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SHADOW OPTICAL METHOD OF CAUSTICS: EXPERIMENTAL SETUP, COMPUTER SIMULATION, CONSTRAINTS

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The shadow optical method of caustics has been presented. Presentation includes the theoretical background, experimental setup as well as computer simulation of the laser ray from laser through the specimen to the screen. Finally certain constraints of the method have been discussed based on an analysis of the experimental results.

1. Introduction

The optical method of caustics which is also known as the shadow spot method has been used by a number of investigators to determine stress intensity factors in crack problems under static and dynamic loading conditions. The method was invented by Manogg (1964) and later developed by Theocaris and Joakimides (1971), Theocaris (1981), Kalthoff (1987), Rosakis end Freund (1982). According to the author's knowledge in Poland this method has not been used yet. Therefore, in this article we will present the first in our country experimental setup to investigate the stress concentrations in front of the cracks, notches or other defects. Moreover, the results of the computer simulation of theoretical caustics, created by the laser ray on the screen and passing from the laser through deformed specimen will be presented. Finally, results of the experiments will be discussed from the method constraints point of view.

2. Short theoretical background

Stresses alter the optical properties of solids. They change the thickness of the body, due to the Poisson effect and refractive index of a material. These changes are utilized in the method of caustics to measure and compute the stress distribution in the vicinity of the crack tip. Basic physical principle of the method is demonstrated in the Fig.1. Mode I is shown, but the similar behavior is observed for Mode II and III.

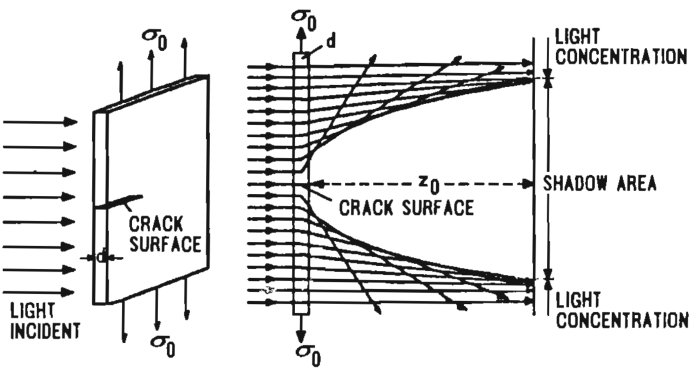

Fig. 1. The scheme of the light rays passing the specimen in front of the stress concentrators

The domain lying in the crack neighborhood acts as a concave mirror for reflected rays or as a divergent lens for transmitted rays. This is due to the fact that the specimen thickness and the refractive index of the material are steadily reduced when the crack tip is approached. As a result of these phenomena a shadow area appears on the screen located at a distance z_0 from the specimen. This shadow area is surrounded by a sharp boundary line between dark area and surrounding areas, of a light concentration. The sharp boundary line is a mapping of the specimen plane onto a screen. The corresponding boundary line on the object plane is called the initial curve. In this article we will concentrate on a Mode I loading case and transparent materials. More general analysis is given by Kalthoff (1987). Each point $P(r)$ along the initial curve is located at a distance r from the crack tip and is mapped onto the point $P'(r')$ according to the formula

$$\boldsymbol{r}' = \boldsymbol{r} + \boldsymbol{w} \tag{2.1}$$

where

$$\boldsymbol{w} = -z_0 \text{gradient} \Delta s(r, \phi) \tag{2.2}$$

where z_0 is a distance between the screen and the specimen and Δs is a change of the distorted wave front with respect to an equivalent wave front that did not pass through the specimen

$$\Delta s_{1,2} = (n-1)\Delta d_{\rm eff} + d_{\rm eff}\Delta n_{1,2} \tag{2.3}$$

where n is refractive index and $n = 1$, $d_{\rm eff} = d$ for transition (through the thickness of the specimen) and $d_{\rm eff} = d/2$, $n = -1$ for reflection. According to the Maxwell-Neumann's law

$$\begin{aligned} \Delta n_1 &= A\sigma_1 + B(\sigma_2 + \sigma_3) \\ \Delta n_2 &= A\sigma_2 + B(\sigma_1 + \sigma_3) \end{aligned} \tag{2.4}$$

where A, B are material constants. For optically isotropic, non-birefringent materials $A = B$ (for reflection $A = B = 0$). The changes in the specimen thickness can be computed from the Hook's law

$$\Delta d_{\rm eff} = \Big[\frac{1}{E}\sigma_3 - \frac{\nu}{E}(\sigma_1 + \sigma_2)\Big] d_{\rm eff} \tag{2.5}$$

Introducing Eqs (2.5), (2.4) into Eq (2.3) one obtains

$$\Delta s_{1,2} = c d_{\rm eff}[(\sigma_1 + \sigma_2) \pm \lambda(\sigma_1 - \sigma_2)] \tag{2.6}$$

with

— for plane stress

$$c = \frac{A+B}{2} - \frac{(n-1)\nu}{E} \qquad\qquad \lambda = \frac{A-B}{A+B-\frac{2(n-1)\nu}{E}}$$

— for plne strain

$$c = \frac{A+B}{2} + \nu B \qquad\qquad \lambda = \frac{A-B}{A+B+2\nu B}$$

the constant c is called "shadow optical constant". The values of A, B, c, λ, n are given for various materials e.g. by Kalthoff (1987). When Eq (2.6) is substituted to Eqs (2.2) and (2.1) and values of σ_1, σ_2, σ_3, are taken in the form of well known formulas for stresses in front of the crack tip, mapping equations can be rewritten in term of the Stress Intensity Factors (SIF). Consequently they are used to compute the Jacobian that vanishes leading to the equation of the initial curve

$$r = \Big[\frac{3}{2}\frac{K_I}{\sqrt{2\pi}}|z_0||c|d_{\rm eff}\Big]^{\frac{2}{5}} \tag{2.7}$$

The shape of initial curve is similar to a circle (in fact it forms an epicycloid) and the maximum dimension of a curve is given by the formula

$$D = 3.17r_0$$

Thus

$$K_I = \frac{-2\sqrt{2}\pi}{3(3.17)^{\frac{5}{2}} z_0 c d_{\text{eff}} m^{\frac{3}{2}}} D^{\frac{5}{2}} \tag{2.8}$$

3. Experimental arrangement

To measure the stress intensity factor from the formula (2.8) one must know optical and elastic constants for a given material (shadow optical constant) i.e. distance between screen and specimen z_0, thickness of the specimen d_{eff} and the maximum dimension of caustics curve D. Parameter m characterizes optical features of the optical arrangement – for parallel a light beam $m = 1$, for a divergent beam $m > 1$ and for a convergent beam $m < 1$, respectively.

The experimental arrangement was designed and built at Kielce University of Technology. It is shown in Fig.2.

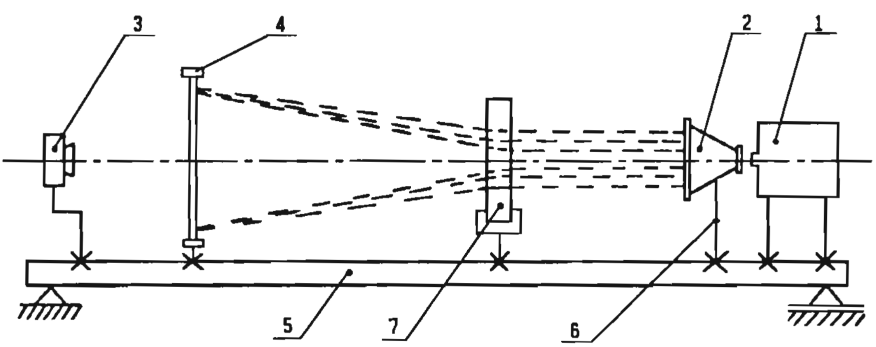

Fig. 2. Optical arrangement

It consists of:

1. laser LHN-15 (15 mV) with the diameter of a light beam 1.5 mm
2. beam expander – magnification 20×, a light beam diameter 30 mm – it can produce parallel, divergent or convergent light beams, respectively

3. camera
4. screen
5. optical bench
6. fixtures
7. specimen.

This arrangement can be used to test specimens made of transparent or opaque materials.

4. Computer simulation

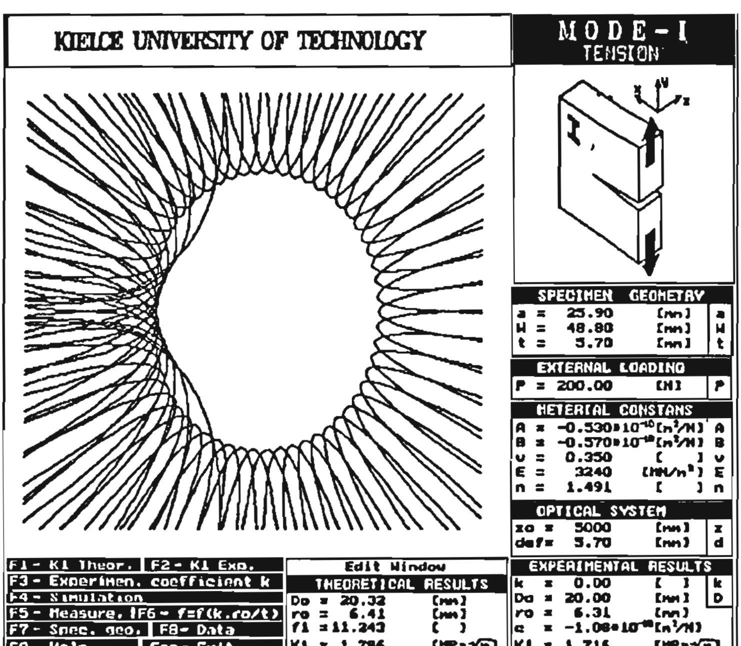

Fig. 3. Computer simulation of the caustics curve (Mode I)

The aim of this computer program is to demonstrate the shape of caustics curve for all three modes of loading and to compute SIF for given D. With this program we can easily compare the experimental and theoretical results. The computer program is very friendly and it allows for geometry and size of the specimen changes as well as both external force and optical arrangements. It contains tables of material and optical constants. It is also very useful in a teaching process.

The computer program is a result of computer simulation of light rays way from the source (laser) to the screen, taking into account all processes within a deformed material. An example screen is shown in the Fig.3.

5. Experimental results, analysis of the method constraints

First experimental results (with PMMA) showed that not always the agreement between theory and experiment is satisfactory. In fact the distance z_0 can not be absolutely arbitrary. Outside a certain range of this distance the contour of caustics is not sharp enough to be measured with a sufficient accuracy. The higher SIF is the shorter z_0 can be selected. For very small SIF a discrepancy between theory and experiment become large. For example: if the ratio between SIF and its critical value is smaller than 0.22 the difference between experiment and theory exceeds 28 per cent for tested specimens. For high values of SIF this difference becomes negligibly small. In this paragraph we will try to explain this observations.

The most important remark that should be made first is that method of caustics has been introduced for a plane stress situation. Thus, the initial curve should be located within a domain where the plane stress dominates. However, the stress situation in vicinity of the crack tip is much more complex. Just at the crack tip we observe a plane strain situation that changes to three-dimensional state of stress and strain when the distance from a crack tip increases, finally at a certain distance we can assume that the plane stress dominates. From numerical investigation of Levy et al. (1971), and experimental results of Rosakis and Ravi-Chander (1986) it is known that one may expect a plane stress situation at the distance grater than $0.5d$ (d – thickness of the specimen). The value of $0.5d$ must be considered as an approximate one. Gdoutos et al. (1992) tested this condition for reflected laser rays (non-transparent material) and found that this value depends on a particular thickness of specimen. However, if we assume, at the time being, that the condition $r_0 > 0.5d$ holds, the following inequality may be obtained

$$r_0 = \Big(\frac{3.384 z_0 c K_I}{m}\Big)^{\frac{2}{3}} > d \tag{5.1}$$

The above condition will be tested in this article for transparent materials. Nevertheless we have already found the origins of discrepancies between theory and experiment mentioned above. For small K_I and given thickness of the specimen the plane stress state was violated.

To characterize three-dimensional state of stresses and strains in front of the crack tip we introduce the coefficient of three-dimensionality k

$$\sigma_1 = k\nu(\sigma_1 + \sigma_2) \qquad\qquad 0 \leq k \leq 1 \tag{5.2}$$

For a plane strain $k = 1$, for a plane stress $k = 0$. With this assumption the relation (2.3) can be rewritten in the form

$$\Delta s_{1,2} = d\sigma_{1,2}\Big[A - (n-1)(k-1)\frac{\nu}{E} + Bk\nu\Big] + d\sigma_{2,1}\Big[B - (n-1)(k-1)\frac{\nu}{E} + Ak\nu\Big] \tag{5.3}$$

The change of light rays way while crossing the specimen thickness can be expressed by the formula

$$\Delta s_f = c_f(\sigma_1 + \sigma_2)d \tag{5.4}$$

where

$$c_f = \frac{A+B}{2} + \nu Bk - \frac{(n-1)(1-k)\nu}{E} \tag{5.5}$$

and the relation for SIF measured experimentally using a transparent material can be written in the form

$$K_{Idos} = \frac{0.0934 D^{\frac{5}{2}}}{c_f z_0 d m^{\frac{3}{2}}} \tag{5.6}$$

If for a given geometry we adopt the formula for SIF (K_{Iteo}) (cf e.g. Murakami, 1986) the value of k can be computed from the relation

$$K_{Idos} = K_{Iteo} \tag{5.7}$$

The similar approach was used by Gdoutos et al. (1992) but for reflected rays. In this case Eqs (5.5) and (5.6) are different.

In order to test a three dimensional effect in front of the crack tip the transparent material PMMA was selected. Geometrical parameters of the specimens were as follows.

	crack length a [mm]	thickness d [mm]	W [mm]
1	25.66	2.95	50.01
2	25.89	5.70	48.81
3	25.40	10.9	50.10

Using Eq (5.7) and changing external loading and z_0, respectively, the curves k vs. r_0/d were plotted for different thicknesses of specimens. They are shown in Fig. 4, 5 and 6. In Fig.7 the relation between specimen thickness and r_0/d was plotted for a situation when the plane stress was reached. From this plot one can notice that the plane stress is reached not necessarily at the distance from the crack tip equal to $0.5d$. For thinner specimens this distance is longer than $0.5d$ and for thick specimens is shorter.

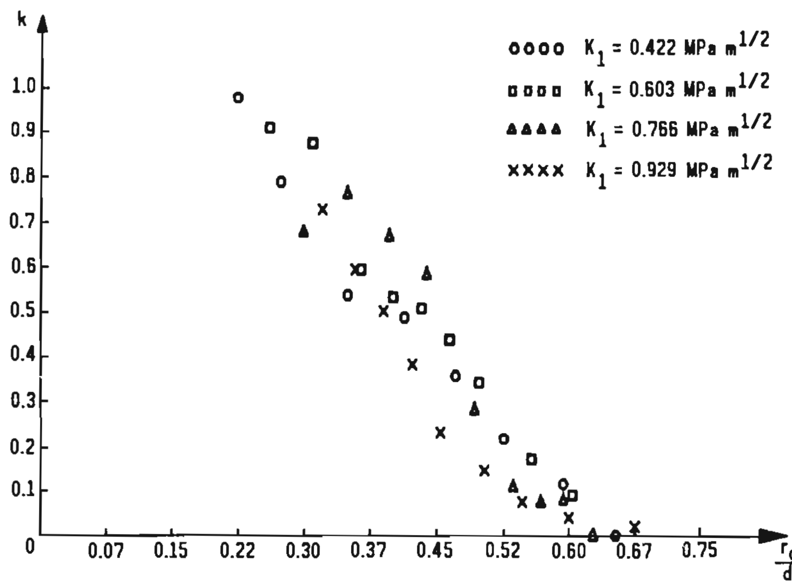

Fig. 4. Correction coefficient k vs. r_0/d for $d = 2.95$ mm

In many practical cases the plane stress is unreachable, thus in these cases curves like in Fig. 4, 5 and 6 might be helpful to introduce an experimental correction factor k to measure more realistic SIF according to Eqs (5.6) and (5.5).

One remark must be made here. In Gdoutos et al. (1992) analysis for reflected rays the value of k when computed from the relation similar to Eq (5.7) has always greater than 0 and smaller than 1 independently of experimental conditions. In our case it is not true. For small external loading, when K is smaller than $0.4K_{Ic}$ the value of k is greater than 1 if we use Eq (5.7) to compute it. It is "unphysical" result and at the time being we are not able to explain it. We simply assume that for this case the plane strain is reached in the region for which k is grater than 1.

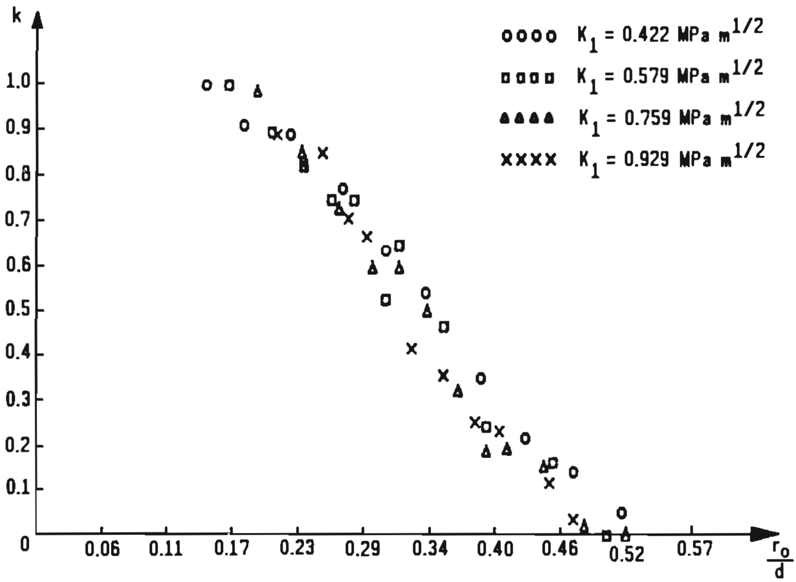

Fig. 5. Correction coefficient k vs. r_0/d for $d = 5.70$ mm

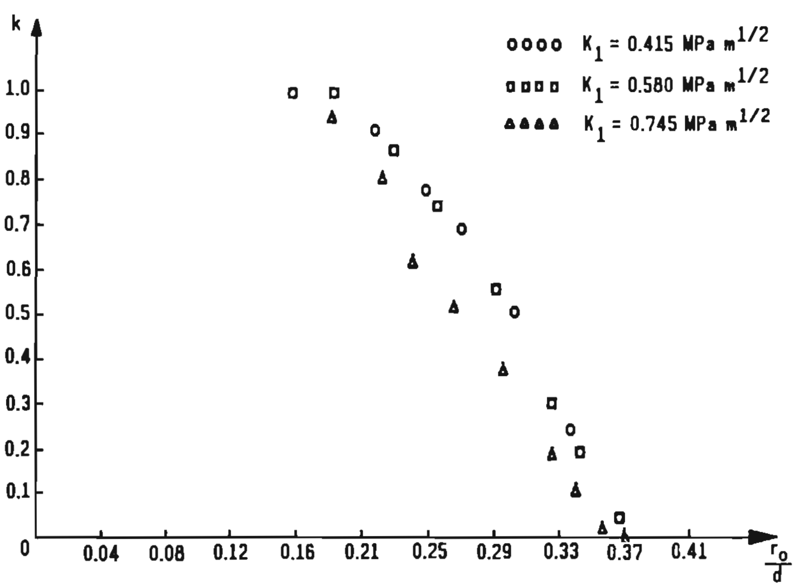

Fig. 6. Correction coefficient k vs. r_0/d for $d = 10.90$ mm

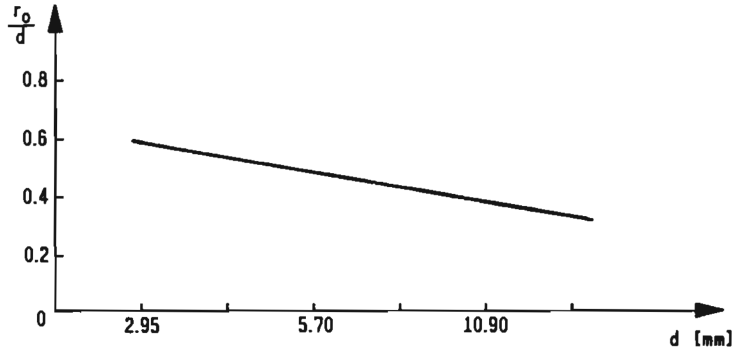

Fig. 7. Relation between thickness d and initial curve radius r_0/d for a moment when the plane stress is reached

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Metoda caustics: stanowisko, symulacja komputrowa, rezultaty doświadczeń dla I sposobu obciążenia

Streszczenie

W pracy dokonano przeglądu metody caustics. Przedstawiono podstawy fizyczne, przeprowadzono analizę matematyczną oraz pokazano sposób pomiaru współczynnika intensywności naprężenia K_I (I – sposób obciążenia) dla materiałów przepuszczających światło.

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