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SIMULATION OF THE ANTI-AIRCRAFT, LONG-RANGE MISSILE GUIDANCE

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A mathematical model of the missile guidance system in terms of the "three point" and the "half-to-half lead guidance law" methods, respectively, has been presented in the paper. A hypothetical anti-aircraft, long-range missile (a-a missile) and the target motion hypothesis have been assumed.

1. Introduction

A number of simulator types are now being employed when investigating the guidance system due to complexity and high exploitation costs of the relevant technical equipment, which should be used for this purposes.

Due to modern armament technology the necessity for proper service training and correct equipment operation and maintenance arises (technical tests, missile launching and propulsion data, presumed flight-paths, crew training etc.)

The majority of the aforementioned tasks, when undertaken practically, demands for the properly developed process of modelling, including all the physical phenomena involved. The role of modelling when solving technical, training and other engineering problems become more and more important due to broad accessibility to modern, effective computers (both software and hardware).

Modelling process plays also the essential role in simulation and investigation into flying objects dynamics. Simulators of such complicated processes as flights, air-fighting, guidance, propulsion data assumption (both launch and airdrop), etc., consist of numerical algorithms based on the corresponding mathematical models.

Notation employed in the present contribution comply with the Polish Standard PN-83 [8].

2. Missile guidance scheme and the target motion hypothesis

The hypothesis of a target constant speed, straight-line flight at a given altitude has been usually accepted when considering the problems of anti-aircraft missile launch-zones determination. The aforementioned assumption appears often to be unrealistic, especially when investigating the target maneuvers effect on the guided anti-aircraft missile dynamics. The target maneuver, defined in terms of the velocity variation (both in the magnitude and direction) influences dominantly dynamical properties of the missile when guided. Therefore the target maneuverability, resulting from its maximal permissible overloads should be taken into account.

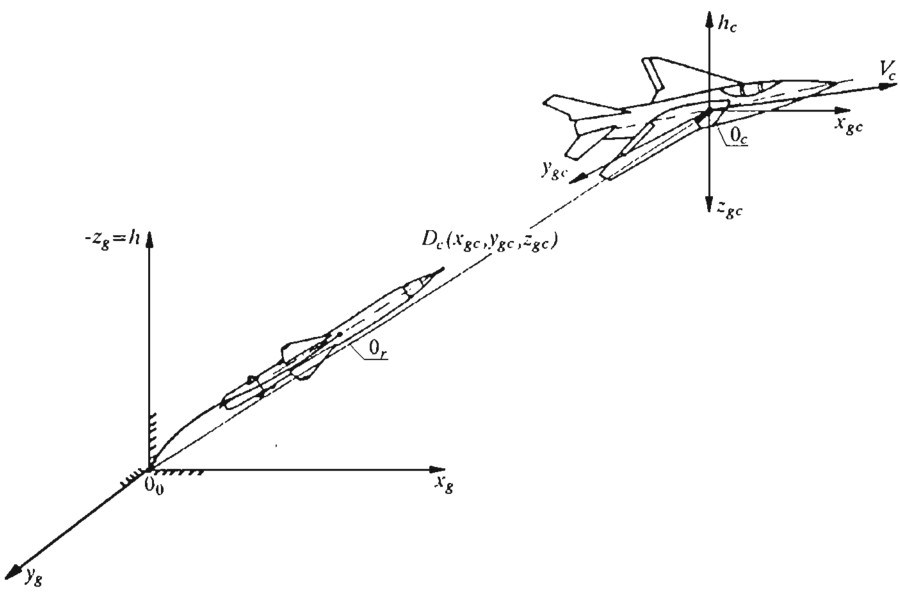

Fig. 1. Assumed scheme of the missile guidance onto the maneuvering target

The assumed guidance scheme of anti-aircraft missile to a maneuvering target is shown in Fig.1 and Fig.2. According to the scheme given in Fig.2 the target velocity vector $\boldsymbol{V}_c$ can be represented in the earth-fixed coordinate system $0_0 x_g y_g z_g$ by means of γ_c and χ_c angles, while the line-of-sight can be described by ε_c and β_c angles, respectively.

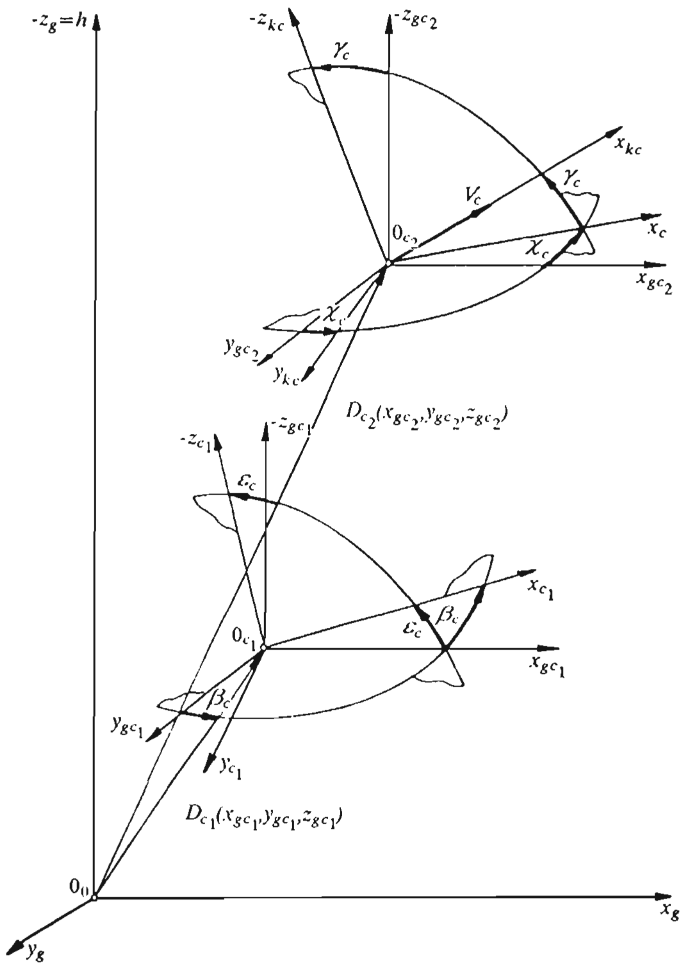

Fig. 2. Determination of the angles $\gamma_c, \chi_c, \varepsilon_c$ and β_c, respectively

The following mathematical model of target motion has been applied. Target translation can be represented by the following equations

$$\begin{aligned}
\frac{dx_{gc}}{dt} &= V_{xgc} = V_c \cos\gamma_c \cos\chi_c \\
\frac{dy_{gc}}{dt} &= V_{ygc} = -V_c \cos\gamma_c \sin\chi_c \\
\frac{dh_c}{dt} &= -V_{zgc} = V_c \sin\gamma_c
\end{aligned} \tag{2.1}$$

The formulae for distance D_c and velocity V_c, respectively, can be taken as

$$\begin{aligned}
D_c &= \sqrt{x_{gc}^2 + y_{gc}^2 + h_c^2} \\
V_c &= \sqrt{\dot{x}_{gc}^2 + \dot{y}_{gc}^2 + \dot{h}_c^2}
\end{aligned} \tag{2.2}$$

The formulae for ε_c and β_c angles read (Fig.2)

$$\sin\varepsilon_c = \frac{h_c}{D_c} \qquad \sin\beta_c = -\frac{y_g}{D_c\cos\varepsilon_c} \tag{2.3}$$

The above formulated mathematical model can be treated as a crucial element of the a-a missile guidance investigations algorithm. The target motion can be presumed in terms of the deliberate initial conditions choice, i.e., the values of x_{gc0}, y_{gc0}, z_{gc0}, V_{c0}, ε_{c0}, β_{c0}, respectively, or basing on the initial conditions choice supplied with the presumed variations of γ_c and χ_c angles, i.e., accepting the forms of functions $\gamma_c(t) = f_{1pr}(t)$, $\chi_c(t) = f_{2pr}(t)$.

3. Complete set of equations of 3D missile motion when guided onto an the airborne target

The earth-fixed $0_0x_gy_gz_g$ coordinate system has been assumed as the inertial frame of reference (Fig.3) when formulating the mathematical model of the missile guidance process relative the missile body-axis coordinate system $0xyz$ [8]. Formulae transferring the vectors from one coordinate system to another have also been established and called *geometrical and kinematic relations* (cf Gacek (1992))

The mathematical model of the rigid missile in 3D constrained motion relative the missile body-fixed coordinate system is represented by Eqs (2.1)÷(2.3) and (3.1)÷(3.3).

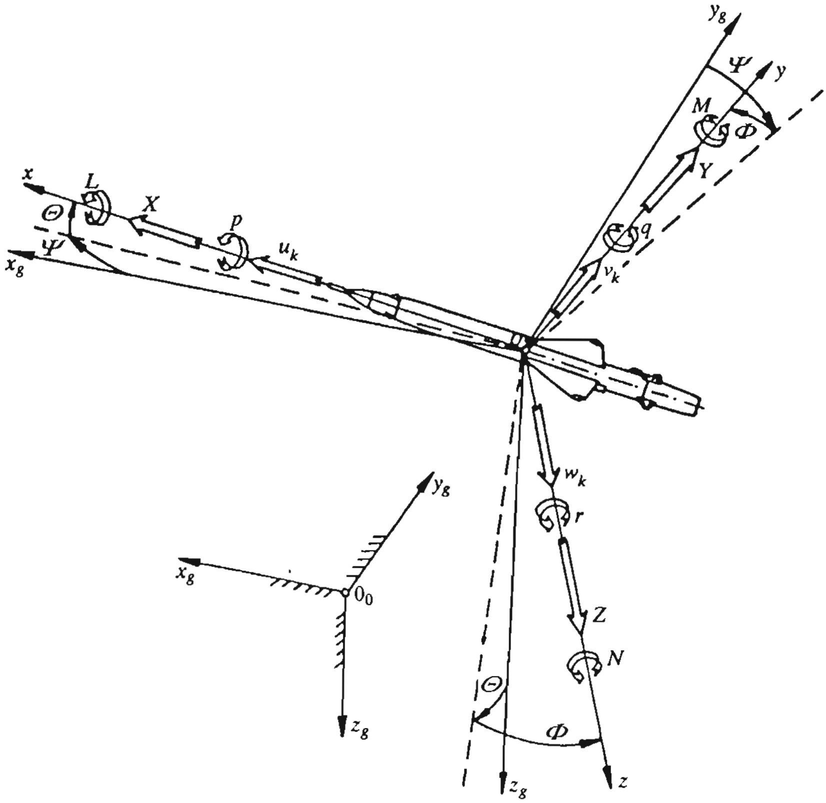

Fig. 3. Assumed linear x_g, y_g, z_g and angular Ψ, Θ, Φ coordinates, respectively, and their rates u_k, v_k, w_k, p, q, r; forces and moments acting upon the missile relative the body-axis system $0xyz$

(a) Dynamic equations of the missile center translation, i.e., longitudinal, lateral and lift motions, respectively, relative the object body-axis system $0xyz$, Fig.3, can be written in the form

$$m\Big(\frac{du_k}{dt} + w_k q - v_k r\Big) - S_x(q^2 + r^2) - S_y\Big(\frac{dr}{dt} - pq\Big) + S_z\Big(\frac{dq}{dt} + pr\Big) =$$

$$= F\cos\phi_{Fy}\cos\phi_{Fz} - mg(B,h)\sin\Theta - \frac{\rho(h)V^2}{2}S(C_x\cos\alpha\cos\beta +$$

$$+C_y\cos\alpha\sin\beta - C_z\sin\alpha) + P_s\cos\phi_{Py}\cos\phi_{Pz} + X_q^* q$$

$$m\Big(\frac{dv_k}{dt} + u_k r - w_k p\Big) + S_x\Big(\frac{dr}{dt} + qp\Big) - S_y(p^2 + r^2) - S_z\Big(\frac{dp}{dt} - qr\Big) =$$

$$= F\sin\phi_{Fz} + mg(B,h)\cos\Theta\sin\Phi + \frac{\rho(h)V^2}{2}S(-C_x\sin\beta + C_y\cos\beta) + \tag{3.1}$$

$$+P_s\sin\phi_{Pz} + Y_r^*r + Y_p^M p$$

$$m\Big(\frac{dw_k}{dt} + v_kp - u_kq\Big) - S_x\Big(\frac{dq}{dt} - pr\Big) + S_y\Big(\frac{dp}{dt} + qr\Big) + S_z(q^2+p^2) =$$

$$= -F\sin\phi_{Fy}\cos\phi_{Fz} + mg(B,h)\cos\Theta\cos\Phi - \frac{\rho(h)V^2}{2}S(C_x\sin\alpha\cos\beta +$$

$$+C_y\sin\alpha\sin\beta + C_z\cos\alpha) - P_s\sin\phi_{Py}\cos\phi_{Pz} + Z_q^*q + Z_p^M p$$

(b) Dynamic equations of the missile rotation about the center of mass; i.e., roll, pitch and yaw motions, respectively, relative the $0xyz$ coordinate system we can write as follows

$$I_x\frac{dp}{dt} - (I_y - I_z)qr - I_{xy}\Big(\frac{dq}{dt} - pr\Big) - I_{xz}\Big(\frac{dr}{dt} + qp\Big) - I_{yz}(q^2 - r^2) +$$

$$+S_y\Big(\frac{dw_k}{dt} - u_kq + v_kp\Big) - S_z\Big(\frac{dv_k}{dt} - w_kp + u_kr\Big) =$$

$$= F(y_F\sin\phi_{Fy}\cos\phi_{Fz} + z_F\sin\phi_{Fz}) + mg(B,h)(y_G\cos\Theta\cos\Phi +$$

$$-z_G\cos\Theta\sin\Phi) + \frac{\rho(h)V^2}{2}S[-y_A(C_x\sin\alpha\cos\beta + C_y\sin\alpha\sin\beta + C_z\cos\alpha) +$$

$$+z_A(C_x\sin\beta - C_y\cos\beta) - l(C_{la}^A\cos\alpha\cos\beta + C_{ma}^A\cos\alpha\sin\beta - C_{na}^A\sin\alpha)] +$$

$$+P_s(y_{ps}\sin\phi_{Py}\cos\phi_{Pz} + z_{ps}\sin\phi_{Pz}) + L_p^*p + L_r^*r$$

$$I_y\frac{dq}{dt} - (I_z - I_x)rp - I_{xy}\Big(\frac{dp}{dt} + qr\Big) - I_{yz}\Big(\frac{dr}{dt} - qp\Big) - I_{xz}(r^2 - p^2) +$$

$$+S_x\Big(\frac{dw_k}{dt} + v_kp - u_kq\Big) + S_z\Big(\frac{du_k}{dt} - v_kr - w_kq\Big) =$$

$$= F(z_F\cos\phi_{Fy}\cos\phi_{Fz} + x_F\sin\phi_{Fy}\cos\phi_{Fz}) +$$

$$-mg(B,h)(z_G\sin\Theta + x_G\cos\Theta\cos\Phi) + \tag{3.2}$$

$$+\frac{\rho(h)V^2}{2}S[-z_A(C_x\cos\alpha\cos\beta + C_y\cos\alpha\sin\beta - C_z\sin\alpha) +$$

$$+x_A(C_x \sin\alpha \cos\beta + C_y \sin\alpha \sin\beta + C_z \cos\alpha) +$$

$$+b_A(-C_{la}^A \sin\beta + C_{ma}^A \cos\beta)] +$$

$$+P_s(x_{ps} \sin\phi_{Py} \cos\phi_{Pz} + z_{ps} \sin\phi_{Py} \cos\phi_{Py}) + M_q^* q + M_p^* p$$

$$I_z \frac{dr}{dt} - (I_x - I_y)pq - I_{zy}\Big(\frac{dq}{dt} + rp\Big) - I_{zx}\Big(\frac{dp}{dt} - rq\Big) - I_{xy}(p^2 - q^2) +$$

$$+S_x\Big(\frac{dv_k}{dt} - w_k p + u_k r\Big) - S_y\Big(\frac{du_k}{dt} + v_k r + w_k q\Big) = F(x_F \sin\phi_{Fz} +$$

$$-y_F \cos\phi_{Fy} \cos\phi_{Fz}) + mg(B,h)(x_G \cos\Theta \sin\Phi + y_G \sin\Theta) +$$

$$+\frac{\rho(h)V^2}{2} S[x_A(-C_x \sin\beta + C_y \cos\beta) + y_A(C_x \cos\alpha \cos\beta + C_y \cos\alpha \sin\beta +$$

$$-C_z \sin\alpha) - b_A(C_{la}^A \sin\alpha \cos\beta + C_{ma}^A \sin\alpha \sin\beta + C_{na}^A \cos\alpha)] +$$

$$+P_s(x_{ps} \sin\phi_{Pz} - y_{ps} \cos\phi_{Py} \cos\phi_{Pz}) + N_r^* r + N_p^* p$$

(c) Kinematical relations between angular velocities (of roll, pitch and yaw, respectively) have the form

$$\begin{aligned}
\frac{d\Theta}{dt} &= q\cos\Phi - r\sin\Phi \\
\frac{d\Psi}{dt} &= (q\sin\Phi + r\cos\Phi)\sec\Theta \\
\frac{d\Phi}{dt} &= p + q\sin\Phi\tan\Theta + r\cos\Phi\tan\Theta
\end{aligned} \tag{3.3}$$

(d) Kinematic relations between linear velocities (in longitudinal, lateral and climb motions, respectively) in the earth-fixed $0_0 x_g y_g z_g$ system

$$\begin{aligned}
\frac{dx_g}{dt} &= u_k \cos\Theta \cos\Psi + v_k(\sin\Phi \sin\Theta \cos\Psi - \cos\Phi \sin\Psi) + \\
&+ w_k(\cos\Phi \sin\Theta \cos\Psi + \sin\Phi \sin\Psi) \\
\frac{dy_g}{dt} &= u_k \cos\Theta \sin\Psi + v_k(\sin\Phi \sin\Theta \sin\Psi + \cos\Phi \cos\Psi) +
\end{aligned} \tag{3.4}$$

$$\begin{aligned} &+ \quad w_k(\cos\Phi\sin\Theta\sin\Psi - \sin\Phi\cos\Psi) \\ \frac{dz_g}{dt} &= -u_k\sin\Theta + v_k\sin\Phi\cos\Theta + w_k\cos\Phi\cos\Theta \end{aligned}$$

where

$x_F, y_F, z_F, x_G, y_G, z_G, x_A, y_A, z_A, x_{Ps}, y_{Ps}, z_{Ps}$ – stand for coordinates of position vectors $\boldsymbol{r}_F, \boldsymbol{r}_G, \boldsymbol{r}_A, \boldsymbol{r}_{Ps}$ representing the points, through which pass the directions of the external forces acting upon the missile resultants (thrust $\boldsymbol{F}$, gravity forces $\boldsymbol{G}$, aerodynamic forces $\boldsymbol{R}_A$, and control forces $\boldsymbol{P}_s$) in the body-axis coordinate system $0xyz$ (Fig.3), respectively

ϕ_{Fy}, ϕ_{Fz} – angular eccentricity of the thrust $\boldsymbol{F}$ in $0xyz$ coordinate system

ϕ_{Py}, ϕ_{Pz} – angular eccentricity of the control force $\boldsymbol{P}_s$ in $0xyz$ coordinate system

b_A – mean aerodynamic chord

S – characteristic area

B – latitude.

Eqs (3.1)÷(3.4) when applied to the missile guidance analysis, should be supplied with the following additional equations:

- Formulae for the current values of parameters describing the missile structure: $m(t)$, $x_{sm}(t)$, $y_{sm}(t)$, $z_{sm}(t)$, $S_x(t)$, $S_y(t)$, $S_z(t)$, $I_x(t)$, $I_y(t)$, $I_z(t)$, $I_{xy}(t)$, $I_{yz}(t)$, $I_{xz}(t)$, $F(t)$, (cf Gacek (1992); Gacek et al. (1993a))
- Formulae for the current values of parameters describing the air (cf Gacek (1992); Gacek et al. (1993a)):
  - acceleration of gravity $g(B,h)$
  - ambient air parameters
- Formulae for the forces and moments acting upon the missile in 3D motion (cf Gacek (1992); Gacek et al. (1993a)).

Having known forces and moments (thrust, aerodynamic and control) appearing in right-hand sides of Eqs (3.1)÷(3.4) one can employ the above mathematical model to a complete dynamic properties analysis and simulation of any rigid missile 3D motion.

4. Conclusions

- The mathematical model presented above can be treated as a crucial element of the theoretical investigations algorithm into the a-a missile guidance.
- The verification process applied to both stages of research i.e., mathematical and physical modelling and computation should be treated very carefully when theoretically investigating the dynamic properties of flying objects. The physical ranges of the model validity should be predetermined. In the present contribution the verification process based on the available experimental results for the real missile, aerodynamic scheme of which was similar to the considered one has been performed.
- Numerical implementation of the above method will be given in the forthcoming paper.

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Model symulacyjny procesu naprowadzania rakiety przeciwlotniczej dalekiego zasięgu

Streszczenie

Dla przyjętej hipotetycznej rakiety przeciwlotniczej (rakiety plot) dalekiego zasięgu i hipotezy ruchu celu powietrznego, opracowano model matematyczny procesu naprowadzania rakiety metodą trzech punktów oraz metodą polowicznego prostowania toru.

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