one-degree-of-freedom system and the function $u(t)$ – the movable analysis window

Fig.2 presents the character of freely decaying signal $y(t)$ and the function $u(t)$ in the form

$$u(t) = \begin{cases} 1 & \text{for} \quad \tau \le t \le T+\tau \\ 0 & \text{for} \quad t < \tau \text{ and } t > T+\tau \end{cases}$$

The Fourier transform at the point ω_n for the $y(t)$ function multiplied by $u(t)$ can be written as

$$Y(\mathrm{j}\omega_n) = \int_{\tau}^{\tau+T} y(t)\exp(-\mathrm{j}\omega_n t)\, dt \qquad (1.2)$$

Hammond and Doggett (1976) present the result of transformation of Eq (1.2) after many simplifications, e.g.

$$\omega_0 \cong \omega_n \qquad \zeta \ll 1$$

ζ^2 – value to be neglected as being small compared to unity and after expanding some functions into the Maclaurin series, which yields

$$\begin{aligned}\ln|Y(\mathrm{j}\omega_n)| &= -\zeta\omega_n\tau + \ln\frac{A}{2\omega_n} + \\ &+\frac{1}{2}\ln\Big[(\omega_n T)^2 + \omega_n T\Big(\sin 2(\omega_n\tau+\varphi) - \sin 2[\omega_n(\tau+T)+\varphi]\Big)\Big] + \\ &-\frac{\zeta}{4}\omega_n T\Big(\frac{2\omega_n T + \sin 2(\omega_n\tau+\varphi) - 3\sin 2[\omega_n(\tau+T)+\varphi]}{\omega_n T + \sin 2(\omega_n\tau+\varphi) - \sin 2[\omega_n(\tau+T)+\varphi]}\Big)\end{aligned} \tag{1.3}$$

On the assumption that

$$T = kT_c \qquad k = 1, 2, 3, ...$$

the formula becomes

$$\ln|Y(\mathrm{j}\omega_n)| = -\zeta\omega_n\tau + \frac{1}{2}\zeta\sin 2(\omega_n\tau+\varphi) + \ln\frac{A}{2\omega_n} + \ln(\omega_n T) - \frac{\zeta\omega_n T}{2} \tag{1.4}$$

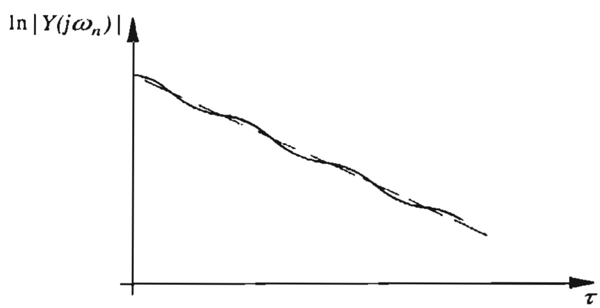

Fig. 3. Transform Y versus the shift of the analysis window

The form of Eq (1.4) indicates that the graph presented in Fig.3 can be obtained by computing $|Y(\mathrm{j}\omega_n)|$ for successive values of τ. The coefficient ζ represents the slope of a curve in Fig.3. Simplifications accompanying inventing of the formula (1.4) give way to some doubts about correctness of the received result and precision of the computation of ζ in this way.

2. A correct formula for the damping coefficient as the ratio of Fourier transforms

The outline of evidence was given by Lenort (1989b). The Fourier transform was there defined for the T-long vibration section, e.g. for $\tau = 0$ and for the T-long vibration section shifted to the right, not for an arbitrary value of τ, but for $\tau = T_c = 2\pi/\omega_n$.

The Fourier transform of the vibration section $y(t)$ for $\omega = \omega_n$, which starts at the point $\tau = 0$ can be expressed as

$$Y_1(\mathrm{j}\omega_n) = \frac{2}{T}\int\limits_0^T y(t)\exp(-\mathrm{j}\omega_n t)\,dt \tag{2.1}$$

By substituting Eq (1.1) into Eq (2.1) we obtain

$$Y_1(\mathrm{j}\omega_n) = \frac{2}{T}\int\limits_0^T A\exp(-\lambda t)\sin(\omega_n t + \varphi)\exp(-\mathrm{j}\omega_n t)\,dt \tag{2.2}$$

where the following notation is used

$$\lambda = \zeta\omega_0 = \frac{\zeta\omega_n}{\sqrt{1-\zeta^2}} \tag{2.3}$$

For the T-long vibration section which is shifted to the right by $\tau = T_c$ Eq (2.2) takes the form

$$Y_2(\mathrm{j}\omega_n) = \frac{2}{T}\int\limits_{T_c}^{T_c+T} A\exp(-\lambda t)\sin(\omega_n t + \varphi)\exp(-\mathrm{j}\omega_n t)\,dt \tag{2.4}$$

Introducing a new variable

$$z = t - T_c$$

into the limits of integration we obtain

$$\begin{aligned} z &= 0 \qquad \text{for} \quad t = T_c \\ z &= T \qquad \text{for} \quad t = T_c + T \end{aligned}$$

Thou Eq (2.4) takes the form

$$Y_2(\mathrm{j}\omega_n) = \frac{2}{T}\int\limits_0^T A\exp[-\lambda(z+T_c)]\sin[\omega_n(z+T_c)+\varphi]\exp[-\mathrm{j}\omega_n(z+T_c)]\,dz \tag{2.5}$$

Bearing in mind that

$$\omega_n T_c = 2\pi$$

we have

$$\sin[(\omega_n z + \omega_n T_c) + \varphi] = \sin(\omega_n z + \varphi) \tag{2.6}$$

and

$$\begin{aligned} \exp[-\mathrm{j}\omega_n(z + T_c)] &= \exp(-\mathrm{j}\omega_n z)\exp(-\mathrm{j}\omega_n T_c) = \\ &= \exp(-\mathrm{j}\omega_n z)(\cos 2\pi - \mathrm{j}\sin 2\pi) = \exp(-\mathrm{j}\omega_n z) \end{aligned} \tag{2.7}$$

Taking into consideration Eqs (2.6) and (2.7) from Eq (2.5) we have

$$Y_2(\mathrm{j}\omega_n) = \exp(-\lambda T_c)\frac{2}{T}\int_0^T A\exp(-\lambda z)\sin(\omega_n z + \varphi)\exp(-\mathrm{j}\omega_n z)\,dz \tag{2.8}$$

Since for

$$y(t) \neq 0 \qquad\qquad Y_2(\mathrm{j}\omega_n) \neq 0$$

we can write the ratio of transforms

$$\frac{Y_1(\mathrm{j}\omega_n)}{Y_2(\mathrm{j}\omega_n)} = \frac{\frac{2}{T}\int_0^T A\exp(-\lambda t)\sin(\omega_n t + \varphi)\exp(-\mathrm{j}\omega_n t)\,dt}{\exp(-\lambda T_c)\frac{2}{T}\int_0^T A\exp(-\lambda z)\sin(\omega_n z + \varphi)\exp(-\mathrm{j}\omega_n z)\,dz} \tag{2.9}$$

from which, substituting for $z = t$ we have

$$\frac{Y_1(\mathrm{j}\omega_n)}{Y_2(\mathrm{j}\omega_n)} = \exp(\lambda T_c) \tag{2.10}$$

hence

$$\lambda = \frac{1}{T_c}\ln\frac{Y_1(\mathrm{j}\omega_n)}{Y_2(\mathrm{j}\omega_n)} \tag{2.11}$$

Let us notice that when inventing the formula (2.11) it was not necessary to assume that $T = nT_c$ where $n = 1, 2, 3, ...$

Eq (2.11) can be generalized

$$\lambda = \frac{1}{kT_c}\ln\frac{Y_1(\mathrm{j}\omega_n)}{Y_{k+1}(\mathrm{j}\omega_n)} \qquad\qquad k = 1, 2, 3, ... \tag{2.12}$$

In general $Y(j\omega_n)$ is a complex number and and using simplified notation we rewrite Eq (2.10) as $Y_1 = Y_2 \exp(\lambda T_c)$ therefore

$$\mathrm{Re}Y_1 = \mathrm{Re}Y_2 \exp(\lambda T_c) \qquad \mathrm{Im}Y_1 = \mathrm{Im}Y_2 \exp(\lambda T_c)$$

Then we have

$$\lambda = \frac{1}{T_c} \ln \frac{\mathrm{Re}Y_1}{\mathrm{Re}Y_2} \tag{2.13}$$

and

$$\lambda = \frac{1}{T_c} \ln \frac{\mathrm{Im}Y_1}{\mathrm{Im}Y_2} \tag{2.14}$$

as well as

$$\lambda = \frac{1}{T_c} \ln \frac{|Y_1|}{|Y_2|} \tag{2.15}$$

Eq (2.15) represents the generalization of the well-known equation

$$\lambda = \frac{1}{T_c} \ln \frac{A_1}{A_2} \tag{2.16}$$

where the amplitudes A_1 and A_2 have been replaced by modules $|Y_1|$ and $|Y_2|$ of the Fourier transform.

After having computed $|Y_1|$ and $|Y_2|$, using Eq (2.15) we can determine λ and then using Eq (2.3) and knowing ω_n, ζ can be determined as well. λ can be computed also from Eq (2.13), even when only the real parts of the Fourier transforms are known or from Eq (2.14) when only imaginary parts of transforms Y_1 and Y_2 are computed. It is obvious that for the real data the best estimate of ζ is obtained from Eq (2.15). To the computation of the Fourier transform Y_1 we take e.g. $N = 1024$ of the $y(t)$ value instead of one value of A_1 and the estimate of the damping coefficient obtained from Eq (2.15) is definitely much more precise than that obtained from Eq (2.16).

Eqs (2.13), (2.14) and (2.15) can be generalized in the same way as Eq (2.12).

Computation of the damping coefficient ζ using Eq (2.15) has been verified on the model (see Fig.4) and the real (see Fig.5) data, respectively.

The presented method enables one to assess the damping coefficient of raw signals, which are noise-distorted or composed of a high number of vibration components, providing that their frequencies are sufficiently far from each other.

It can be proved that the shift τ can appear not only in the form

$$\tau = kT_c \qquad k = 1, 2, 3, ...$$

but also

$$\tau = \frac{1}{2}kT_c \qquad k = 1, 2, 3, ...$$

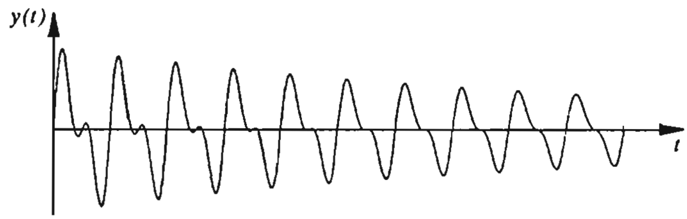

Fig. 4. Example of analysis of the damped model vibrations, composed of two components: $f_1 = 10$ Hz, $\zeta_1 = 0.01$ and $f_2 = 20$ Hz, $\zeta_2 = 0.01$. The damping coefficients for such a simple case are computed precisely. The analyzed signal denoted by dots overlaps the approximating curve denoted by a solid line

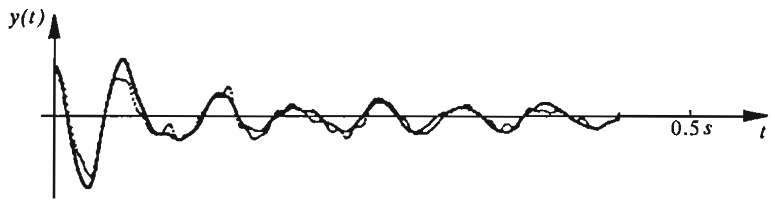

Fig. 5. Example of record of the real signal of aircaft vibrations measued in flight at the speed of 870 kph – denoted by dots and the result of analysis – the approximating solid line. Three main components are: $f_1 = 15.3$ Hz, $\zeta_1 = 0.037$; $f_2 = 19.2$ Hz, $\zeta_2 = 0.092$; $f_3 = 24.2$ Hz, $\zeta_3 = 0.041$

The above holds true when the level of damping has to be determined for short vibration sections. In practice the value $\tau = T_c$ has been used most often. The width of an analysis window T depends on features of the object under consideration. When evaluating the flutter tendency, the time T is, as a rule, less than 0.5 sec (see Fig.5). When employing the Fast Fourier Transform (FFT) there should be in the section T at least two cycles $2T_c$ of vibrations with the known frequency. For small values of T, near to 0.5 sec in order to obtain the satisfactory resolution it would be useful apply the so called "making up" with zeros. That is the reason why for short vibration sections (cf Lenort (1989a)) the algorithm for quick computation of the discrete Fourier transform has been worked-out. It enables one to determine the natural frequency ω_n and the damping coefficient value for decaying signals $y(t)$.

This method opens the way to improves the resolution by increasing the frequency of sampling evaluated signals. It enables the quick computation of individual transform points, which is useful when determining the damping level.

As it is well known the FFT is presumed to compute the whole set of transform points independently of current needs of the user.

3. Conclusions

The presented method of computing the damping coefficient by means of the Fourier transformation is very useful when applied to the analysis of real signals, rough, noise-distorted signals, signals distorted by random disturbances, as well as signals composed of a high number of vibration components.

The method has become the part of the computer-based system of vibration analysis and is currently being used in the analysis of flutter characteristics of an aircraft in flight.

References

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Zastosowanie przekształcenia Fouriera w badaniach flatterowych

Streszczenie

W pracy Hammond i Doggett (1976) opisano metodę określania współczynnika tlumienia drgań w czasie badań flatterowych przy pomocy przksztalcenia Fouriera. Wyprowadzona tam zależność na współczynnik tlumiennia jest skomplikowana i w

czasie jej wyprowadzania poczyniono wiele założeń upraszczających. W niniejszym artykule podano dowód bez uproszczeń i wyprowadzono prosty i ścisły wzór na współczynnik tłumienia przez zastosowanie transformaty Fouriera. Metoda ta została przetestowana na danych modelowych i doświadczalnych. Jest stosowana do analizy własności flatterowych samolotu w locie.

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