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FATIGUE LIFE OF STEEL UNDER SYNCHRONOUS AND ASYNCHRONOUS COMBINED BENDING AND TORSION WITH VARIABLE AMPLITUDES ACCORDING TO CRITERIA OF BIAXIAL RANDOM FATIGUE

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Fatigue lives, calculated according to three criteria of multiaxial random fatigue, were compared with lives obtained from tests of cylindrical specimens made of 30CrNiMo8 steel and subjected to synchronous and asynchronous combined bending and torsion with variable amplitudes. The expected fatigue fracture plane positions were determined with the aid of variance method. The equivalent stress histories were schematized with the rain flow method and fatigue damages were cumulated according to the Palmgren-Miner hypothesis.

1. Introduction

Some criteria of multiaxial cyclic fatigue were applied to correlation of experimental data obtained for cylindrical specimens made of 30CrNiMo8 steel, subjected to synchronous and asynchronous combined bending and torsion with variable amplitudes (Sanetra, 1991). Unfortunately, the results obtained are rather poor. Thus, it seems to be necessary to search for new fatigue criteria.

In this paper the authors have analysed experimental data given by Sanetra (1991) once again, using three linear criteria of multiaxial random fatigue and they have tried to choose the criterion leading to the most realistic estimations of a fatigue life. The analysis is carried out by numerical simulation in which histories of loadings with variable amplitudes are generated in the same way as in specimens on the test stand.

2. The aim and course of fatigue tests

Cylindrical specimens made of 30CrNiMo8 steel were tested. The following constants of the S-N curve were determined for cyclic:

- bending $(\tau_a = 0)$ $\sigma_{af} = 549$ MPa $N_0 = 5 \cdot 10^5$ $m = 11.04$
- torsion $(\sigma_a = 0)$ $\tau_{af} = 370$ MPa $N_0 = 3 \cdot 10^6$ $m = 28.80$

The analysed program of fatigue tests under variable amplitudes included bending $(\widehat{\tau}_a = 0)$, three combinations of in-phase $(\varphi = 0)$ bending with torsion $(\widehat{\tau}_a = 0.19\widehat{\sigma}_a,\ \widehat{\tau}_a = 0.50\widehat{\sigma}_a,\ \widehat{\tau}_a = 0.73\widehat{\sigma}_a)$, torsion $(\widehat{\sigma}_a = 0)$ and two-phase displacements under combined bending and torsion $(\widehat{\tau}_a = 0.5\widehat{\sigma}_a$ and $\varphi = \pi/2$, $\widehat{\tau}_a = 0.5\widehat{\sigma}_a$ and $\varphi = 5 \cdot 2\pi)$.

The standardized histories of fatigue loadings were generated with the matrix method and a special computer program presented by Huck et al. (1976). The following parameters of generation were assumed:

- Frequency of generation $f = 5$ Hz
- Time of observation $T_0 = 200$ s
- Irregularity factor $I = 0.99$
- Mean values $\sigma_m = \tau_m = 0$
- Crest factor $\widehat{\sigma}_a/\sigma_{RMS} = \widehat{\tau}_a/\tau_{RMS} = 5.26$
- Number of cycles in the block $H_0 = 10^6$.

An example of the generated histories of stresses is shown in Fig.1.

3. Algorithm for fatigue life estimation

The block diagram of algorithm for fatigue life estimation is shown in Fig.2.

In Block 1 standard histories of stresses $\sigma_{xx}(t)$ and $\tau_{xy}(t)$ are generated according to the subroutine presented by Huck et al. (1976).

In Block 2 the expected fatigue fracture plane position is determined with the method of equivalent stress variance (Będkowski and Macha, 1985). The equivalent stress was determined according to:

- The linearized criterion of ellipse quadrant – C1

$$\sigma_{eq}(t) = \sigma_{xx}(t)\cos\beta + \frac{\sigma_{af}}{\tau_{af}}\tau_{xy}(t)\sin\beta \tag{3.1}$$

where

$$\tan\beta = \frac{2\tau_a}{\sigma_a}\frac{\sigma_{af}}{2\tau_{af}} = \frac{\sigma_{af}}{2\tau_{af}}\tan 2\alpha = \text{const.}$$

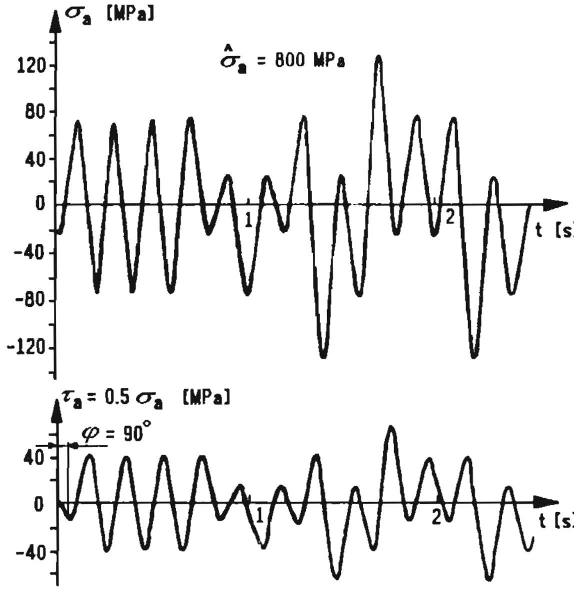

Fig. 1. Standarized stress histories generated in tests of out-of-phase $(\pi/2)$ bending with torsion under combination of the maximum amplitudes $\widehat{\tau}_a = 0.5\widehat{\sigma}_a$

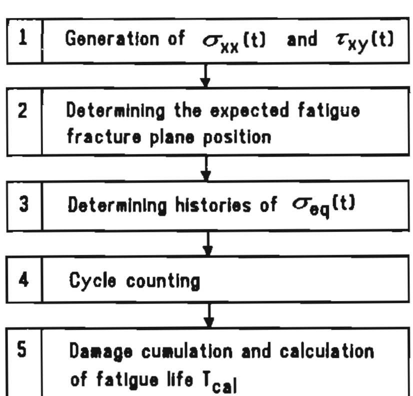

Fig. 2. Block diagram of the algorithm for calculations of fatigue life T_{cal}

α – angle between normal to the fracture plane and direction of the stress σ_a, overlapping axis x.

- Criterion of the maximum normal stress in the fracture plane (Macha, 1979) – C2

$$\sigma_{eq}(t) = \widehat{l}_1^2\sigma_{xx}(t) + 2\widehat{m}_1\widehat{l}_1\tau_{xy}(t) \tag{3.2}$$

where in the plane stress state

$$\widehat{l}_1 = \cos\alpha \qquad \widehat{m}_1 = \sin\alpha$$
$$\widehat{l}_3 = -\sin\alpha \qquad \widehat{m}_3 = \cos\alpha$$

- Criterion of the maximum shear and normal stresses in a fracture plane with a modified part of shear stresses – C3

$$\sigma_{eq}(t) = (\widehat{l}_1^2 - \widehat{l}_1\widehat{l}_3)\sigma_{xx}(t) + (2\widehat{m}_1\widehat{l}_1 - \widehat{l}_1\widehat{m}_3 - \widehat{m}_1\widehat{l}_3)\frac{\sigma_{af}}{\tau_{af}}\tau_{xy}(t) \tag{3.3}$$

Variance of the equivalent stress according to criteria C1 and C2 is

$$\mu_{\sigma_{eq}C1} = \mu_{11}\cos^2\beta + \Big(\frac{\sigma_{af}}{\tau_{af}}\Big)^2\mu_{22}\sin^2\beta + \frac{\sigma_{af}}{\tau_{af}}\mu_{12}\sin 2\beta \tag{3.4}$$

$$\mu_{\sigma_{eq}C2} = \widehat{l}_1^4\mu_{11} + 4\widehat{l}_1^2\widehat{m}_1^2\mu_{22} + 4\widehat{l}_1^3\widehat{m}_1\mu_{12} \tag{3.5}$$

Searching for maxima of variances $\mu_{\sigma_{eq}C1}$ and $\mu_{\sigma_{eq}C2}$ consisted in calculating the values of quantities given in Eqs (3.4) and (3.5) for all the considered combinations of bending and torsion at a step $\Delta\alpha = \pi/180$ in the interval $0 \div \pi$. The value of α, for which the equivalent stress variance reaches its maximum, determines the expected fatigue fracture plane position. For criterion C3 the expected fatigue fracture plane position is assumed to be the same as that for C2. The values of α calculated with the variance method are shown in Table 1. For comparison, the values of α obtained from experiments are presented as well. From Table 1 it results that the variance method gives a good agreement between theoretical and experimental results.

Table 1. Calculated and experimental values of α determining the expected fatigue fracture plane position

Combination of stresses		α according to (3.4) and (3.5)	α from experiment	Fig.
A	$\hat{\tau}_a = 0$	$0° \pm 1°$	$0°$	3
B	$\hat{\tau}_a = 0.19\hat{\sigma}_a$	$10° \pm 1°$	$10°$	4
C	$\hat{\tau}_a = 0.50\hat{\sigma}_a$	$22° \pm 1°$	$25°$	5
D	$\hat{\tau}_a = 0.73\hat{\sigma}_a$	$28° \pm 1°$	$33°$	6
E	$\hat{\sigma}_a = 0$	$45° \pm 1°$	$45°$	7
F	$\hat{\tau}_a = 0.50\hat{\sigma}_a,\ \varphi = \pi/2$	$3° \pm 1°$	$0°$	8
G	$\hat{\tau}_a = 0.50\hat{\sigma}_a,\ \varphi = 5 \cdot 2\pi$	$22° \pm 1°$	$25°$	9

In Block 3 histories of the equivalent stress are calculated according to criteria C1, C2, C3 for combinations of stresses A $\div$ G and stress levels applied to experiments (Sanetra, 1991).

In Block 4 histories of stresses $\sigma_{eq}(t)$ are schematized with the rain flow method and a histogram of cycle and half-cycle amplitudes is worked out.

In Block 5 fatigue damages are cumulated according to the Palmgren-Miner hypothesis.

4. Analysis of calculation results

In Fig.3 $\div$ Fig.9 the fatigue lives calculated according to criteria C1 $\div$ C3 are compared with the experimental data. The figures show also scatter bands of the results with the factor of two ($T_{cal}/T_{exp} = 2$ and $1/2$) and three ($T_{cal}/T_{exp} = 3$ and $1/3$), respectively. For combination of stresses including bending (A), synchronous bending and torsion (B $\div$ D) and torsion (E) in the scatter band with the factor of 2 criterion C1 is effective; for the factor of 3 also criterion C3 is effective. For combination of out-of-phase bending with torsion (F, G) and the factor of 3, criterion C3 is effective. Thus, for the factor of 3 criterion C3 can be applied to description of all combinations of stresses (A $\div$ G) and for factor of 2 it can be used for description of five combinations (A, C $\div$ F).

Criterion C1 can be used for six combinations of stresses (apart from F) in the scatter band with the factor of 3; for the factor of 2 it gives estimations for five combinations (A $\div$ E).

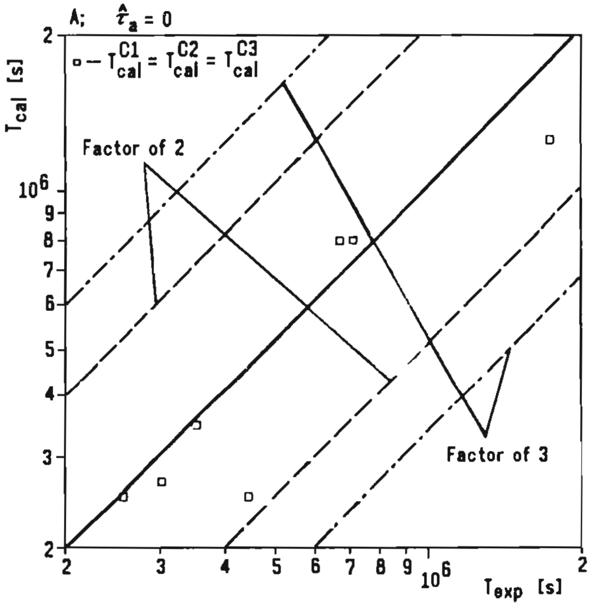

Fig. 3. Comparison of calculated fatigue lives according to criteria C1 ÷ C3 with experimental fatigue lives

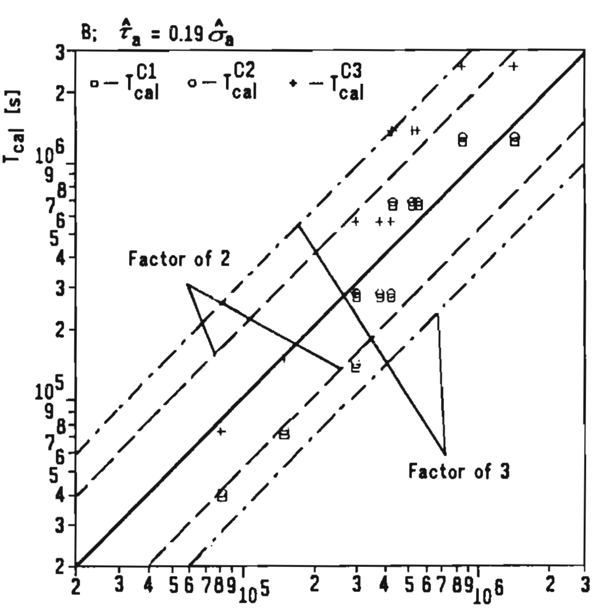

Fig. 4. Comparison of calculated fatigue lives according to criteria C1 ÷ C3 with experimental fatigue lives

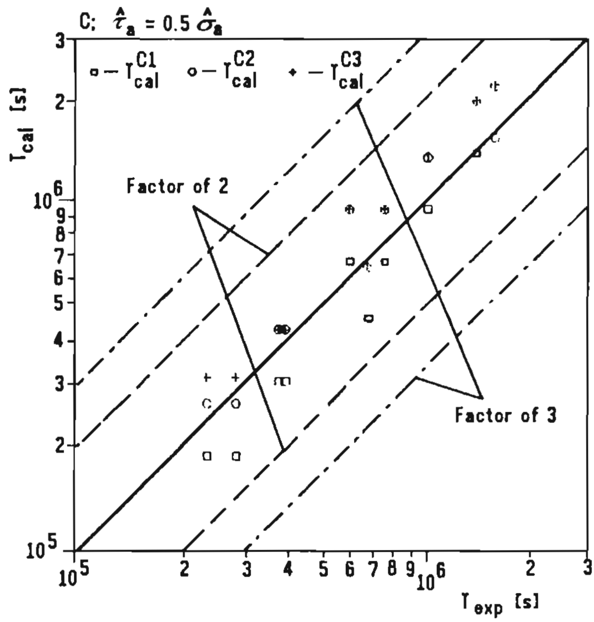

Fig. 5. Comparison of calculated fatigue lives according to criteria C1 ÷ C3 with experimental fatigue lives

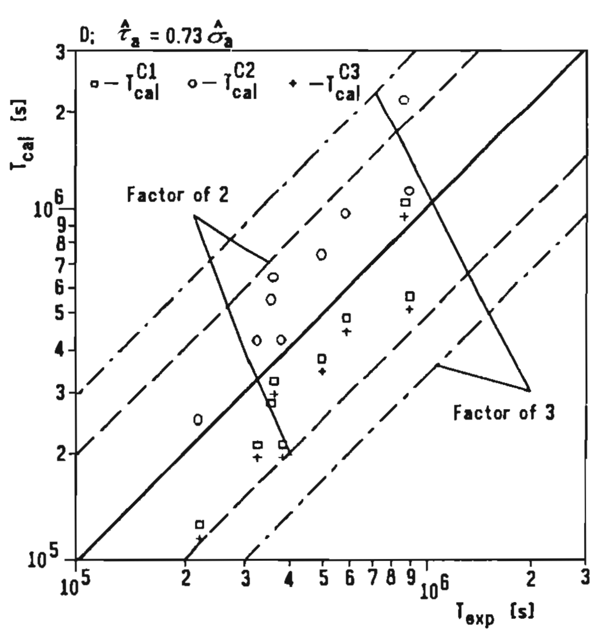

Fig. 6. Comparison of calculated fatigue lives according to criteria C1 ÷ C3 with experimental fatigue lives

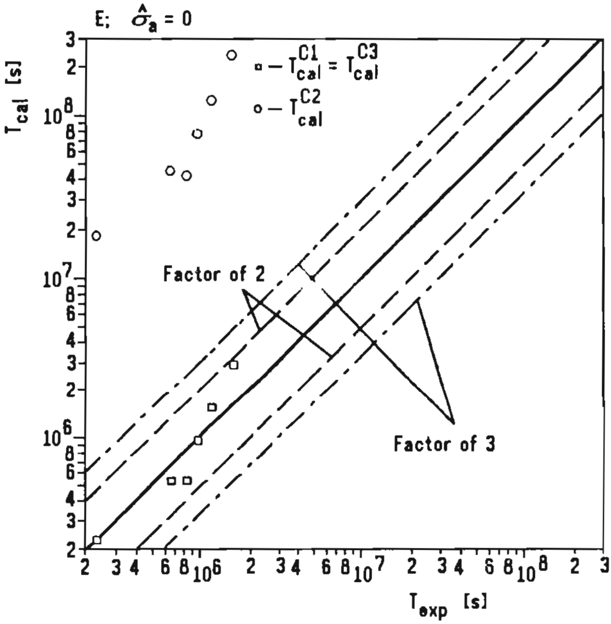

Fig. 7. Comparison of calculated fatigue lives according to criteria C1 ÷ C3 with experimental fatigue lives

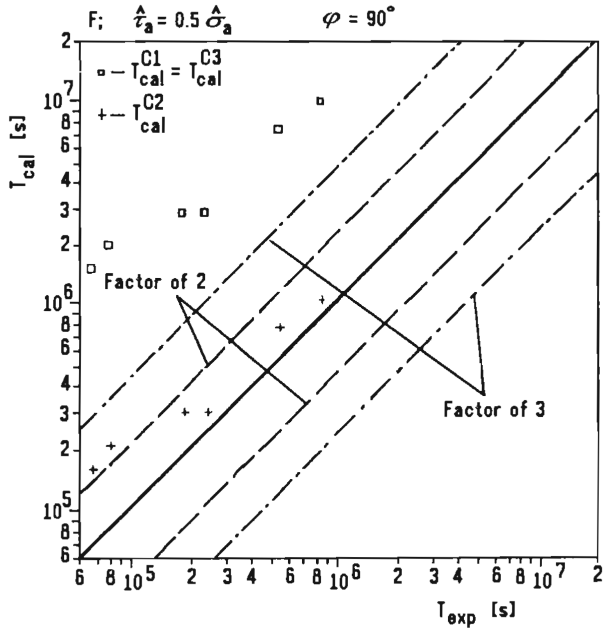

Fig. 8. Comparison of calculated fatigue lives according to criteria C1 ÷ C3 with experimental fatigue lives

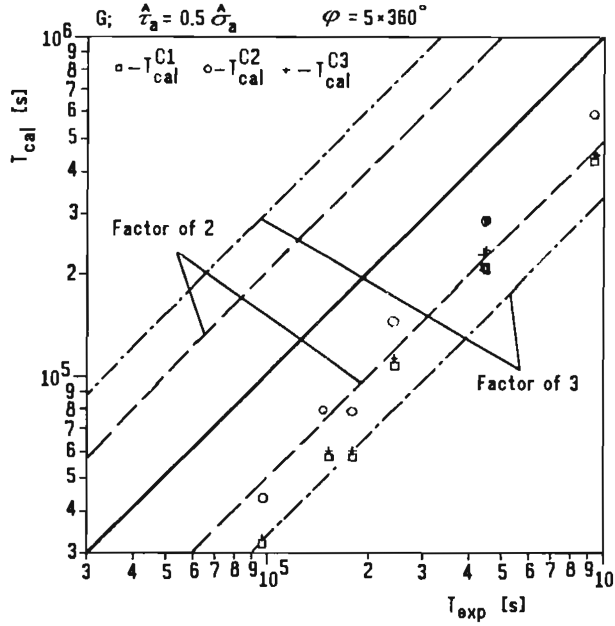

Fig. 9. Comparison of calculated fatigue lives according to criteria C1 ÷ C3 with experimental fatigue lives

Criterion C2 gives estimations for five combinations (apart from E, F) in the case of factor of 2; for combination G some estimations slightly exceed this range.

Fatigue life estimations given by criterion C2 for torsion (E) and out-of-phase bending with torsion ($\varphi = \pi/2$) (F) and by criterion C1 for combination of stresses (F) seem to be nonrealistic.

5. Conclusions

- The most realistic fatigue life estimations are obtained after the criterion of maximum shear and normal stresses in the fracture plane (with a modified participation of shear stress), C3. Here, the equivalent stress history was schematized with the rain flow method and damages were cumulated according to the Palmgren-Miner hypothesis. This criterion can be used for descripton of all the experimental data within the scatter band of results with the factor of 3; for the factor of 2 it can be applied for five – from among seven – considered stress combinations.

- For stress combinations including bending, synchronous bending and torsion as well as torsion and for the scatter band with the factor of 2, the linearized criterion of ellipse quadrant (C1) seems to be the most effective.

- The criterion of maximum normal stress in the fracture plane (C2) gives realistic fatigue life estimations in the scatter band with the factor of 2 for bending and combination of synchronous bending with torsion.

- For all the considered stress combinations the expected fatigue fracture plane positions can be effectively determined with the method of equivalent stress variance according to criterion C2.

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Trwałość stali w warunkach synchronicznego i asynchronicznego zginania ze skręcaniem o zmiennych amplitudach według kryteriów dwuosiowego zmęczenia losowego

Streszczenie

Porównano trwałości obliczone za pomocą trzech kryteriów wieloosiowego zmęczenia losowego z trwałościami uzyskanymi w badaniach próbek walcowych ze stali 30CrNiMo8 w warunkach synchronicznego i asynchronicznego zginania ze skręcaniem o zmiennych amplitudach. Oczekiwane położenie płaszczyzn złomu wyznaczono metodą wariancji. Historie naprężenia ekwiwalentnego schematyzowano metodą płynącego deszczu, a uszkodzenia zmęczeniowe kumulowano według hipotezy Palmgrena-Minera.

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