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PROBABILISTIC EVALUATION OF FATIGUE LIFE OF STRUCTURAL COMPONENTS IN PRESENCE OF CRACK PROPAGATION PROCESS

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What is to be shown hereafter is the capability of predicting fatigue life of randomly loaded structural components, using a two-dimensional model. Fundamental for the description are partial differential equations of parabolic type with variable coefficients. The method discussed has been illustrated with a practical example of evaluating fatigue life of an aircraft undercarriage under operational conditions.

1. Introduction

The conception of forecasting fatigue life of structural components presented in this paper has been based on the grounds of probabilistic description of a crack propagation process. It differs, however, in some details from multiplicity of various stochastic methods of fatigue calculations. Difference equations that describe accumulation of changes due to fatigue are the core of the conception. Appropriate transformations of these equations have resulted in differential equations of parabolic type with the description of a possible rapid catastrophic failure included. In some measure the paper surmounts the series of publications on the above-mentioned grounds. The publications have been appearing for a number of years and include one- and two-dimensional approaches to cracking processes (cf Kocańda and Tomaszek, 1989, 1990, 1992a,b; Kocańda et al., 1990). A two-dimensional model of half- and quarter-elliptical cracks has been used in the present paper. The model differs, however, from the previous ones. The paper is a generalized form of the previous one (Kocańda and Tomaszek, 1992b) and at the same time amplifies (Kocańda et al., 1990). To recall: in case of fissures or cracks mostly of half- and quarter-elliptical shape there is probability that cracks propagate in two directions, perpendicular to each other-hence, two-dimensional the model postulated.

Bibliography of the subject includes quite a rich set of works treating crack propagation process as a random one. Part of it that covers a period of 1985 ÷ 90 has been referred to in references to our previous works (cf Kocańda and Tomaszek, 1989, 1990, 1992a,b; Kocańda et al., 1990) (35 entries). Valuable monographs with comprehensive references have also been published by Sobczyk and Spencer (1992), Prowan edit. (1987). Besides the introduction into fatigue problems on an experimental level and description of these problems, and initiation into mathematical grounds for stochastic approach in formulating fatigue questions, the first monograph reviews methods and models of probabilistic approach to fatigue cracking processes. The work mentioned presents its co-author's, Sobczyk's achievements in crack propagation modelling using cumulative discrete stochastic processes. The second monograph has been composed of works of fifteen authors who give their own analyses, variously approached, of random fatigue cracking, mainly in aircrafts and products of nuclear engineering. Among quite recently published works of Polish authors the work by Doliński (1992) should be mentioned. The author includes plastic zone in front of the cracking and material heterogeneity into statistic modelling of cracking propagation under constant-amplitude loading.

Limits set to the contents of this paper are the reasons for reductions in mathematical derivations and comments on recently (i.e. 1991 ÷ 93) published works. These are to be reflected in a far more comprehensive and extensive publication.

2. Description of the method

The following equation has been applied by Kocańda and Tomaszek (1989) to describe crack propagation process in a general case of random loading

$$
\begin{aligned}
\frac{\partial U(l_1,l_2,t)}{\partial t} &= -C(l_1,l_2)U(l_1,l_2,t) - \frac{\partial b_1(l_1,l_2)U(l_1,l_2,t)}{\partial l_1} + \\
&- \frac{\partial b_2(l_1,l_2)U(l_1,l_2,t)}{\partial l_2} + \frac{\partial^2 \mu_1(l_1,l_2)U(l_1,l_2,t)}{\partial l_1 \partial l_2} + \\
&+ \frac{1}{2}\frac{\partial^2 a_1(l_1,l_2)U(l_1,l_2,t)}{\partial l_1^2} + \frac{1}{2}\frac{\partial^2 a_2(l_1,l_2)U(l_1,l_2,t)}{\partial l_2^2}
\end{aligned}
\tag{2.1}
$$

where

$U(l_1,l_2,t)$ – density function of crack lengths in two directions perpendicular to each other, at the instant t

$C(l_1,l_2)$ – coefficient that describes possibility of a catastrophic failure of a component at crack lengths: l_1 in one direction and l_2 in the other one

$a_1(l_1,l_2)$, $a_2(l_1,l_2)$ – mean square of crack increments in respective directions in the assumed time unit

$b_1(l_1,l_2)$, $b_2(l_1,l_2)$ – mean value of crack increments in respective directions in the assumed time unit

$\mu(l_1,l_2)$ – correlation moment in the assumed time unit, $\mu(l_1,l_2) = r\sqrt{a_1(l_1,l_2)}\sqrt{a_2(l_1,l_2)}$

r – a correlation coefficient.

To apply Eq (2.1) to evaluate fatigue life is a difficult task, due to the lack of analytical solution. However, Eq (2.1) can be simplified if the following assumptions are taken into account

- There are such fatigue crack lengths (in two directions, square to each other) that probability of a catastrophic failure of a structural component equals zero for a certain assumed interval (or for a certain number of loading cycles)

- In deterministic approach fatigue cracking rates are described with the Paris equation

- Loading cycles of Δt duration may not occur in a continuous way; instead, they can occur randomly at a rate of λ, i.e. $\lambda\Delta t \leq 1$.

Taking such assumptions into consideration the following difference equation can be formulated

$$
U_{l_1,l_2,t+\Delta t} = (1-\lambda\Delta t)U_{l_1,l_2,t} + \lambda\Delta t U_{l_1-\Delta l_1,l_2-\Delta l_2,t}
\tag{2.2}
$$

where

$U_{l_1,l_2,t}$ – probability that the cracks in two directions, perpendicular to each other, at the instant t, are l_1, l_2

Δl_1, Δl_2 – crack increments in directions l_1 and l_2 in time interval Δt

λ – loading cycle rate.

Transition to functional notation and the Taylor series expansion enable us to derive the following equation from Eq (2.2)

$$\frac{\partial U(l_1,l_2,t)}{\partial t} = -\lambda\Big(\frac{\partial U(l_1,l_2,t)}{\partial l_1}\Delta l_1 + \frac{\partial U(l_1,l_2,t)}{\partial l_2}\Delta l_2\Big) + \\ +\lambda\frac{\partial^2 U(l_1,l_2,t)}{\partial l_1 \partial l_2}\Delta l_1 \Delta l_2 + \frac{1}{2}\lambda\Big(\frac{\partial^2 U(l_1,l_2,t)}{\partial l_1^2}\Delta l_1^2 + \frac{\partial^2 U(l_1,l_2,t)}{\partial l_2^2}\Delta l_2^2\Big) \tag{2.3}$$

Increments Δl_1, Δl_2 will be determined using the Paris formula for a one-dimensional case

$$\frac{dl}{dN} = C\Delta\sigma^m M_k^m \sqrt{(\pi l)^m} \tag{2.4}$$

In the above derived formula

$\Delta\sigma$ – range of stress amplitude σ_a, $\Delta\,\sigma \;=\; 2\sigma_a$

C, m – coefficient and exponent that depend mainly on material

M_k – geometric coefficient that expresses dimensions of structural components.

Eq (2.4) assumes the following form for $m = 2$

$$l_N = l_0\, \mathrm{e}^{C_1 \Delta\sigma^2 N}$$

where: $C_1 = CM_k^2\pi$.

The exponent value $m = 2$ assumed for the calculations may be taken as a mean value for most of metals (it ranges from 1.2 to 4.0). The value refers mainly to low-alloy steel, various grades of steel of high strength, and Titanium alloys as well. The value of m depends in general on the loading structure and conditions.

Therefore, formula (2.4) for $m = 2$ may be written down as

$$\frac{\Delta l}{\Delta N} = C_1\Delta\sigma^2 l \qquad\qquad \Delta l = C_1\Delta\sigma l\Delta N$$

for $\Delta N = 1$, $\Delta l = C\Delta\sigma l$.

One-cycle crack increment can be described, using Eq (2.4), with the relationship

$$\Delta l = P_{th} C_1 \Delta\sigma^2 l_N \tag{2.5}$$

where

P_{th} – probability that the crack growth will occur if stress range $\Delta\sigma$ exceeds threshold value $\Delta\sigma_{th}$ i.e. $\Delta\sigma > \Delta\sigma_{th}$

l_N – crack length determined with the relationship

$$l_N = l\mathrm{e}^{P_{th}C_1\Delta\sigma^2 N} \tag{2.6}$$

Series expansion applied again allows Eq (2.6) to be expressed with

$$l_N \approx l_0(1 + P_{th}C_1\Delta\sigma^2 N)$$

If the expression $P_{th}C_1\Delta\sigma$ is substituted for b then

$$l_N = l_0(1 + bN)$$

Substitution applied to formula (2.5) results in

$$\Delta l = bl_0(1 + bN) = bl_0 + l_0b^2N \tag{2.7}$$

Then, with Eq (2.7) used in a one-dimensional aspect the following equation is obtained

$$\Delta l_i = b_il_i = b_il_{0i}(1 + b_i\lambda t) = l_{0i}b_i + l_{0i}b_i^2\lambda t \tag{2.8}$$

where

$b_i = P_{th}C_i\Delta\sigma^2$

l_{0i} – initial crack length in l_i direction

$i = 1,2$

$N = \lambda t$

Again, substitution of formula (2.8) into Eq (2.3) gives

$$\frac{\partial U}{\partial t} = -\dot{b}_1(t)\frac{\partial U}{\partial l_1} - \dot{b}_2(t)\frac{\partial U}{\partial l_2} + \dot{\mu}(t)\frac{\partial^2 U}{\partial l_1 \partial l_2} + \frac{1}{2}\dot{a}_1(t)\frac{\partial^2 U}{\partial l_1^2} + \frac{1}{2}\dot{a}_2(t)\frac{\partial^2 U}{\partial l_2^2} \tag{2.9}$$

where

$$\dot{b}_i(t) = \lambda(l_{0i}b_i + l_{0i}b_i^2\lambda t) \qquad \dot{a}_i(t) = \lambda(l_{0i}b_i + l_{0i}b_i^2\lambda t)^2 \qquad i = 1,2$$

$$\dot{\mu}(t) = \lambda(l_{01}b_1 + l_{01}b_1^2\lambda t)(l_{02}b_2 + l_{02}b_2^2\lambda t)$$

A dot above the coeffiecients in Eq (2.9) denotes the time derivative. Solution to Eq (2.9) takes the following form

$$U(l_1,l_2,t) = \frac{1}{\sqrt{2\pi a_1(t)}}\frac{1}{\sqrt{2\pi a_2(t)}}\frac{1}{\sqrt{1-r^2}}\exp\Big\{-\frac{1}{2(1-r^2)}\cdot \\ \cdot\Big[\frac{\Big(l_1 - b_1(t)\Big)^2}{a_1(t)} - 2\frac{r\Big(l_1 - b_1(t)\Big)\Big(l_2 - b_2(t)\Big)}{\sqrt{a_1(t)}\sqrt{a_2(t)}} + \frac{\Big(l_2 - b_2(t)\Big)^2}{a_2(t)}\Big]\Big\} \tag{2.10}$$

where

$$
\begin{aligned}
b_i(t) &= \lambda l_{0i} b_i t + \frac{1}{2} l_{0i} b_i^2 \lambda^2 t^2 \qquad\qquad i = 1,2 \\
a_i(t) &= \lambda l_{0i}^2 b_i^2 t + l_{0i}^2 b_i^3 \lambda^2 t^2 + \frac{1}{3} l_{0i}^2 b_i^4 \lambda^3 t^3 \\
\mu(t) &= \int\limits_0^t \mu'(t)\, dt \\
r &= \frac{\mu(t)}{\sqrt{a_1(t)}\sqrt{a_2(t)}}
\end{aligned}
\tag{2.11}
$$

Two-dimensional Gaussian distribution of crack lengths in two, perpendicular to each other, directions has been arrived at in this way, with the centre of this distribution shifting. The parameters of the centre of the distribution have been determined with dependences (2.11).

Operational fatigue crack propagation depends on a great deal of various factors described in many publications. The most important factors include: the nature of random loading, geometry and properties of material, etc.

Therefore, let us assume that the actual crack propagation process is known (its nature comprises full probabilistic description of mechanical properties of a given structural component). Let it be a general notation in form

$$
\Big[(l_{i0}, t_0), (l_{i1}, t_1), (l_{i2}, t_2), ..., (l_{in}, t_n)\Big] \qquad i = 1,2 \tag{2.12}
$$

The likelihood function for the density function (2.10) and data (2.12) can be described as

$$
\begin{aligned}
L &= \frac{1}{(2\pi)^2} \prod_{k=0}^{n-1} \frac{1}{\sqrt{a_1(\Delta t_k)}} \frac{1}{\sqrt{a_2(\Delta t_k)}} \frac{1}{\sqrt{1-r^2}} \cdot \\
&\cdot \exp\Big\{ -\frac{1}{2(1-r^2)} \Big[\frac{\Big[(l_{1,k+1} - l_{1,k}) - b_1(\Delta t_k)\Big]^2}{a_1(\Delta t_k)} + \\
&- 2\frac{r\Big[(l_{1,k+1} - l_{1,k}) - b_1(\Delta t_k)\Big]\Big[(l_{2,k+1} - l_{2,k}) - b_2(\Delta t_k)\Big]}{\sqrt{a_1(\Delta t_k)}\sqrt{a_2(\Delta t_k)}} + \\
&+ \frac{\Big[(l_{2,k+1} - l_{2,k}) - b_2(\Delta t_k)\Big]^2}{a_2(\Delta t_k)} \Big]\Big\}
\end{aligned}
\tag{2.13}
$$

where

$$b_i(\Delta t_k) = \lambda l_{i0} b_i (t_{k+1} - t_k) + \frac{1}{2} l_{i0} b_i^2 \lambda^2 (t_{k+1}^2 - t_k^2) \qquad i = 1,2$$

$$a_i(\Delta t_k) = \lambda l_{i0}^2 b_i^2 (t_{k+1} - t_k) + l_{i0}^2 b_i^3 \lambda^2 (t_{k+1}^2 - t_k^2) + \frac{1}{3} l_{i0}^2 b_i^4 \lambda^3 (t_{k+1}^3 - t_k^3)$$

The likelihood function (2.13) has been applied to estimate the distribution parameters in the density function (2.10). The formulae (2.11) define the form of dependences of these parameters of distribution upon each other. The right sides of the formulae (2.11) show coefficients b_1 and b_2. To estimate the parameters of distribution (2.11) it will be good enough to estimate the coefficients b_1 and b_2 and the correlation coefficient, using the likelihood function.

The formulae to estimate these coefficients are as follows

$$\begin{aligned} b_i^* &= \frac{\sqrt{1 + 2\frac{l_{in}}{l_{i0}}} - 1}{\lambda t_n} \qquad i = 1,2 \\ r^* &= \frac{\frac{1}{n}\sum\limits_{k=0}^{n-1} \frac{\Big[\Big(l_{1,k+1} - l_{1,k}\Big) - \Big(b_1^*(t_{k+1} - b_1^*(t_k)\Big)\Big]\Big[\Big(l_{2,k+1} - l_{2,k}\Big) - \Big(b_2^*(t_{k+1} - b_2^*(t_k)\Big)\Big]}{t_{k+1} - t_k}}{\sqrt{a_1^*(t)_m}\sqrt{a_2^*(t)_m}} \end{aligned} \tag{2.14}$$

where

$$a_i^*(t)_m = \frac{1}{n}\sum_{k=0}^{n-1} \frac{\Big[\Big(l_{i,k+1} - l_{i,k}\Big) - \Big(b_i^*(t_{k+1} - b_i^*(t_k)\Big)\Big]^2}{t_{k+1} - t_k} \qquad i = 1,2$$

What has been obtained is a probability density function of crack lengths in two directions perpendicular to each other, with shifting centres and parameters of distribution known. Let it be written down as

$$U(l_1, l_2, t) = U\Big(l_1, l_2, b_1^*(t), b_2^*(t), \sqrt{a_1^*(t)}, \sqrt{a_2^*(t)}, r^*\Big) \tag{2.15}$$

Probability of not exceeding the permissible crack lengths in both the directions can be defined using this function

$$R(t) = R\Big(l_1 \le l_{1d}; l_2 \le l_{2d}; t\Big) = \int\limits_{-\infty}^{l_{1d}} \int\limits_{-\infty}^{l_{2d}} U(l_1, l_2, t)\, dl_1 dl_2 \tag{2.16}$$

Assuming that $R(t) \ge R_0$ where R_0 is a permissible minimum probability of exceeding l_{1d} and l_{2d}, one can evaluate a fatigue life of a component.

Relationship between l_1 and l_2 makes calculations related to the two-dimensional Gaussian distribution highly difficult. Notable simplifications are to be obtained as a result of transforming the coordinates

$$\begin{aligned} l_1' &= l_1 \cos\alpha + l_2 \sin\alpha \\ l_2' &= -l_1 \sin\alpha + l_2 \cos\alpha \end{aligned} \tag{2.17}$$

where angle α results from the formula

$$\tan 2\alpha = \frac{2r^* \sqrt{a_1^*(t)_m} \sqrt{a_2^*(t)_m}}{a_1^*(t)_m a_2^*(t)_m} \tag{2.18}$$

Such being the transformation, the modified crack lengths l_1' and l_2' become independent random variables for every t assumed. Average values, standard deviation and permissible crack lengths are also subjects to transformation by the formula (2.17). In a new coordinate system formula (2.16) takes the following form

$$R(t) = R\Big(l_1 \leq l_{1d}; l_2 \leq l_{2d}, t\Big) = R_1\Big(l_1' \leq l_{1d}'; t\Big) R_2\Big(l_2' \leq l_{2d}'; t\Big) \tag{2.19}$$

where $R_1\Big(l_1' \leq l_{1d}', t\Big)$ and $R_2\Big(l_2' \leq l_{2d}'; t\Big)$ have been determined from the one-dimensional Gaussian distributions.

To take advantage of the tables of normal distributions calls for the standardization of random variables l_1' and l_2'.

Probability $R(t)$ determined by the formula (2.19) can also be understood as probability of that the growing probability of a catastrophic crack of a component P_t (together with the formulae for) will not exceed the assumed hazard level while determining the permissible crack lengths l_{1d} and l_{2d}. The fact is to be written down as

$$R(t) = P\Big\{P_t \leq P_r\Big\} \tag{2.20}$$

where P_r is the probability of a catastrophic failure of a component for $l_1 \leq l_{1d}$ and $l_2 \leq l_{2d}$.

Here is the place to draw and introduce some definitions from the field of equipment reliability. Let us assume that the state of serviceability has been determined with the probability $\bar{R}(t)$ of the event $\Big\{l_1 \leq l_{1d}$ and $l_2 \leq l_{2d}$ and $P_t \leq P_r\Big\}$ whereas the state of unserviceability is the probability $\bar{Q}(t)$ of the event $\Big\{l_1 > l_{1d}$ or $l_2 > l_{2d}$ or $P_t > ; P_r\Big\}$. Hence,

$$\bar{R}(t) = P\Big\{\sigma \leq \sigma_{cal}\Big\} R_1\Big(l_1 \leq l_{1d}, t\Big) R_2\Big(l_2 \leq l_{2d}, t\Big)$$

$$\bar{Q}(t) = P\Big\{\sigma > \sigma_{cal}\Big\} + P\Big\{\sigma \leq \sigma_{cal}\Big\}\Big\{1 - R_1\Big(l_1 \leq l_{1d}, t\Big) R_2\Big(l_2 \leq l_{2d}, t\Big)\Big\} \tag{2.21}$$

In the formulae appears σ_{cal} – here the analytical value of stresses included in the stress intensity factor K for permissible lengths l_{1d} and l_{2d}. The notations of probability in both the formulae i.e. (2.21) and (2.22).

$P\Big\{\sigma \leq \sigma_{cal}\Big\}$ and $P\Big\{\sigma > \sigma_{cal}\Big\}$ follow from the condition $P_r = P\Big\{\sigma > \sigma_{cal}\Big\}$.

It should be pointed out that σ_{cal} can also be treated as a stress due to misestimation of the greatest operational loadings possible.

Various instances could take place in the practice of calculating fatigue lives of structural components under random loadings, depending on the contribution of these components to the reliable performance of a machine/instrument. The hazard of a catastrophic failure of a component during its life has often been required to equal – in practice – zero. The requirement will be met if:

- The hazard of component failure assumed while determining permissible crack lengths equals zero

- Probability of exceeding the permissible crack lengths in course of operational life l_{1d} and l_{2d} equals zero.

The component life will then be the shortest time (or the lowest cycle/flight number), for which R_1 and R_2 equal in practice unity. This condition can be expressed in the following form

$$t = \min(T_1, T_2)$$

where T_1, T_2 – times in which probability of not exceeding permissible crack lengths l_{1d} and l_{2d} equals in practice unity.

The second possible way of calculating fatigue life under random loading consists in allowing the hazard of a catastrophic failure of a component. Such being the case, the following practical assumptions are made while determining component's life:

- The hazard of a catastrophic failure of the component cannot exceed the hazard assumed while determining permissible crack lengths in directions perpendicular to each other

- Permissible crack lengths have been determined for the hazard that equals zero. Cracks in course of operational life are allowed then
- A certain level of hazard is assumed while determining permissible crack lengths in two directions square to each other.

3. Numerical example

A crack of quarter-elliptical shape was propagating within a structural component of an aircraft undercarriage, depending on a number of flights N in a way illustrated by Table 1.

Table 1

Measurement no.	Number of flights N	Crack length in direction l_1 [mm]	Crack length in direction l_2 [mm]
1	0	0.3	0.1
2	500	3.0	2.0
3	1300	5.0	3.0
4	1400	6.0	5.0
5	1500	9.0	6.0
6	1612	10.0	7.0
7	1721	16.0	8.0
8	1904	20.0	10.0
9	2105	22.0	12.0
10	2200	23.0	14.0

After taking into consideration, among other factors, the component geometry, random loadings, and crack shape, and adequately assuming the hazardous level of catastrophic fatigue of a component permissible crack lengths in two directions: $l_{1d} = 23.0$ mm and $l_{2d} = 14.0$ mm have been determined. Using the formulae (2.14) coefficients b_1^* and b_2^* have been calculated. Their values are: $b_1^* = 0.0052$, $b_2^* = 0.0072$. Table 2 shows mean crack lengths $b_1^*(t)$ and $b_2^*(t)$ calculated by the formulae (2.11).

Table 2

N	500	1000	1500	2000
l_1 [mm]	3	4.2	9.0	20.6
$b_1^*(t)$	1.6	6.0	11.3	19.5
l_2 [mm]	2.0	2.7	6.0	11.0
$b_2^*(t)$	1.0	3.6	6.9	11.8

The calculated correlation coefficient is $r = 0.725$, angle $\alpha = 19°$.

Now we are going to transform by the formula (2.17) the data necessary to evaluate fatigue life.

The results obtained have been put together in Tables 3 ÷ 5.

Table 3

N	500	1000	1500	2000
l_1 [mm]	3.0	4.2	9.0	20.6
l_1' [mm]	3.4877	4.850	10.4631	23.500
l_2 [mm]	2.0	2.7	6.0	11.0
l_2' [mm]	0.9142	1.1853	2.7426	3.6931

Table 4

N	500	1000	1500	2000
$b_1^*(N)$	1.6	6.0	11.3	19.5
$b_1'^*(N)$	1.8384	6.845	12.93	22.28
$b_2^*(N)$	1.0	3.6	6.9	11.8
$b_2'^*(N)$	0.4245	1.45	2.845	4.81

Table 5

N	500	1000	1500	2000
$a_1^*(N)$	0.0071	0.0370	0.1059	0.231
$a_1'^*(t) = \left(\sqrt{a_1^*(t)}\cos\alpha + \sqrt{a_2^*(t)}\sin\alpha\right)^2$				
$a_1'^*(N)$	0.0091	0.04814	0.1387	0.304
$a_2^*(N)$	0.002317	0.01325	0.0395	0.08796
$a_2'^*(t) = \left(-\sqrt{a_1^*(t)}\sin\alpha + \sqrt{a_2^*(t)}\cos\alpha\right)^2$				
$'^*a_2(N)$	0.00033	0.00213	0.0067	0.0154

Parameters determined in such a way, shown in Tables 2 ÷ 5, make calculation of probability that the crack will not exceed the permissible values in both the directions l_{1d} and l_{2d} possible.

Results are shown in Table 6.

Table 6

N	500	1000	1500	2000	2200
$l'_{1d} = 26.31$	$l_{2d} = 5.748$ (ac. to Eq (2.17))				
$\frac{l'_{1d} - b'^*(N)}{\sqrt{a_2'^*(N)}}$	256.5	88.7	36.162	7.307	0.00
$\frac{l'_{2d} - b'^*(N)}{\sqrt{a_2'^*(N)}}$	294.4	93.43	35.84	7.85	0.00
$R_1(N)$	1.000	1.000	1.000	1.000	0.50
$R_2(N)$	1.000	1.000	1.000	1.000	0.5
$R_1(N)R_2(N)$	1.000	1.000	1.000	1.000	0.25

Having the results of the above presented calculations acquired curves shown in Fig.1 have been plotted. A "dotted" line illustrates crack propagation in two directions perpendicular to each other, in a transformed system. The full lines illustrate the calculated crack lengths l'_1 and l'_2 for estimated values of N. Horizontal dashed lines indicate the permissible crack lengths l'_{1d} and l'_{2d} calculated by the formula (2.17). Vertical segments on full-line curves show confidence intervals for the hazard assumed.

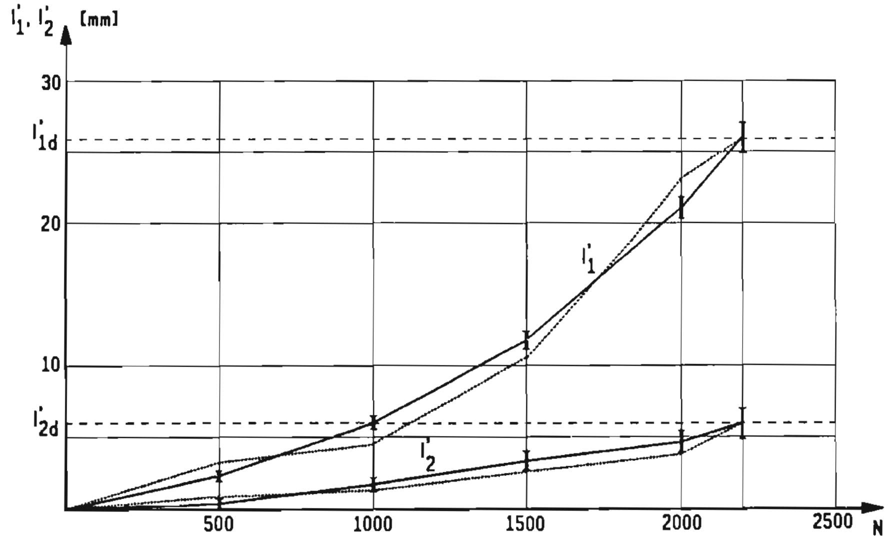

Fig. 1.

Reliability requirements to be met by aircraft structure components are very high. It means that probability of a component crack/failure should approach zero. With the data assumed in the example discussed and calculations made one can admit that fatigue life of the undercarriage component under investigation must not exceed a number of 2000 flights if the hazard level assumed while determining permissible crack lengths is to be preserved.

4. Final remarks

Dynamics of crack propagation process according to a two-dimensional approach is described by Eq (2.1). Applying Eq (2.1) directly to evaluate fatigue life is not simple. The paper contains some simplifications that enable the physical and mathematical meaning of this equation to be replaced with a solution that makes evaluation of fatigue life of a component possible.

The above presented considerations are simplification of the previous papers by Kocańda and Tomaszek (1992a,b) and resolve themselves into two-dimensional description of cracking phenomena. They enable life of a component with initial cracks to be determined in a simplified way, with two directions of their propagation distinguished. The two directions are quite sufficient to describe dynamics of growth of a crack of a given shape. It should be noted, however, that instances of propagation of cracks of so many and various shapes may occur, that their description cannot be limited to two directions only. In practical calculations of fatigue life of components with selected two-dimensional cracking process initiated some other instances of crack propagation may also occur. They have been described by Kocańda, Smoliński and Tomaszek (1990).

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Probabilistyczna ocena trwałości zmęczeniowej elementów konstrukcyjnych w warunkach rozwoju pęknięć

Streszczenie

Wykazano możliwość przewidywania trwałości zmęczeniowej losowo obciążonych elementów przy wykorzystaniu dwuwymiarowego modelu rozwoju pękania. Podstawę opisu stanowią równania różniczkowe cząstkowe typu parabolicznego ze zmiennymi współczynnikami. Proponowaną metodę zilustrowano praktycznym przykładem określenia trwałości zmęczeniowej podwozia statku powietrznego w warunkach eksploatacji.

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