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INVESTIGATION INTO FATIGUE CRACK PROPAGATION IN WT-9 TITANIUM ALLOY

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The investigation into basic characteristics of fatigue crack propagation in titanium alloy WT-9 has been presented. The tests have been done on compact tension specimens manufactured from the element of compressor of aircraft engine. The measurement results of the fatigue crack growth rate have been described. The numerical analysis of the results has been provided in order to estimate parameters of the fatigue crack growth rate description.

1. Introduction

Some cases of fatigue cracking of a jet turbine engine compressor discs and blades made of WT-9 alloy created a need of experimental study of fatigue crack propagation in the as-received state of these components. In such cases, for protection purposes, it is very important to get some estimation of a fatigue crack propagation time between detectable and critical crack sizes.

2. Experimental procedure

2.1. Material and specimen

WT-9 is a martensitic titanium alloy chemical composition of which is (in weight percent): 6.4Al, 3.3Mo, 2.5Fe, 1.0C, 0.15H, 0.5N and 1.5O. Basic mechanical properties (after tensile test) are: $R_m = 1600$ MPa, $R_{0.2} = 964$ MPa, the Young modulus $E = 1.28 \cdot 10^5$ MPa, the Poisson ratio $\nu = 0.244$.

11 CT specimens marked T1 to T11 had been cut off a compressor disk in the as-received state (after disassembly). Dimensions of the specimen are: $B = 3.7$ mm (disc thickness), $W = 50$ mm, $H = 60$ mm. The specimens were situated on the disk in a position ensuring that the crack propagated in radial direction.

2.2. Fatigue test

All the tests were carried out on the servo hydraulic testing machine type INSTRON 8502, in four sequences described bellow.

2.2.1. Sequence (a)

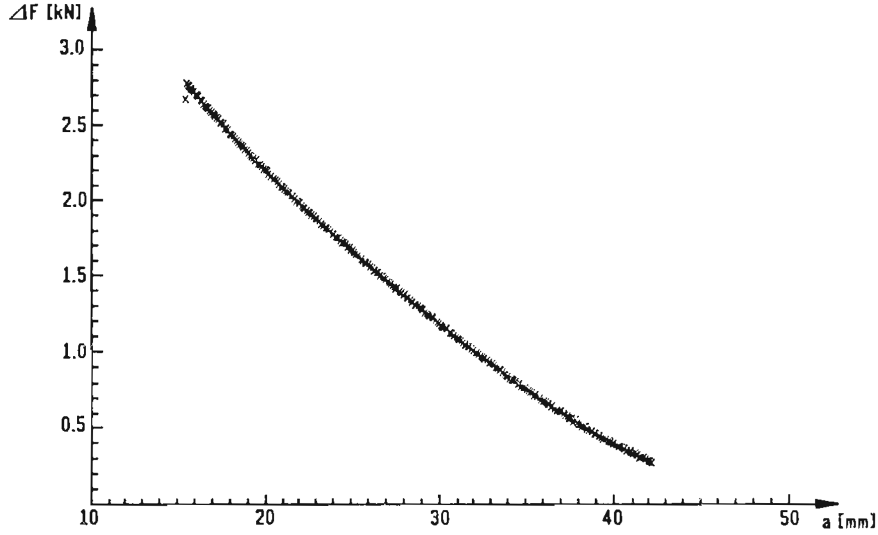

Fig. 1. Experimentaly recorded load amplitude – crack length relation for T5 specimen

Specimens T1 and T5 have been subjected to load at a fixed stress intensity factor (SIF): 1000 and 600 $\mathrm{MPa}\sqrt{\mathrm{mm}}$, respectively, and the stress ratio $R = 0.3$. Fig.1 presents an example of the load amplitude – crack length relation for $\Delta K = \mathrm{const}$. A constant value of the range of SIF resulted in a constant da/dN value during test, i.e. in this example $2.2 \cdot 10^{-3}$ mm/cycle for T1 and $2 \cdot 10^{-4}$ mm/cycle for T5, respectively. Table 1 (cf Bukowski and Kłysz, 1993) shows some experimental data recorded in the test.

Table 1. Several quantities measured during sequence (a) tests

Quantity measured		T5	T1
range of SIF	ΔK [MPa$\sqrt{mm}$]	600	1000
initial values of loading	F_0 [kN]	2.77	5.36
critical values of loading	F_{cr} [kN]	0.27	0.41
initial values of compliance	C_0 [ ]	25.7	21.3
critical values of compliance	C_{cr} [ ]	649	724
minimal values of observed crack rate	da/dN_{min} [mm/cycle]	$9.08 \cdot 10^{-5}$	$1.38 \cdot 10^{-3}$
maximal values of observed crack rate	da/dN_{max} [mm/cycle]	$2.94 \cdot 10^{-4}$	$3.32 \cdot 10^{-3}$
mean values of observed crack rate	da/dN_{aver} [mm/cycle]	$2.00 \cdot 10^{-4}$	$2.20 \cdot 10^{-3}$
critical crack size	a_{cr} [mm]	42.05	43.50
number of cycle to failure	N_f [cycles]	131.349	14.682

2.2.2. Sequence (b)

Each measuring point (specimen T2) was subject to cycling loading with constant load range ΔF and the stress ratio $R = 0.3$. Under such conditions the range of SIF increased continuosly within the range $415 \div 1340$ MPa$\sqrt{mm}$ together with the crack rate ranged $7 \cdot 10^{-5} \div 9 \cdot 10^{-3}$ mm/cycle. A crack length – the range of SIF curve is presented in Fig.2a.

2.2.3. Sequence (c)

These tests were done to determine the influence of stress ratio R on the $da/dN - \Delta K$ relation. 5 specimens were loaded with step by step changing ΔK within the range $300 \div 1700$ MPa$\sqrt{mm}$ (or to failure), and at different stress ratios for each specimen, viz. $R = 0$, 0.2, 0.4, 0.6 and 0.8. Each test was preceded by the following precracking procedure. At first, the specimen was subjected to a relatively high value of ΔK (about 600 MPa$\sqrt{mm}$) to get a precrack about $a/W = 0.34$ of length, as quick as possible. After that ΔK was diminished to 100 MPa$\sqrt{mm}$. When the fatigue crack crossed the plastic zone (created by previous higher loading), at a distance of 1 mm, a new value of ΔK (400 MPa$\sqrt{mm}$) was being set up and so on, for subsequently increasing values of the SIF range. A crack rate for each step was measured on the basis of 1 mm increment of crack. Fig.2b presents an example of this procedure. The results of experiment are presented in Table 2 and, in a

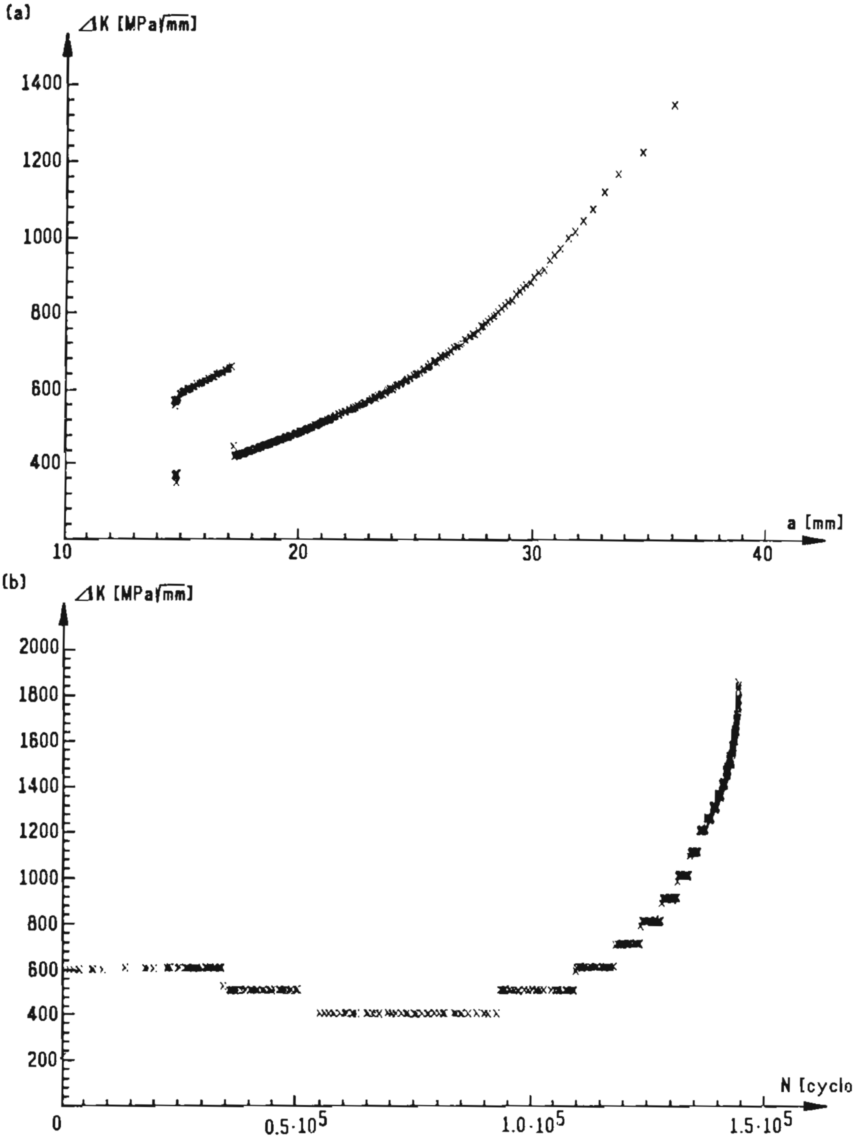

Fig. 2. (a) Experimentaly recorded SIF range – crack length relation for T2 specimen; (b) changes of SIF range characteristic for sequence (c) tests

graphic form, in Fig.3 (in log-log coordinates). As one can see, all the results are situated partly in the midle (linear) and partly (particularly for $R = 0$ and $R = 0.2$) in the critical (non linear) part of the crack rate curve. But it is to be emphasized that for higher R passage from the midle range of fatigue cracking to the critical one is more rapid and for this reason – more difficult to observe.

Table 2. Experimental results of sequence (c) tests

ΔK	$da/dN \cdot 10^3$ [mm/cycle]				
[MPa$\sqrt{\text{mm}}$]	R = 0	R = 0.2	R = 0.4	R = 0.6	R = 0.8
300			0.0197	0.0300	0.0527
400	0.0279	0.0586	0.0647	0.0945	0.1791
500	0.0657	0.1149	0.1336	0.1943	0.3554
600	0.1406	0.1836	0.2375	0.3794	0.6496
700	0.2205	0.2826	0.3634	0.6716	
800	0.2824	0.3927	0.5903	1.0640	
900	0.3959	0.5749	0.9358	1.7420	
1000	0.5229	0.8194	1.6240	2.8970	
1100	0.7882	1.1590	2.8290	4.4920	
1200	0.9933	1.7340	3.9970		
1250	1.3020	2.1910			
1300	1.3540	2.3820	5.4170		
1350	1.6130		6.7200		
1400	2.0140	3.3660	6.8240		
1450	2.5400				
1500	3.0370	6.3660			
1550	3.5060				
1600	4.3430				
1650	5.4140				
1700	8.3480				

2.2.4. Sequence (d) tests

These tests were conducted to observe fatigue life of specimens T7, T8, T9, T11 and T2. The last one, for comparision, was cycled at a constant amplitude up to failure. The others were subjected to modified spectrum of loading: after each ΔN cycles at a constant amplitude a single peak $k_{ov}F_{max}$, where $k_{ov} = 1.4$ – overloading ratio, has been aplied. The results of the test – number of cycles to failure, are shown in Table 3.

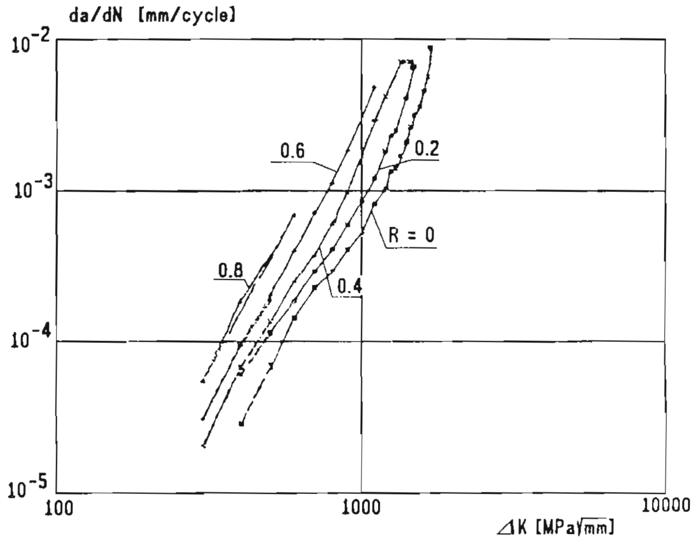

Fig. 3. Experimentaly measured da/dN vs. ΔK results for sequence (c) tests

Table 3. Number of cycles to failure for different distances between overloading peaks

Specimen	a_0	ΔN	$2N_{f,exp}$
T2	17.20	-	72.339
T7	20.29	5.000	64.600
T8	17.33	10.000	93.000
T9	17.31	15.000	89.200
T11	16.78	2.000	147.900

3. Relation between crack propagation rate and SIF range

SIF ranges ΔK have been calculated from the formula (cf Murakami edit., 1987)

$$\Delta K = \frac{\Delta F}{B\sqrt{W}}\frac{2+\alpha}{\sqrt{(1-\alpha)^3}}\Big(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4\Big) \qquad (3.1)$$

where

B, W – specimen thickness and width, respectively
α – nondimensional crack length, $\alpha = a/W$
ΔF – range of loading, $\Delta F = F_{max} - F_{min}$

Fatigue crack rate has been described by the Paris

$$\frac{da}{dN} = C(\Delta K)^m \tag{3.2}$$

and Forman

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K} \tag{3.3}$$

formulas, respectively.

Within the linear part of crack rate curve in log-log coordinates all experimental points may be approximated by linear regression

$$y = a_i x + b_i$$

where

$$y = \log \frac{da}{dN} \qquad x = \log \Delta K$$
$$a_i = m_i \qquad b_i = \log C_i$$

a_i, b_i – parameters of the Paris equation.

The results of calculation for seqence (c) are shown in the Table 4.

Table 4. Parameters of the Paris equation calculated for sequence (c) tests

i	R	$a_i = m_i$	b_i	$C_i \cdot 10^8$
1	0	3.1299	−7.9292	1.1771
2	0.2	2.8828	−7.4158	3.8388
3	0.4	3.5730	−8.1696	0.6767
4	0.6	3.7707	−8.2209	0.6013
5	0.8	3.5958	−7.7613	1.7326

As one can see in Table 4, for different R we have different parameters of the Paris equation, which are not practical in use. To improve it we can get one (single) formula for R within the range $(0, 0.8)$ using the following procedure.

A function which satisfies N consecutive equations

$$\Big(\frac{da}{dN}\Big)_i = 10^{b_i}(\Delta K)^{a_i}$$

is

$$\frac{da}{dN}=\sum_{i=1}^{N}\Big[\prod_{k=1}^{N}\frac{R-R_k}{R_i-R_k}\Big]\Big(\frac{da}{dN}\Big)_i \qquad k\neq i \tag{3.4}$$

The coefficient factor in square parenthesis meets equals 0 unless $R=R_i$, when it is equal to 1.

It means that for $R=R_i$ we have only the component $(da/dN)_i$, however for intermediate values of R it consists of superposition of components $(da/dN)_i$ multiplied by weight factors expressed by polynominal of $(N-1)$th degree

$$\Big(\alpha_{N-1,i}R^{N-1}+\alpha_{N-2,i}R^{N-2}+...+\alpha_{1,i}R+\alpha_{0,i}\Big) \tag{3.5}$$

Thus for arbitrary R we can obtain the following formula for fatigue crack rate

$$\frac{da}{dN}=\sum_{j=0}^{N-1}\Big[\sum_{i=1}^{N}\Big(\alpha_{j,i}\cdot 10^{b_i}(\Delta K)^{a_i}\Big)R^j\Big] \tag{3.6}$$

For equal slopes of all curves from the Fig.3, it is

$$a_1=a_2=...=a_N=a=\text{const} \tag{3.7}$$

the equation of crack rate takes the form

$$\begin{aligned}\Big(\frac{da}{dN}\Big)&=f(R)(\Delta K)^a=\\&=\frac{(\Delta K)^{3.05195}}{10^8}(-41.193R^4+68.469R^3-28.498R^2+7.217R+1.496)\end{aligned} \tag{3.8}$$

which is presented in Fig.4.

The final form of crack rate – SIF range relationship for R within the range $(0,0.8)$ (applying the aforementioned approach) is shown in Fig.5.

All experimental data shown in Fig.3 may be approximated by the Paris and Forman equations by means of the least square method as well

$$\begin{aligned}\log\frac{da}{dN}&=\log C+m\log\Delta K\\ \log\Big[\frac{da}{dN}\Big((1-R)K_c-\Delta K\Big)\Big]&=\log C+m\log\Delta K\end{aligned} \tag{3.9}$$

respectively.

For all experimental points the Paris equation takes the form

$$\frac{da}{dN}=10^{-9.10548}(\Delta K)^{3.06259} \tag{3.10}$$

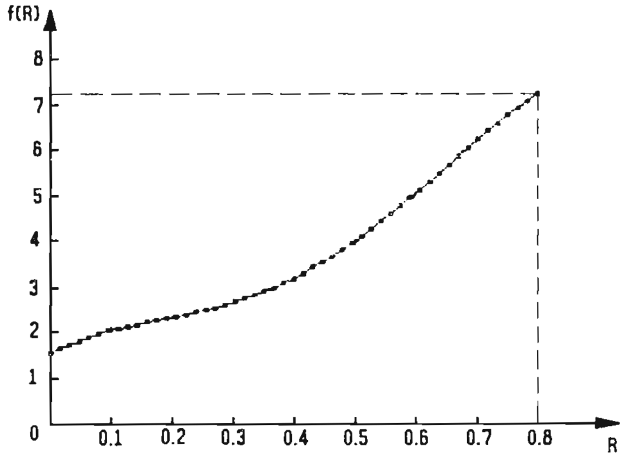

Fig. 4. $f(R)$ relation course – Eq (3.8)

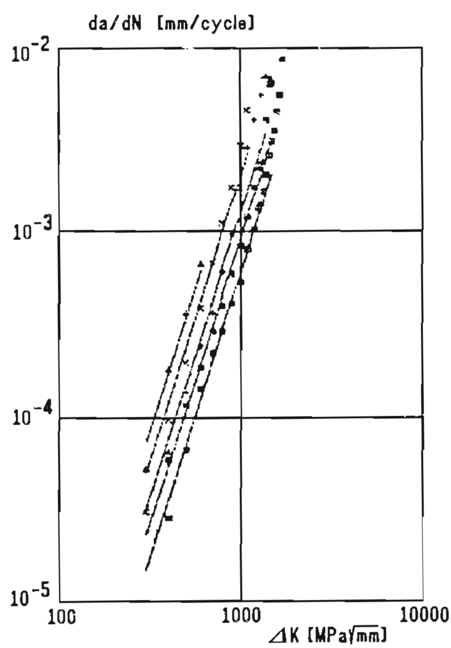

Fig. 5. Approximation of experimental data da/dN vs. ΔK – Eq (3.8)

As can one see, both exponents in Eq (3.10) differ from those in Eq (3.8) because in former approximation the experimental points only for $\Delta K < 1200$ $\mathrm{MPa}\sqrt{\mathrm{mm}}$ have been taken into account.

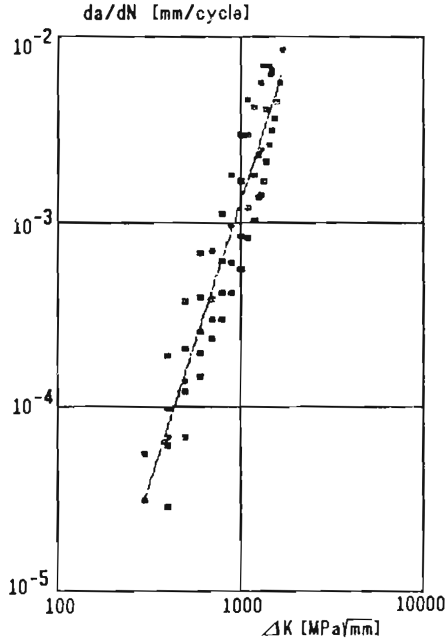

Fig. 6 Approximation of experimental data da/dN vs. ΔK – Eq (3.10)

Since Eq (3.10) does not include the stress ratio R it is not suitable for approximation of particular curves – Fig.6. The Forman equation rewritten similarly is of the form

$$\frac{da}{dN} = \frac{10^{-6.81465}(\Delta K)^{3.34216}}{(1-R)K_c - \Delta K} \tag{3.11}$$

and enables approximation of all curves in Fig.7.

Eq (3.11) has been obtained on the assumption that the value of K_c remains the same for all tests (for different R). From the physical point of view the denominator of right-hand side of Eq (3.11) has to be positive. For this reason the value of 3500 $\mathrm{MPa}\sqrt{\mathrm{mm}}$, as K_c, has been taken in the above described calculation. Due to small thickness of tested specimens the value of K_c should not be identified with the fracture toughness K_{1c}. Besides it should change for different values of R, otherwise for $R = 0$ the asymptotic convergance of $da/dN = f(\Delta K)$ in the critical range of fatigue cracking (i.e.

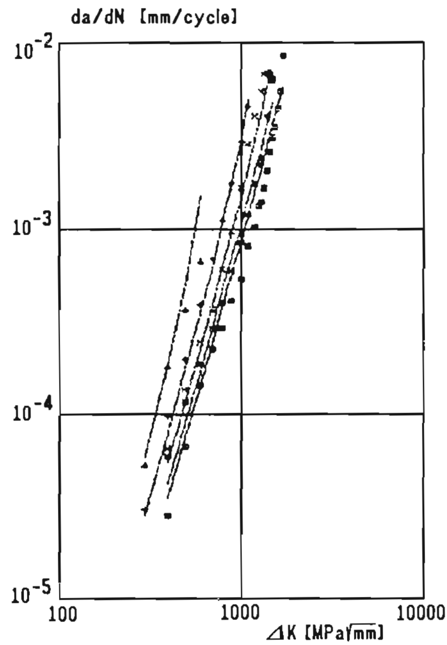

Fig. 7. Approximation of experimental data da/dN vs. ΔK - Eq (3.11)

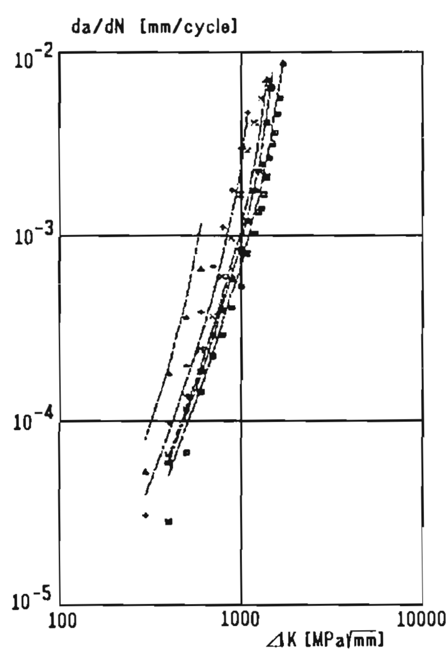

Fig. 8. Approximation of experimental data da/dN vs. ΔK - Eq (3.12)

when $(da/dN) \rightarrow \infty)$ should take place at $\Delta K = 3500$ MPa$\sqrt{\text{mm}}$, while in each test on the level of 1750 MPa$\sqrt{\text{mm}}$ specimen may fail after several cycles. Due to this, all the experimental results have been approximated by the Forman equation on the assumption that K_c is estimated on the basis of data optained in a particular test. For example, assuming that for $R = 0, 0.2, 0.4, 0.6, 0.8$ $K_c = 1900, 2100, 2600, 3100$ and 3700 MPa$\sqrt{\text{mm}}$, respectively, after the linear analysis of regression we can get the following the Forman equation

$$\frac{da}{dN} = \frac{10^{-4.0862}(\Delta K)^{2.27929}}{(1-R)K_c - \Delta K} \tag{3.12}$$

which is presented in Fig.8.

By means of the least square method on the basis of experimental results obtained from sequence (d) tests the Paris equation parameters (C, m) and the value of exponent n entering the Wheeler retardation model (cf Fuchs and Stephens, 1980) have been calculated. The results are presented in Table 5.

Table 5. Paris equation parameters and exponent of the Wheeler retardation model determined for sequence (d) tests results

Specimen	$C \cdot 10^7$	m	n
T2	0.9552	2.8904	1.50
T7	3.0422	2.4123	1.50
T8	1.7254	2.6472	2.60
T9	1.3223	2.7249	2.10
T11	5.0814	2.2079	1.60

As can one see in Table 5 the scatter of obtained parameters is distinct, which makes the uniform description of crack propagation process for all specimens difficult.

It is interesing that an arbitrary assumed Wheeler exponent n of the form (cf Klysz, 1991)

$$n = \frac{2F_{ov}}{Fmax} \tag{3.13}$$

gives quite good approximation for all tests (Table 6).

Table 6. Paris equation parameters for Wheeler exponent n calculated according to Eq (3.13)

Specimen	$C \cdot 10^7$	m	n
T2	2.2431	2.6105	3.17
T7	9.2834	2.1090	2.80
T8	2.6397	2.5122	2.80
T9	2.7835	2.4891	2.80
T11	11.3770	2.0683	2.80

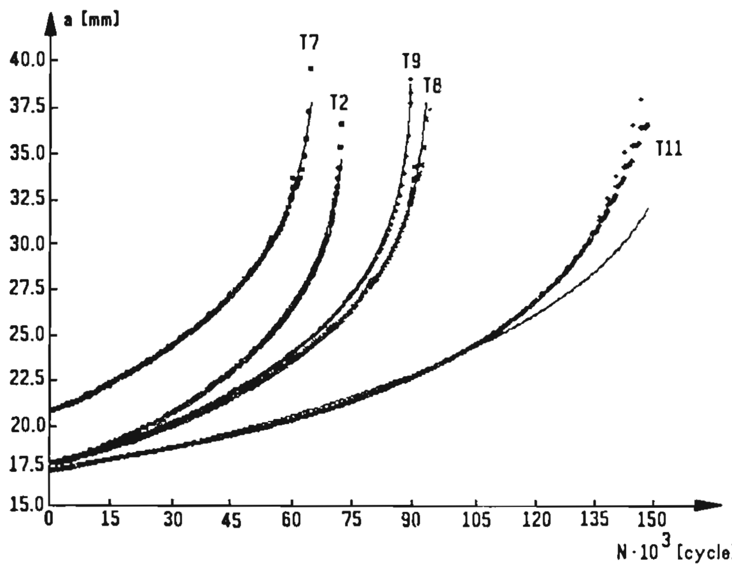

Fig. 9. Theoretical approximation of crack length – number of cycles relation for sequence (d) tests

In Fig.9 the theoretical curves based on parameters from Table 6 have been compared with the experimental results. Taking into account some dissimilarity between T7 and T11 tests (different initial crack lengh and a misfit to experimental data), one can state that parameters C, m and n, respectively, from Table 6 show good agreement with characteristics for the test at rest. Calculating the relative fatigue life δ (for specimens T2, T8 and T9) defined as proportion of $N_{f,cal}$ to $N_{f,exp}$ we obtain the results shown in Table 7. E.g. for parameters C, m, n obtained for specimen T2 the relative fatigue lives $\delta = 1.01$, 1.14 and 1.15 for specimens T2, T8 and T9 respectively, were calculated.

Table 7. Calculated relative fatigue lives for different specimens and chosen C, m, n parameters

Parameters for specimen	Relative fatigue lives for specimen		
	T2	T8	T9
T2	1.01	1.14	1.15
T8	0.86	0.98	1.00
T9	0.87	0.98	1.00

Thus, in this case we can take into account the formula

$$\frac{da}{dN} = C_pC(\Delta K)^m \tag{3.14}$$

where

$$C = 2.55 \cdot 10^{-7} \pm 12\% \qquad\qquad m = 2.54 \pm 3\%$$

and the Wheeler coefficient of retardation as follows

$$C_p = \Big(\frac{r_{p,i}}{r_{p,ov} - r_{p,i}}\Big)^{2\frac{F_{ov}}{F_{max}}} \tag{3.15}$$

where $r_{p,i}$ and $r_{p,ov}$ – radii of plastic zones created by subsequent basic and overloading cycles, respectively.

4. Conclusions

The presented analytical rate of crack – SIF range relationships given above based on the Paris and Forman equations for different stress ratios R, have shown good correlation to experimental points within the range of R from 0 to 0.8.

The numerical analysis by means of computer models of fatigue crack propagation enabled an optimum estimation of Paris and Forman equations parameters and the exponent of the Wheeler retardation model. This models have been experimentaly verified on specimens subjected to a constant amplitude loading with repeated overloading peaks. Maximal difference between analytical and experimental fatigue lives for specimens tested with overloading peaks has not exceeded 30%.

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Analiza rozwoju pęknięć zmęczeniowych w stopie tytanu WT-9

Streszczenie

W pracy przedstawiono wyniki badań podstawowych charakterystyk propagacyjnych stopu tytanu WT-9, stanowiącego materiał, z którego wykonane są m.in. tarcze turbin i sprężarek współczesnych silników odrzutowych. Z tarczy sprężarki wycięto 11 próbek typu zwartego *CT* i przeprowadzono badania propagacji pęknięć zmęczeniowych. Wyniki pomiarów poddano analizie numerycznej, w celu określenia parametrów opisu prędkości rozwoju pęknięć zmęczeniowych w badanym materiale.

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