COMPARISON OF SIMPLIFIED METHODS OF DYNAMIC STRESS INTENSITY FSCTOR EVALUATION

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Numerical methods of determination of the dynamic stress intensity factor based on modal decomposition are considered. These methods are introduced to processing impact bending tests data in order to replace conventional but controversial quasi-static methodology. The influence of type of bend specimen model and method of determination of the model parameters on accuracy of the solution is investigated. Numerical results are compared with one- and three-point bending tests data reported in the literature.

1. Introduction

The objective of dynamic fracture tests of quasi brittle materials is to determine both the critical (corresponding to the beginning of unstable crack propagation) value of dynamic stress intensity factor (DSIF) and its growth rate which characterizes the strain rate in the process zone. Determination of a history of DSIF variation $K_I(t)$ and monitoring of the moment of crack initiation are necessary for this purpose. Methods of $K_I(t)$ determination can be classified into two groups. First group consists of so called direct experimental methods such as: method of caustics, photoelasticity or application of the strain gauges cemented near the crack tip. It is a local approach and the only theoretical assumption, one needs to make applying such an analysis is to accept the asymptotic distribution of stresses or strains provided by theory.

In contrast, the second group consists of methods in which DSIF is evaluated from the global load-time or load-deflection diagrams. It requires full theoretical analysis including assumptions concerning proper modelling of the specimen response to external loading. The simplest assumption is to consider

deformation of a specimen during impact test as a quasi-static one. It makes possible to evaluate $K_I(t)$ from the load-time diagram directly using the well known static formulae. However, in many theoretical and experimental works it was found that this assumption had been sometimes too strong. As a rule, the time interval between first contact of a tup with a brittle specimen and crack initiation is too short for dynamic processes in specimen to vanish. Thus, the quasi-static approach is inapplicable to one-point bend testing conditions when crack starts before the specimen comes into contact with the supports.

Dynamic effects are taken into account most completely when two dimensional (2D) modelling of the impact test is performed using finite or boundary element methods. Results obtained from such an analysis agree well with experimental data. But these methods are too complex and expensive to be used as a simple tool for the tests data processing. From practical point of view, and to the author's best knowlege, the simplified methods of dynamic analysis employing modal decomposition technique are more convenient. Accuracy of some of these methods is examined in this paper.

2.   Theoretical background

Let us consider a test specimen as a linear vibrating system and interpret $K_I(t)$ as a form of the specimen response to external excitation. For one-point bending conditions (only tup force $F(t)$ acts upon the specimen) one can use the following formula in order to compute DSIF

$$K_I(t) = \int\limits_0^t F(\tau) h_K^{(1)}(t - \tau) \, d\tau \qquad (2.1)$$

where $h_K^{(1)}(t)$ is the DSIF-response of the unsupported specimen to unit impulse from the tup (cf Andreikiv and Rokach, 1989; Rokach, 1990a,b). This formula can also be used in three-point bending conditions replacing $h_K^{(1)}(t)$ by $h_K^{(3)}(t)$ (DSIF-response of the supported specimen) and assuming that the specimen is in continuous contact with the anvil (cf Kishimoto et al., 1980 and 1984). However, in such an approach the specimen "bouncing" effect is not taken into account, therefore, results obtained are not precise. DSIF can be calculated more exactly when the anvil load $R(t)$ is recorded during test and

the following formula is used

$$K_I(t) = \int_0^t F(\tau) h_K^{(1)}(t - \tau) \, d\tau + \int_0^t R(\tau) h_K^{(2)}(t - \tau) \, d\tau \qquad (2.2)$$

where $h_K^{(2)}(t)$ is the DSIF-response of the specimen to two simultaneous unit impulses from the supports.

Eqs (2.1) and (2.2) can be transformed into more convenient for calculations forms taking into account specific methods of approximation of the recorded loading. For piece-wise linear and the Fourier series approximations of $F(t)$ and $R(t)$ the corresponding formulae are so simple that require a pocket computer for evaluation of $K_I(t)$ only (cf Andreikiv and Rokach, 1989; Rokach, 1990a,b).

It can be shown that the only difference between all existing simplified procedures of DSIF determination is a method and/or accuracy of deriving of $h_K^{(i)}(t)$ $i = 1, 2, 3$ functions. As a rule, these functions are expanded into series with respect to the vibrations eigenforms of the specimen model and only a few first terms of the series are used in calculations. When specimen is modelled within a framework of plane linear elasticity the most general form of DSIF-response functions is (Rokach, 1990a)

$$h_K^{(i)}(t) = k_s^{(i)} \sum_j \eta_j^{(i)} \omega_j^{(i)} \sin(\omega_j^{(i)} t) \qquad (2.3)$$

where $\omega_j^{(i)}$ is the jth eigenfrequency of the specimen model ($i = 1, 2$ corresponds to an unsupported specimen, $i = 3$ corresponds to a supported one), $k_s^{(i)}$ are SIF values for the following static loading conditions: one-point bending ($i = 1$, see Fig.1a) – specimen is loaded by a unit uniformly distributed forces, two-point bending ($i = 2$, see Fig.1b) – of the same form but twice greater loading, and conventional three-point bending by a unit concentrated force ($i = 3$, see Fig.1c). For each of the above-mentioned cases of static loading displacements of the specimen can also be expanded into series with respect to corresponding eigenmodes. These expansions are valid for any point of specimen including the points situated near the crack tip. Therefore, $k_s^{(i)}$ can be expanded into similar series of SIFs corresponding to normalized eigenmodes too. Weight coefficients that are proportional to the contribution of each normalized mode into $k_s^{(i)}$ are denoted by $\eta_j^{(i)}$ ($\sum_j \eta_j^{(i)} \equiv 1$) in Eqs (2.3) (see Rokach (1992) for details).
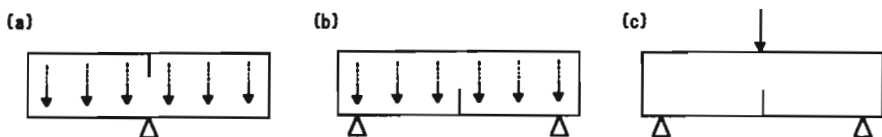
(a)   (b)   (c)

Fig. 1. Schemes of cracked specimen loading: one- (a), two- (b) and
three-point-bending (c)

Values of $k_s^{(1)}$, $\omega_j^{(1)}$, $\eta_j^{(1)}$, $\eta_j^{(2)}$ (for $j = 1, 2, 3$) have been determined, for a wide range of relative specimen and crack lengths and values of the Poisson ratio, by finite element analysis for 2D (plane stress) model of specimen. The numerical results were later fitted by polynomials (cf Andreikiv and Rokach, 1989; Rokach, 1990a,b). For the Euler-Bernoulli beam model of specimen, where the crack is modelled by elastic hinge, located in the midspan, the frequency equations for determination of $\omega_j^{(1)}$, $\omega_j^{(3)}$ have been derived (cf Andreikiv and Rokach, 1989; Kishimoto et al., 1980, 1984 and 1990) and relations for $\eta_j^{(1)}$, $\eta_j^{(2)}$ obtained (Andreikiv and Rokach, 1989).

3.   Experimental data processing

In order to perform calculations using Eqs (2.1) ÷ (2.3) the following parameters must be determined

– Number $N^{(i)}$, $i = 1, 2$ of terms of the series which are taken into account

– Values of eigenfrequences $\omega_j^{(i)}$ and weight coefficients $\eta_j^{(i)}$ ($j = 1, N^{(i)}$)

– SIF values $k_s^{(i)}$.

Applying the finite element analysis to 2D model of the specimen makes it possible to evaluate these parameters with high accuracy. However, the following simplifications of the formulae (2.3) as well as methods to determine their parameters can also be introduced

• Taking into account only a few lowest modes of vibration. When single mode approach ($N^{(i)} = 1$) is used, true values of $\eta_j^{(1)}$ are often neglected assuming $\eta_j^{(1)} \equiv 1$

- Using simpler than the 2D models of specimen (e.g., one dimensional beam) (cf Andreikiv and Rokach, 1989; Kishimoto et al., 1980, 1984 and 1990). This assumption considerably simplifies determination of $\omega_j^{(i)}$ and $\eta_j^{(i)}$ but effects in essential errors in their values

- Calculation of $k_s^{(1)}$ and $k_s^{(2)}$ using known formulae for three-point static bending of the specimen with the standard normalized span $S/W = 4$, where S is the span length, W is the width of the specimen. In this case the equivalent load has to be chosen in order to obtain the value of bending moment in the midspan equal to the corresponding values for one- and two-point bending schemes. Thus we arrive at

$$k_s^{(1)} \simeq k_s^{(3)} \frac{L}{2S} \tag{3.1}$$

$$k_s^{(2)} = 2(k_s^{(3)} - k_s^{(1)}) \simeq k_s^{(3)}\left(2 - \frac{L}{S}\right) \tag{3.2}$$

where L is the specimen length. It was found by Andreikiv and Rokach (1989) that for the most important for practical applications values of the relative specimen length $L/W > 4$ and relative crack length $\alpha/W = 0.45 \div 0.55$ the accuracy of these formulae is better than 9%. But it becomes worse with decreasing value of L/W ratio.

Let us analyze the consequences of those simplifications by comparing corresponding numerical and experimental results. The latter have been obtained during testing large-scale Araldite B specimens by the method of caustics (Böhme, 1985).

In Fig.2a the piece-wise linear approximation of the load-time diagram of the tup force recorded during one-point bend test of the specimen with $L = 0.412$ m, $W = 0.1$ m, and crack length $\alpha = 0.03$ m, is presented. Numerical results for the Euler-Bernoulli beam model of specimen, using this diagram as an external excitation, are compared with experimental ones in Fig.2b. Lines 2 and 3 represent the most complete solution for the beam model (i.e. $k_s^{(1)}$ is obtained from 2D static finite element analysis, values of $\omega_j^{(1)}$ and $\eta_j^{(1)}$ are determined without any simplifications) (cf Andreikiv and Rokach, 1989). They differ only in number of vibration modes that were taken into account. It effects in differences in DSIF values corresponding to early stage of the specimen deformation.

Less precise approach to the same type of specimen model has been proposed by Kishimoto et al. (1990). Using approximation (3.1) for $k_s^{(1)}$ and
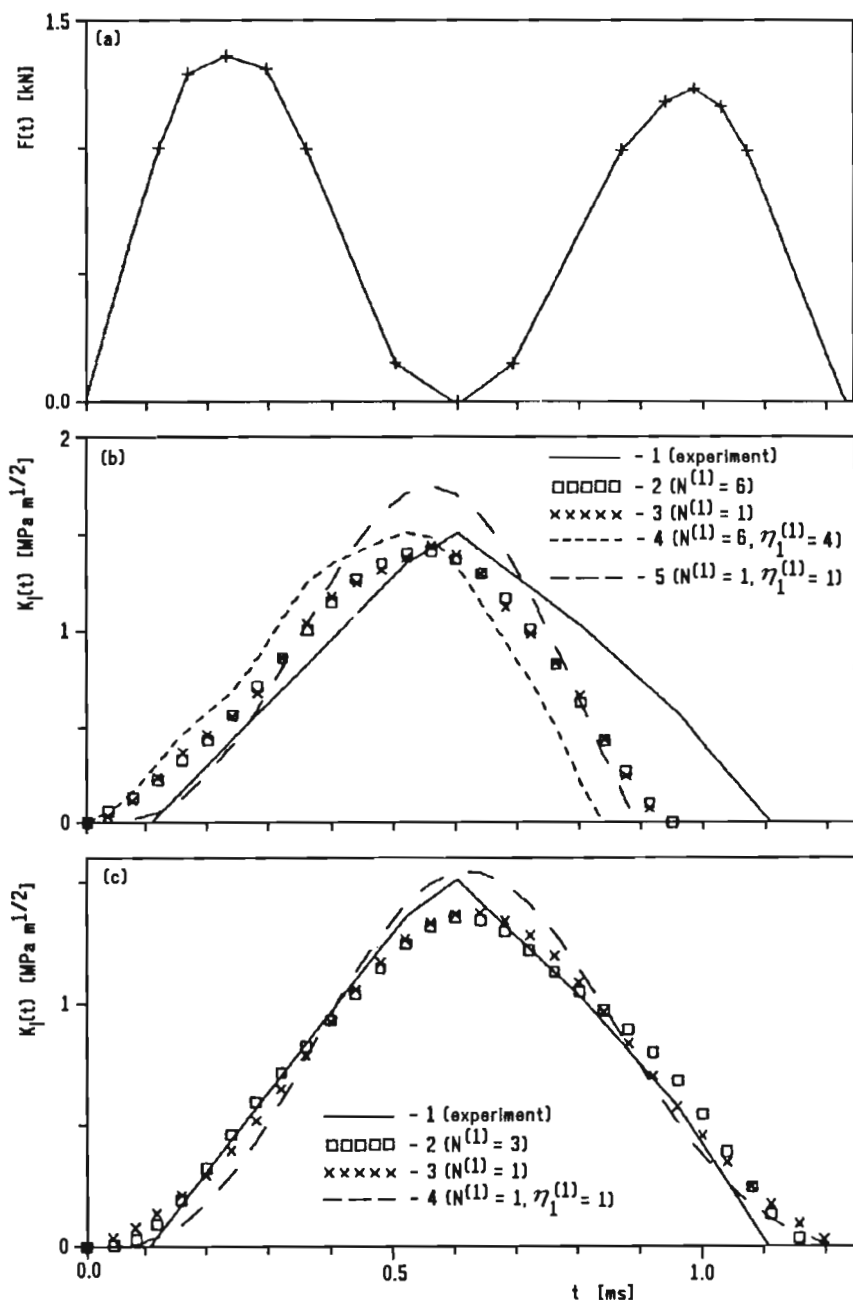
Fig. 2. (a) – force-time diagram recorded during one-point bend test (cf Böhme, 1985); time variations of DSIF obtained experimentally and numerically for: (b) the beam model of specimen, (c) the 2D model of specimen
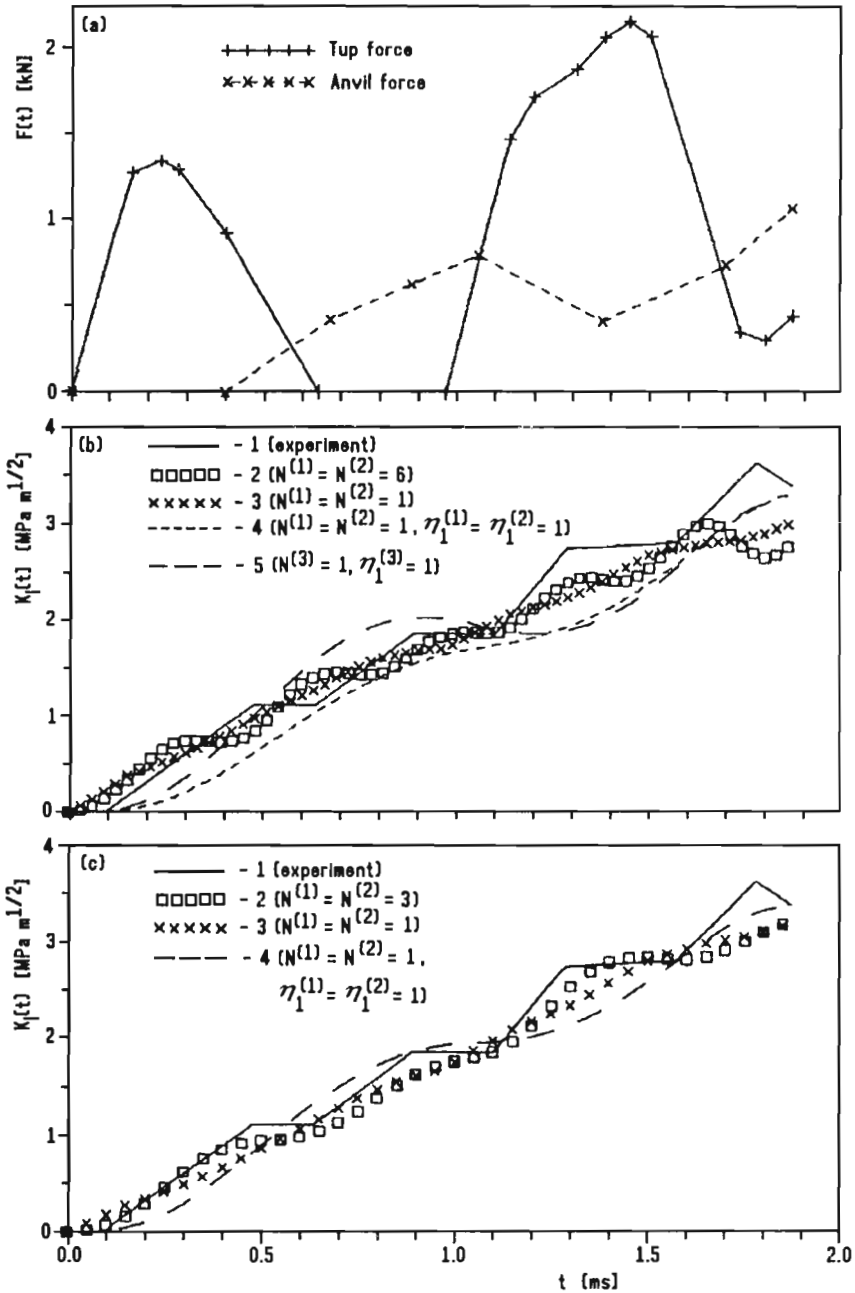
Fig. 3. (a) – force-time diagram recorded during three-point bend test (cf Böhme, 1985); time variations of DSIF obtained experimentally and numerically for: (b) the beam model of specimen, (c) the 2D model of specimen

the Tada's formula of a poor accuracy for the elastic hinge compliance cau-
ses the increase in discrepancy between the corresponding numerical (line 4)
and experimental data. The worst results were obtained with a single mode
approach with assumption that $\eta_1^{(1)} \equiv 1$ (line 5).

Unfortunately all beam models ignore the fact that the value of specimen
width is finite, therefore, the time delay between the beginning of loading
signal and the beginning of DSIF growth is ignored too. This is the main
reason for discrepancy between numerical and experimental data in Fig.2b.

When the 2D model of specimen is used the results obtained are sufficiently
close to experimental data (Fig.2c). Influence of the number of modes that
were taken into account is again important for the early stage of DSIF growth
only (line 2 and 3). Single mode approach with $\eta_1^{(1)} \equiv 1$ effects in a noticeable
drop in accuracy of initial DSIF values (line 4).

In Fig.3a time variations of tup and anvil forces for three-point bend test
are presented. The specimen geometry was as follows: $L = 0.55$ m, $W = 0.1$
m, $a = 0.03$ m. Comparison of the experimental DSIF-time curve with nume-
rical results (Fig.3b) shows an improvement of accuracy of the beam model
with increasing relative length of a specimen. The most complete solution to
the problem (lines 2 and 3) gives reasonable but rather "smoothed" (especially
for a single mode approach) information about DSIF growth. Assumption that
$\eta_1^{(1)} = \eta_1^{(2)} \equiv 1$ leads to noticeable distortions in results (line 4). Additional
assumption that specimen is in continuous contact with the anvil during the
test causes underestimation of DSIF (line 5).

Results obtained for 2D model of specimen (Fig.3c) better represent the
oscillations of experimental $K_I(t)$ curve. The best correlation is achieved
when the first three specimen vibration modes are taken into account (line 2).

Because normalized specimen length is sufficiently large there is only a few
per cent difference between $k_s^{(1)}$ and $k_s^{(2)}$ and their approximations given
by the relations (3.1) and (3.2), respectively. Thus, such a replacement has
practically no influence on the results of calculations.

Single mode approximation which neglect true $\eta_j^{(1)}$ and $\eta_j^{(2)}$ values leads
to a noticeable discrepancy between the theoretical and experimental results
again (line 4).

4.   Conclusions

It was shown that the modal decomposition method is cheep and sufficien-

tly accurate for evaluation of DSIF from impact testing data. The method can be applied to specimen models with different levels of sophistication without any essential changes in procedure of calculations. The best results were obtained using the most powerful 2D multi-modal model of specimen. Our analysis suggests that the following consequences of simplification of this model appear:

- Using the one-dimensional Euler-Bernoulli model of specimen leads to less precise results than those for the 2D model. Although this difference decreases with increasing of the specimen relative length (cf Rokach, 1992), application of the beam model is not recommended even in this case for the following reason. The results of a test may be accepted if DSIF deviations from mean linear increase line are sufficiently small. Results obtained for the beam model usually underestimate these deviations representing rather smoothed approximation of DSIF-time curve than the real one. Therefore, it is difficult to check the validity of a test using such results.

- The single mode approach for both specimen models leads to smoothing of results too. It is acceptable with caution together with the 2D model for one-point bend data processing only, because for this case DSIF growths almost linearly.

- Determination of $k_s^{(1)}$ and $k_s^{(2)}$ using the approximate formulae (3.1) and (3.2) leads to large errors only for specimens with low relative length.

- The highest level of simplification of the formulae (2.3) (single mode approach together with neglecting true $\eta_1^{(1)}$ and $\eta_1^{(2)}$ values) effects in noticeable errors.

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Porównanie uproszczonych metod wyznaczenia dynamicznego współczynnika intensywności naprężeń

Streszczenie

W pracy są rozpatrywane metody numeryczne wyznaczenia dynamicznego współczynnika intensywności naprężeń oparte na dekompozycji modalnej. Metody te zastosowano do obliczania wyników prób na zginanie udarowe zamiast zwykłego lecz kontrowersyjnego podejścia quasi-statycznego. Bada się wpływ typu modelu próbki oraz metody wyznaczania parametrów tego modelu na dokładność otrzymywanych rozwiązań. Rezultaty obliczeń są porównywane ze wziętymi z literatury wynikami eksperymentów na zginanie jedno- i trójpunktowe.