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CLASSICAL PANEL METHODS – A ROUTINE TOOL FOR AERODYNAMIC CALCULATIONS OF COMPLEX AIRCRAFT CONFIGURATIONS: FROM CONCEPTS TO CODES

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Among all the methods of computational fluid dynamics perhaps most significant are the panel methods (PMs). This paper not only reviews existing techniques for calculating aerodynamic loads for realistic aircraft configurations, but also provides an account of mathematical ideas needed to understand them. The scope of this paper covers the main issues of problems of engineering analysis and design by means of PMs. Results of example applications of these methods to particulary complex configurations encountered in aircraft industry are also included. A selection is given at both subsonic and supersonic speeds.

Notation

BC	–	boundary condition
PM	–	panel method
VLM	–	vortex lattice method

1. Prologue

1.1. Introduction

As is known, Euler and even Navier-Stokes models can be today solved using field methods (mainly the finite difference method) with the aid of supercomputers. Nevertheless, applying the Navier-Stokes model to all problems is not only impractical today, but may not satisfy other necessary criteria such as timeliness, accuracy, reliability, availability of computing resources, and management confidence. E.g., in the Euler or Navier-Stokes formulation the shed wake becomes a natural part of the solution. In these methods the wakes are naturally convected downstream. However, this does not automatically guarantee a better solution than the simplified approach taken in potential methods (Tinoco (1990), p.567).

PMs based on potential flow models are still most popular tools in numerical aircraft-oriented aerodynamics. Application of PMs seems to be especially reasonable at preliminary design stage of complex configurations when high speed and low cost of computations are most important.

The primary aim of this paper is to provide a clear account of the mathematical ideas needed to understand these methods in engineering analysis and design. For this reason each section includes the basic concepts of advance in the field required to implement the techniques in practical engineering activity.

The paper is divided into five sections. The first two ones have an introductory character, while next two section contain the bulk of the paper. Section 3 presents a general methodology for the solution to potential flows problems, whereas Section 4 provides a global look at the state of the art in PMs. The final Section includes some examples of applications of selected codes to particulary complex configurations at both subsonic and supersonic speeds.

1.2. Brief disscusion of previous reviews

Review papers on PMs have been published for a long time on several occasions. A complete review of these papers is beyond the scope of this paper; only publications that appeared in the last decade and concern formulation in the field of PMs in aerodynamics are discussed here. So, we limit our considerations to reviews written by Hess (1985) and (1990), Hunt and Hewitt (1986), Wagner (1987), Erickson (1990), Morino (1993).

Hess (1985) presented the comprehensive review of the historical development of surface source methods. He makes also a comparison between first-order and higher-order methods and notices that in higher-order methods both the local surface curvature and the singularity variation over a panel must be taken into account. Since analytic integrations can be performed only over flat panels even for variable singularity strengths, so many authors incorrectly refer to such methods with flat panel as to higher-order ones. It is worthy to emphasize that paper by Hess (1985) contents extremely valuable discussion of numerical efficiency and presents (following Slooff (1981)) a number of challenges and problems to solve in the future.

Hess (1990) pointed out the important role of the Kutta-Joukowski condition. According to him theoretical considerations concerning this condition are even more important than the so-called *singularity "mix"*, i.e., the choice of particular relation between source and dipole distributions. It may also be noted that selection of the PMs generalizations applied to 3D fluid-dynamic problems is briefly discussed. This discussion includes propellers, transonic applications, and free-surface applications (surface-ship and underwater-structure problems).

An extensive paper of Hunt and Hewitt (1986) gives a comprehensive look at three aspects of the boundary integral method; namely,

- A discussion of the fundamental physics involved shows how one of the practical boundary integral models employed in routine aerodynamic applications to incompressible, irrotational flows may be interpreted in pure physical terms
- A detailed mathematical analysis of the linearised elliptic formulation results in the so-called "indirect" boundary integral formulation which allows source and vorticity distributions of very low order to be used to give accurate and economical solution
- Detailed discussion of the linearized hyperbolic equation indicates how methodologies have been developed which allow boundary integral formulations to be successfully applied to supersonic flow problem. A number of computed examples are presented as well. The paper is rather difficult to follow because of too much mathematical consideration introduced.

Wagner (1987) presents interesting classification of flows for sharp leading-edge delta wing according to the angle of attack and Mach number normal to the leading edge. Then, the main fundamentals of panel methods are discussed. Some examples for the aerodynamic analysis of complex configuration by PMs are presented that show the versatility of these methods in application. On the other hand these indicate that further developments towards a hybrid method, e.g., an integral equation method with the embedded Euler domain will be necessary.

Erickson (1990) discusses the PM capabilities and limitations, basic concepts common to all PM codes, different choices that have been made in the implementation of these concepts into working computer programs, and various modeling techniques involving boundary conditions, jump properties, and trailing wakes. Three appendixes supplement the main text giving details of computer program (PANAIR), showing how to evaluate analytically the fundamental surface integrals and presenting the so-called finite part of improper integrals, respectively.

The extensive review of Morino (1993) presents a unified methodology for the analysis of potential and viscous flows. Worthy to note is that Morino understands aerodynamics as inclusive of unsteady flows. Hence, in that paper he always starts with the formulations for unsteady flows and recovers the steady-state solution from a transient response as time goes to infinity. The use of the Helmholtz and Poincaré decompositions to examine viscous flows is described. The distinguishing feature of Morino's review is the fact that aspects of the formulations that require additional work have been pointed out.

Comparing these reviews one can say that some of them are more practically oriented (cf Erickson (1990); Hess (1985) and (1990)) and contain working formulae and valuable results, useful in numerical package testing. Other present mathematical foundation and even philosophical considerations (cf Hunt and Hewitt (1986); Morino (1993); Wagner (1987)). All of them are helpful in understanding panel methods, their state-of-the-art and challenges for the future.

As follows from the above considerations each review paper reveals rather a specialized interest and not very wide look. If a paper contains mathematical framework, so does not even give any information about specialized problems, such as unsteady effects, e.g., Erickson (1990).

Our main purpose is to present the PMs as full, comprehensive, coherent and relatively simple ones, therefore we decided to include formulation of the problem, mathematical background, methodology of solution, presentation of some codes, selected numerical results and future challanges. Such intention is ambitious and difficult to realize in a reasonable volume, so we decided to divide our work into two separate parts: the first one devoted to classical PMs, whereas the second one to modified (advanced) PMs. This paper contains the first part, the second one is entitled: "Modified Panel Methods with Examples of Applications to Complex Flowfield Calculations", [13].

1.3. Problem formulation

Classical PMs are numerical schemes for solving the *Laplace equation*

$$\varphi_{xx} + \varphi_{yy} + \varphi_{zz} \equiv \nabla^2\varphi = 0 \tag{1.1}$$

where $\{x, y, z\}$ denotes a movable (body fixed) reference frame and φ is the perturbation velocity potential.

Some authors refer classical PMs to the *Prandtl-Glauert equation*

$$(1 - \mathrm{Ma}_\infty^2)\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \tag{1.2}$$

where Ma_∞ is the Mach number of the uniform flow. However, we will limit our further considerations to Eq (1.1) because the *Prandtl-Glauert transformation* to new movable (body fixed) reference frame $\{x_N, y_N, z_N\}$ in the form

$$x = x_N\sqrt{1 - \mathrm{Ma}_\infty^2} \qquad\qquad y = y_N \qquad\qquad z = z_N \tag{1.3}$$

converts Eq (1.2) into Eq (1.1).

Set up the connection between the descriptions in the unmovable (space fixed) reference frame (denoted by K_0) and movable (body fixed) reference frame (denoted by K), Fig.1. Consequently, we shall denote by the subscript 0 all measurements and operations made with respect to the frame K_0 and denote without any subscript all measurements and operations made with respect to the frame K.

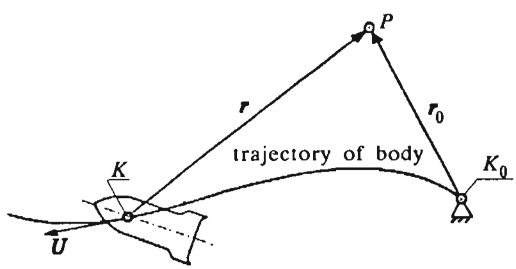


Fig. 1. Relation between space-fixed and body-fixed descriptions

It can be proved (cf Karamcheti (1966), p.284) that the various differentiation operations in the two frames are related as follows

$$\nabla_0 = \nabla \tag{1.4}$$

$$\nabla_0^2 = \nabla^2 \tag{1.5}$$

$$\frac{\partial}{\partial t_0} = \frac{\partial}{\partial t} - \boldsymbol{U}(t)\nabla \tag{1.6}$$

From Eqs (1.5) and (1.3) it follows directly that the Laplace equation for the perturbation velocity potential is valid in both the reference frames. Since full velocity potential can differ from the perturbation velocity potential only by a linear function, so from Eq (1.5) it follows that full potential satisfy the Laplace equation relative both the reference frames also. Moreover, the important formulae for pressure calculations, on the basis of Eq (1.6), are different in both the reference frames, i.e.

$$\begin{aligned} p_0(\boldsymbol{r}_0,t_0) &= p(\boldsymbol{r},t) = p_\infty - \rho\Big[\frac{\partial\varphi_0}{\partial t_0} + \frac{1}{2}(\nabla_0\varphi_0)^2\Big] = \\ &= p_\infty - \rho\Big[\frac{\partial\varphi}{\partial t} - U(t)\nabla\varphi + \frac{1}{2}(\nabla\varphi)^2\Big] \end{aligned} \tag{1.7}$$

Because for a rigid body the function specifying the surface of the body becomes independent of time if described from a movable reference frame, so all further considerations will be carried out just in the body fixed (movable) reference frame.

In order to complete the problem we need to formulate the proper BCs on the body surface, at the trailing edge (*Kutta-Joukowski condition*), and at infinity, respectively.

The first BC requiring zero normal velocity across the body solid boundaries

$$\nabla\varphi\,\boldsymbol{n} = 0 \tag{1.8}$$

where $\boldsymbol{n}$ is a unit vector normal to the body.

Along the wing trailing edges the velocity has to be limited in order to fix the rear stagnation line and therefore

$$\nabla\varphi < \infty \tag{1.9}$$

The third BC requires that the flow disturbance, due to the body motion through the fluid, should diminish far from the aircraft

$$\lim \nabla\varphi = 0 \tag{1.10}$$

In the PM approach, a governing differential equation is converted to an integral one over the configuration surface by means of the Third Identity of Green (cf Kellog (1967), Chap.VIII, Sec.4), that from here will be call simply the Green Theorem. This integral equation is then solved by means of a discretization process. The configuration under consideration is divided into panels to which a certain distribution of singularities of unknown strength is assigned. Thus, PMs should be more precisely called as the *surface-singularity methods.*

In classical PM codes a combination of source and doublet distributions on the panels is used. Some codes use elementary horseshoe vortices instead of doublets. The strength of the singular elements are determined by satisfying the proper boundary conditions. Once these are known, the surface velocity components and pressures can be calculated by using

$$C_p = 1 - \frac{V^2}{V_\infty^2} \tag{1.11}$$

where V is the velocity at the panel surface due to the freestream and the perturbation velocity potentials.

When one writes a review of PMs, one has to bear in mind that the main feature of classical PMs consist in their capability to predict correctly the aerodynamics in a linear approach only. In this context it is worth noting that the field of computational fluid dynamics may be conveniently divided into two major areas:

1. Subsonic and supersonic regimes, where the nonlinear terms in the differential equation are negligible

2. The transonic regime, where the nonlinear terms are essential to describe the phenomenon.

Mathematically speaking, a problem is called a *nonlinear problem* if the governing equation is nonlinear and/or if the boundary conditions are nonlinear. So, for inviscid and low-subsonic flows at high angles of attack, the problem is nonlinear, although the governing equation is linear (Laplace or Prandtl-Glauert model). The nonlinearity is due to the BCs, that are formulated on the separated flow surfaces. For inviscid and high – subsonic flows at low angle of attack, the problem is also nonlinear, because the governing equation is nonlinear (the full potential equation; see Eq (6) in Goraj and Pietrucha (1993)). Of course, for compressible flows at high angle of attack, the problem is nonlinear due to both factors.

Suming up: starting from the *Navier-Stokes model*, which is the most general model representing the flow, the following assumptions are necessary to derive Eq (1.1):

- No viscosity terms

- Irrotationality of the flow

- Steady state
- Small perturbation approximation
- Incompressible flow.

Therefore, it is clear that PMs are applicable to the flow conditions for which, at least in a global sense, the flow can be considered inviscid, irrotational, and stationary in respect to time. Such methods we will call the "classical PMs". Thus, it is essential to note that in practice numerous existing PMs incorporate processes which allow some account to be taken of the finite Reynolds number (by including boundary layer effects), the influences of compressibility (e.g., by considering the Prandtl-Glauert equation), and unsteady effects. Such and the like PMs we are calling the "modified PMs" and will be considered in the next paper [13].

Now, we are able to give a bit more full determination of classical PMs. So, we make the following assumptions:

1. Linearity of governing equations and BCs
2. Flateness of the vortex surface
3. Shedding-up of the wake from trailing edge only
4. Steadness of the flow.

2. Mathematical background of the panel methods

2.1. Boundary-value problems

A combination of a partial differential equation and BCs is known as a *boundary value problem.* A boundary value problem is a *wellposed* one if:

1. There exists a solution to the problem
2. This solution is unique
3. The solution depends continuously on the source term and on the boundary data.

Sometimes a physical phenomenon is described by a boundary value problem that is not well posed, but in such cases there are serious implications as to the instability of the physical problem (cf Stakgold (1968), p.89). A thorough investigation is then needed to decide whether it is the physical problem which is in some sense unstable or whether, instead, an error has been made in translating the physical problem into its mathematical formulation. Worthy of note is an opinion: "knowing that a solution exists and is unique is important becauce one would not find himself fallowing a blind alley." (cf Smith (1990), p.7).

Theory of the Laplace equation (and the more general case of Poisson equation) is referred to as a potential theory. Both equations are the partial differential equations of elliptical type. There are two main forms of boundary-value problems, namely the *Dirichlet form* or the *Neumann form*. Both forms can be formulated using *internal* or *external* approaches.

In internal formulation a potential field in region B, bounded by a closed surface S, is considered. In aircraft aerodynamics the external Neumann conditions are usually considered, since in most cases we do not know the potential distribution, whereas we know the potential derivatives, normal to the surface and equal to the velocity components.

For clarity we limited our considerations to the external conditions. To do it let us define the region

$$V := E^3 \setminus (B + S) \tag{2.1}$$

laying in unlimited space, out of the body B.

The boundary value problem for the Laplace equation consists in finding φ, satisfying this equation in the region (2.1) and such that for the Dirichlet condition we have

$$\varphi|_S = f(x, y, z) \tag{2.2}$$

and for the Neumann condition we have

$$\frac{\partial \varphi}{\partial n}\Big|_S = g(x, y, z) \tag{2.3}$$

where f and g are functions given *a priori*.

In both kinds of boundary value problem the potential should vanish at infinity. One should emphasize that the conditions (2.2) and (2.3) could not be given simultaneously. However, it is possible to require that the Dirichlet condition is satisfied in one region of the boundary while the Neumann condition is satisfied in another region. Such form is usually called as *the mixed* (sometimes Poincaré) boundary value problem.

A powerful method for the study of boundary problems makes use of the so-called *fundamental solution*. The modern definition of this solution (that is denoted by E) is as follows (cf Marcinkowska (1972), p.48)

$$\nabla^2 E = \delta(\boldsymbol{r}_i) \tag{2.4}$$

where

δ – Dirac delta function

$\boldsymbol{r}_i$ – distance between the "source" point and the "observation" point, namely

$$r = |\boldsymbol{r}_i| = |\boldsymbol{x} - \boldsymbol{x}_i| \tag{2.5}$$

The fundamental solution to the Laplace equation for 3D problem is

$$E = \frac{1}{4\pi r} \tag{2.6}$$

whereas in 2D case is

$$E = \frac{1}{2\pi} \ln \frac{1}{r} \tag{2.7}$$

It is very important for further considerations to note that the function E is a function of two points: the source point $\boldsymbol{x}_i$ at which arises the singularity of δ, and the observation point $\boldsymbol{x}$ which is the variable involved in Eq (2.4). By the way, it should be noted that one also faces the notions "the influencing point" and "the influenced point", respectively.

2.2. Boundary integral formulation

In order to formulate the Dirichlet and Neumann problems in terms of boundary integral equations, we will use the so-called fundamental formula for harmonic functions (cf Marcinkowska (1972), p.140) in the form

$$\varphi(P) = \int\limits_S \Big(\frac{\partial E}{\partial n}\varphi - E\frac{\partial \varphi}{\partial n}\Big)\, dS \tag{2.8}$$

where E is the fundamental solution given by Eq (2.6) or (2.7), and $\boldsymbol{n}$ is the outward normal to the surface. According to Eq (2.8) and taking into account the conditions (2.2) and (2.3) we have for the 3D case

$$\varphi(P) = \frac{1}{4\pi} \int\limits_S \Big[g(Q)\frac{1}{r(P,Q)} - f(Q)\frac{\partial}{\partial n}\frac{1}{r(P,Q)}\Big]\, dS(Q) \tag{2.9}$$

where P is a point outside B, and Q is a point on S. As was mentioned in Section 2.1 the conditions (2.2) and (2.3) cannot be given simultaneously, and thus Eq (2.9) is not a solution! In order to find it, consider the asymptotic process $P \to R \in S$. As a result we have the equation

$$f(R) - \frac{1}{2\pi} \int\limits_S \Big[g(Q)\frac{1}{r(P,Q)} - f(Q)\frac{\partial}{\partial n}\frac{1}{r(P,Q)}\Big]\,dS(Q) = 0 \tag{2.10}$$

which is the *boundary integral equation*, because it establishes the relation between f and g on the boundary S. From the mathematical point of view this is the Fredholm equation of the first and second kind for a functions g and f, respectively.

After resolving Eq (2.10) we can already obtain the solution to the Laplace equation from Eq (2.9). Realization of a procedure for obtaining of a numerical solution to Eq (2.10) is termed a *panel method.* The name "panel method" comes from the approximating treatment of the configuration surface by a set of panels on which unknown "singularity strength" are defined.

It is known from Lamb (1932) that under fairly general circumstances the disturbance potential due to a body may be expressed as an integral over the body surface of a source distribution σ and a doublet distribution μ, i.e.

$$\varphi = \frac{1}{4\pi} \int\limits_S \Big[-\frac{1}{r}\sigma + \frac{\partial}{\partial n}\Big(\frac{1}{r}\Big)\mu\Big]\,dS \tag{2.11}$$

This is the main formula in aerodynamic calculations.

2.3. Jump properties

We will now give some important properties of the singularity distributions, that are commonly used in aerodynamic PMs. Let us denote two sides of the surface S as U (the upper) and L (the lower) sides, respectively. Also denote $\boldsymbol{n}$ versor, normal to the surface and considered to be positive if directed from the lower side to the upper one.

By virtue of Eq (2.11), the potential φ_P and velocity $\boldsymbol{V}_P$ induced at an arbitrary (external) point P by a source distribution σ on an arbitrary surface S are given by

$$4\pi\varphi_P^s = -\int\limits_S \frac{1}{r}\sigma\,dS \tag{2.12}$$

$$4\pi\frac{\partial\varphi_P^s}{\partial \boldsymbol{n}} = -n\int\limits_S \frac{\partial}{\partial n}\Big(\frac{1}{r}\Big)\sigma\,dS = \boldsymbol{n}\int\limits_S \frac{1}{r^2}\sigma\,dS \tag{2.13}$$

where r is drawn to the point P from the variable point Q on S.

For a point $P \in S$, a limit process shows that the contribution to $4\pi\partial\varphi/\partial n$ due to the infinitesimal portion of S is $2\pi\sigma_P$ if P lies on the face U, whereas derivative $\partial\varphi/\partial n$ is equal to $-2\pi\sigma_P$ if P lies on the face L. Therefore, the discontinuity across the surface from L face to the U one is given by

$$\sigma = \Delta\frac{\partial\varphi}{\partial n} \tag{2.14}$$

where the symbol Δ denotes the difference between the limiting values of derivative $\partial\varphi/\partial n$. Positive derivative $\partial\varphi/\partial n$ means the potential growth towards the outward vector $\boldsymbol{n}$ and corresponds to the positive σ.

It is also worth noting that the local singular contribution to $4\pi\varphi_P$ is equal to zero and thus the potential φ is continuous across S.

In the same way we can obtain the discontinuity for doublets of strenght μ (the doublet axis being orientated from the "sink" to the "source"). Once again by virtue of Eq (2.11), the potential φ_P induced at an arbitrary (external) point P by a doublet distribution μ on an arbitrary surface S is given by

$$4\pi\varphi = \int\limits_S \frac{\partial}{\partial n}\Big(\frac{1}{r}\Big)\mu\, dS \tag{2.15}$$

A limit process is required as a point P approaches S. This shows that the singular contribution to $4\pi\varphi$ due to the infinitesimal portion of S is $-2\pi\mu_P$, if P lies on the face U whereas it takes the value $2\pi\mu_P$ if P lies on the face L. Therefore, the discontinuity in potential crossing S in the direction $\boldsymbol{n} = \boldsymbol{n}_U = -\boldsymbol{n}_L$ is

$$\mu = -(\varphi_U - \varphi_L) \tag{2.16}$$

It can be also shown that the normal velocity component $\boldsymbol{V}_n$ is continuous across S, whilst the tangential velocity $\boldsymbol{V}_t$ jumps by the value equal to the tangential derivative of the jump in potential (cf Hunt (1980), Sec.4.3).

3. Implementation to aerodynamics

3.1. Origin of the concept

The PM idea in aerodynamics was created in Douglas Aircraft Company (Long Beach, California) in 1952. It was connected with the task of computing the flow around the aircraft body, designed according the area rule (introduced by Richard Whitecomb in 1952). The primary algorithm and program in machine code was written for IBM/701 by A.M.O.Smith and J.Pierce. In 1956 dr John L. Hess joined the Douglas aerodynamic group. After a short time he became the creator and author of most of the algorithms. So, the PM is usually associated with the Hess name (cf Hess (1985)).

3.2. Preliminary considerations

The idea presented in Section 2.2 is very universal and can be applied to various engineering problems of continuum mechanics (such as stress analysis, heat conduction, electromagnetic field analysis, etc.). Using this idea in fluid mechanics requires some improvements, due to

- Body movements

  There is considered the fluid movement (or body movement) and the choice of coordinate system is important. In the system connected with the body the velocity potential $\phi = \phi_\infty + \varphi$, where $\phi_\infty = U_\infty x$, whilst in the unmovable system $\phi_\infty = 0$.

- Wake existance

  Behind the body there is created the so-called *wake*, which should be described somehow. So, the surface S has to be divided into sub-areas

$$S = S_B + S_W + S_\infty \tag{3.1}$$

  that are shown in Fig.2 (we have taken a cut through a wing and its semi-infinite wake).

- Interior potential

  An actual surface panel model of a body generally produces a set of panels that separates space into two or more distinct regions: enclosed interior volumes and an external volume extending from infinity to the external side of the panels. The flow in the external volume corresponds to the physical flow field being modeled. Although the flow in the interior volumes is ficticious but can be used to advantage. It must be remembered that we are using surface-singularities and BCs to create the flow fields, and therefore flow exists on either side of the panels. The internal-external fields are, in general, independent of one another.

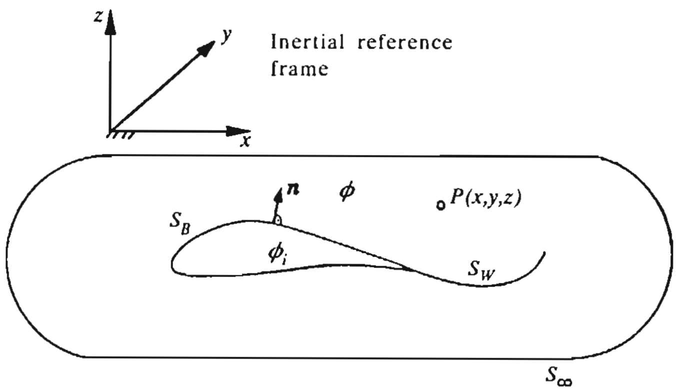


Fig. 2. Notation for definition of the potential flow problem

We assume the existence of velocity potentials ϕ in the flowfield and ϕ_i inside the wing. Applying Green's Third Identity (cf Kellog (1967), Chapt.VIII, Sec.4) to the inner and outer regions, and combining the resulting expressions, the velocity potential at the point P on the inside surface can be written

$$\phi(P) = \phi_\infty(P) - \frac{1}{4\pi}\int\limits_{S_B}\Big(\sigma\frac{1}{r} - \mu\frac{\partial}{\partial n}\frac{1}{r}\Big)\;dS + \frac{1}{4\pi}\int\limits_{S_W}\mu\frac{\partial}{\partial n}\frac{1}{r}\;dS \tag{3.2}$$

where, according to Eq (2.16)

$$\mu = -(\phi - \phi_i) \tag{3.3}$$

$$\sigma = \frac{\partial\mu}{\partial n} \tag{3.4}$$

In principle, an infinite number of combinations of doublet and source distribution will give the same external flowfield, but different internal flowfields. To render a unique combination of singularities we can either specify one of the singularity distributions (e.g., $\sigma = 0$ in the doublet-only formulation) or we can specify the internal flow. It is possible to require that

$$\phi = \phi_i \qquad \text{on} \qquad S_B \tag{3.5}$$

and in this case the doublet term on S_B vanishes and the problem will be modeled by a source distribution on the boundary.

3.3. Methodology of solution

The basic concept of PM is illustrated in Fig.3. The configuration is modeled by a large number of elementary panels lying either on the actual aircraft surface, on some mean surface, or a combination thereof. To each elementary panel there is attached one or more types of singularity distribution, such as sources, doublets, vortices, and so on. These singularities are determined by specifying some functional variation across the panel (e.g., constant, linear, quadratic, etc.), actual value of which is set by corresponding strength parameters.

Eq (1.3) is solved by an integral approach (see Eq (2.11)). The perturbation velocity is then given by

$$\boldsymbol{V} = \frac{\partial \varphi}{\partial \boldsymbol{n}} = \frac{1}{4\pi} \int\limits_{S} \Big[-\Big(\frac{\partial}{\partial n}\frac{1}{r}\Big)\sigma + \frac{\partial}{\partial n}\Big(\frac{\partial}{\partial n}\frac{1}{r}\Big)\mu \Big] \, dS \tag{3.6}$$

where r is the distance between the (x, y, z) coordinates of a field point P, and the (ξ, η) coordinates of a point Q on a surface S (see Fig.3). If sources of strength $\sigma(\xi, \eta)$ and doublets of strength $\mu(\xi, \eta)$ are distributed on the surface (with the doublet axis normal to S), Eqs (2.11) and (3.6) give the formulas for calculating the potential and velocity that these distributions produce at point P.

Once σ and μ are specified, Eqs (2.11) and (3.6) can then be integrated so that $\varphi(x, y, z)$ and $V(x, y, z)$ are expressions involving only the unknown singularity parameters. If point P is assumed to be a control (collocation) point on the wing surface (there is one control point per panel – see Fig.3), both equations give the potential and velocity at this point in terms of the source and doublet distributions of a single panel. Summing the effects from all panels on the wing surface gives the potential and velocity at the point P in terms of the total number N of singularity parameters. Repeating the process for all N points P, and imposing BCs at these control points, leads to the linear algebraic equation

$$[AIC]\{\lambda\} = \{RHS\} \tag{3.7}$$

In this equation, $[AIC]$ is called the *matrix of aerodynamic influence coefficients* (of $N \times N$), $\{\lambda\}$ is the vector of unknown singularity parameters, and $\{RHS\}$ is the so-called *right-hand-sides vector* that expresses the BCs of flow tangency on the surface. Once the singularity parameters are obtained from Eq (3.7), the corresponding potential and velocity field, and next the pressure field, can be computed.

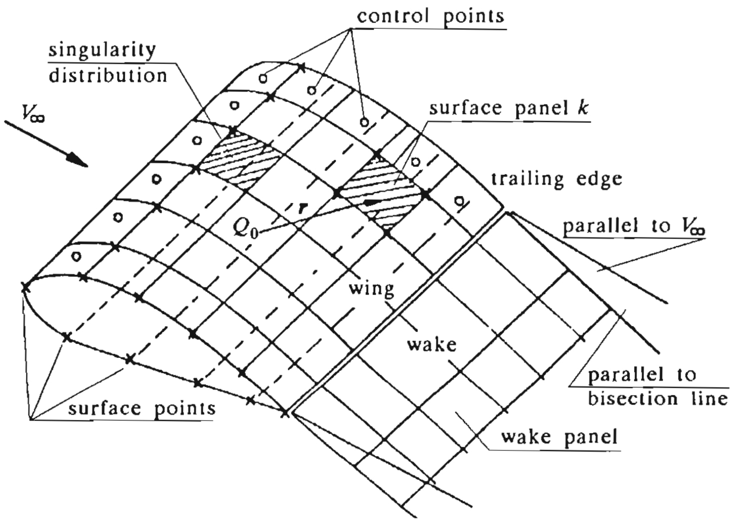


Fig. 3. Panel arrangement – influence of panel k on control point Q

The above described process, in one or another form, is common to all PMs. The choice of particular relations between σ and μ that produce a numerically well-behaved $[AIC]$ matrix is the key problem. In this context it is useful to note that there exists an important equivalence between doublet and vortex distributions on a body surface (cf Katz and Plotkin (1991), p.86). This equivalence also helps to explain such items as the inability of the constant-doublet – strength model to yield good velocity fields by summing the velocities induced by the individual panels and its lack of an explicit Kutta – Joukowski condition (cf Maskew (1982), p.277).

3.4. Example of 2D numerical solution

To present briefly this method, we shall consider, following Yeo et al. (1992), an airfoil with known boundary S_B, submerged in a potential flow (Fig.4). The flow of interest is in the outer region where Laplace's equation in terms of the total velocity potential is fullfiled. Following Eq (2.11) and taking u as $\ln r$ (for the two-dimentional case) the general solution to Eq (2.7) can be obtained using source σ and doublet μ distributions placed on the surface

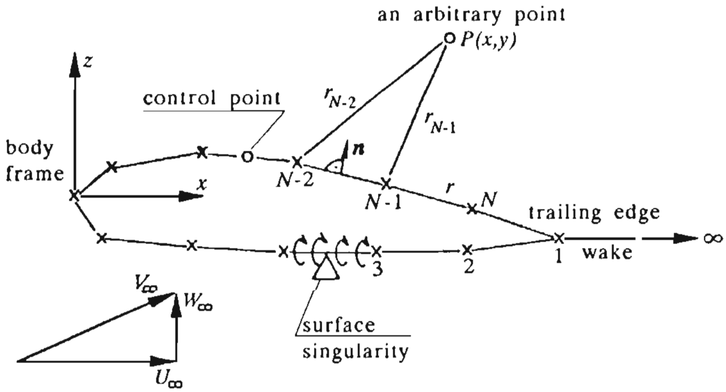


Fig. 4. Notation for a panel on an airfoil

of boundary S (which includes S_B and possibly a wake surface S_W)

$$\phi(x,z) = \phi_\infty - \frac{1}{2\pi} \int_S \left(\sigma \ln r - \mu \frac{\partial \ln r}{\partial n}\right) dS \tag{3.8}$$

Here r is the distance between the singularity elements (source or doublet) and a field point, and ϕ_∞ is the freestream potential

$$\phi_\infty = U_\infty x + W_\infty z \tag{3.9}$$

Place the point (x,z) inside the surface S_B. Then the inner potential ϕ_i in terms of the surface singularity distribution is obtained as

$$\phi_i(x,z) = \phi_{\infty i} - \frac{1}{2\pi} \int_S \left(\sigma \ln r - \mu \frac{\partial \ln r}{\partial n}\right) dS \tag{3.10}$$

Note that if for the enclosed boundary $\partial\varphi_i/\partial n = 0$ as required by the BC, then the potential inside the body will not change (cf Lamb (1932), p.41)

$$\phi_i(x,z) = \text{const} \tag{3.11}$$

By seting $\phi_i = (\varphi + \phi_\infty)_i = \phi_\infty$ and assuming that $\phi_{\infty i} = \phi_\infty$ we obtain

$$\varphi_i = 0 \tag{3.12}$$

So, from Eqs (3.10) and (3.12) it follows that

$$\varphi_i(x,z) = -\frac{1}{2\pi} \int_S \left(\sigma \ln r - \mu \frac{\partial \ln r}{\partial n}\right) dS = 0 \tag{3.13}$$

From the Neumann BC in the form

$$\frac{\partial \phi_i}{\partial n} = \frac{\partial \varphi_i}{\partial n} + \frac{\partial \phi_\infty}{\partial n} = 0 \tag{3.14}$$

and the surface source definition (see Eq (2.13))

$$\sigma = \frac{\partial \phi}{\partial n} - \frac{\partial \phi_i}{\partial n} \tag{3.15}$$

one obtains

$$\sigma = \frac{\partial \phi}{\partial n} \tag{3.16}$$

Since $\partial\phi/\partial n$ is equal to $\boldsymbol{Q}_\infty \boldsymbol{n}$ (accounting that the positive derivative $\partial\phi/\partial n$ corresponds to the normal vector $\boldsymbol{n}$ which is pointed outwards of S_B, Fig.2 and Fig.4), we obtain

$$\sigma = \boldsymbol{Q}_\infty \boldsymbol{n} \tag{3.17}$$

To proceed with the solution, S_B is divided into discrete elements and at each of these elements Eq (3.13) is evaluated. This results in a set of algebraic equations for the unknown μ distribution

$$\sum_{k=1}^{N} C_{pk}\mu_k + C_{p,N+1}\mu_{N+1} + \sum_{k=1}^{N} B_{pk}\sigma_k = 0 \tag{3.18}$$

where

$$C_{pk} = \frac{1}{2\pi} \int\limits_{S_k} \frac{\partial \ln r}{\partial n}\, dS_k \tag{3.19}$$

$$C_{p,N+1} = \frac{1}{2\pi} \int\limits_{S_w} \frac{\partial \ln r}{\partial n}\, dS_{N+1} \tag{3.20}$$

$$B_{pk} = -\frac{1}{2\pi} \int\limits_{S_k} \ln r \, dS_k \tag{3.21}$$

and S_k, S_w denote elements of integration between edges of the kth panel; $p = 1, 2, ..., N$; N is the number of body panel; (note that in 2D solution we have only one wake panel, so $S_w = S_{N+1}$).

To define the problem uniquely, the wake doublet distribution should be known or related to the unknown doublets on S_B (Kutta-Joukowski condition).

From this condition we have

$$\mu_{N+1} = \mu_N - \mu_1 \tag{3.22}$$

and consequently the influence of the wake elements becomes

$$C_{p,N+1}\mu_{N+1} = C_{p,N+1}(\mu_N - \mu_1) \tag{3.23}$$

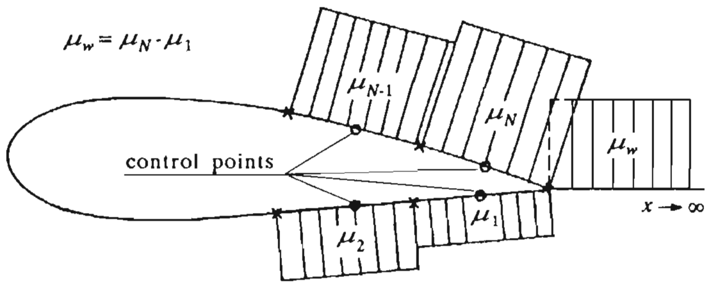


Fig. 5. Representation of body and wake surfaces by a set of doublets

Setting up

$A_{pk} = C_{pk}$ if panel is not at the trailing edge

$A_{pk} = C_{pk} + C_{p,N+1}$ if panel is at trailing edge (up)

$A_{pk} = C_{pk} - C_{p,N+1}$ if panel is at trailing edge (down)

a linear algebraic equation in N unknown variables μ_k can be derived in the form

$$\sum_{k=1}^{N} A_{pk}\mu_k = -\sum_{k=1}^{N} B_{pk}\sigma_k \tag{3.24}$$

Evaluating Eq (3.24) at each of the N control points $(p = 1, ..., N)$ results in N equations in the N unknown μ_k, as follows

$$\mathsf{A}\mu = \mathsf{B}\sigma \tag{3.25}$$

The solution to the matrix equation provides the doublet values of each of the N panels and the pressure coefficient for each panel can be calculated.

4. Getting on to the Codes

4.1. Main features of the Panel Methods

4.1.1. Discretisation of geometric surfaces

Hunt (1978) showed that the surface on which the singularities are arranged need not necessarily be identical to the configuration surface. Fictitious surfaces inside the true configuration surface may instead be used, provided the BC themselves are fulfilled on the appropriate surface. The use of such arbitrary internal surfaces, or of a number of them without geometric continuity (e.g., flat panels representing a twisted and cambered surface), does not by itself necessarily imply any error in the mathematical formulation.

The usual practice to reduce the problem is to approximate a curved surface by a large number of plane panels, each of which passes as closely as possible through usually four points of the actual surface. A single point on each panel is picked up as control point which can be the centroid or the point at which a source panel induces zero tangential velocity.

The main reason for using plane panels is that for low order singularity distributions the induced velocity due to a single panel of arbitrary shape can be computed analytically. Thus, the computing effort is reduced.

4.1.2. Choice of singularity distribution

The mathematical model of PMs may use any combination of sources and doublets. However, source singularities must be applied if there is an overall flux of fluid through the closed boundary of the domain of interest that can include problems in which a boundary layer is simulated by the surface transpiration. If wakes have to be simulated, then doublicity or vorticity must be placed on that surface. Thus, only in special cases can a source-only or a doublicity-only formulation be chosen. The numerical problem should consist in determining the best mix of such singularity distributions that not only satisfy the BCs but that also minimizes the numerical errors in that domain. In the mixed formulation, no approximation is in principle implied if locally or globally uniform distributions of one type of singularities are applied provided the other singularity can vary adequately to satisfy the BCs sufficiently not only at control points but also between them.

4.1.3. Discretisation of boundary conditions

The choice of combination of singularieties is not a trivial matter. Although many different combinations are in principle mathematically equivalent, their numerical implementation may yield significantly different results from the numerical stability, computational economy, accuracy, and overall code robustness, respectively, point of view.

The uniqueness theorem (see Section 3.1) requires that the BCs should be satisfied everywhere on the boundary S (see Eq (3.1)). The numerical process, however, usually has to satisfy the BC at a discrete set of points on this surface. The approximate fulfillment of the BC stems from the *leakage* between collocation points and the approximate representation of the geometric boundary. This is the reason why the control points do not generally lie on the configuration surface (inaccurate fulfillment of the Dirichlet BC) or the computed panel normals are not identical with the true ones (inaccurate fulfillment of the Neumann BC). One means of trying to reduce the leakage is to represent the surface itself more accurately and to use higher order singularity distributions on these surfaces.

4.2. Getting into the Classification

4.2.1. Higher-order versus lower-order panel methods

Since numerous codes using PM approach have been developed, there is the need to distinguish between them. The variations depends mainly on the geometric layout of the elementary panels, the choice of type and form of singularity distribution, and the type of BC imposed, respectively.

In a general classification, PMs can be devided into *lower order* and *higher order* PMs, reffering to the order of singularity distributions and geometry representation applied. However, there is some confusion as to what constitutes a truly higher order method. As Hess pointed out "since analytical integrations can be performed only over flat panels even for variable singularity strenght, the temptation is to use this formulation, and many authors incorrectly refer to it as higher order." (cf Hess (1985), p.32).

In subsonic flow, lower order PMs usually achieve sufficient accuracy, sometimes just by increasing the number of panels. Even strong vortex flows can be modeled by this methods, e.g., Kandil et al. (1977). Only in some cases, such as when the compatibility condition which relates properties of the flowfield at the trailing edge of a lifting wing to the derivative of the circulation in span direction should be fulfilled, special care must be taken.

In supersonic flow, two essential problems arise when surface singularity methods are extended to that flow regime. Both of them stem from the hyperbolic character of the linearized differential Eq (1.2) for the perturbation velocity potential.

The first problem is that lower order singularities can propagate virtual waves causing unrealistic fluctuations of the surface pressure. A new singularity, called *triplet*, has been developed by Woodward and Landrum (1980), to alleviate this problem.

4.2.2. The method of Hess and Smith

Most recent PMs are based on a combination of sources and doublets oriented to the surface. For this reason we have started with this approach. In this subsection we shortly consider the pioneering PM due to Hess and Smith (1964), which employes constant strength distributions of sources on straight line panels. For this reason it is very often called "surface source method" (cf Hess (1985)). This method has been named by its creators the "Douglas-Neumann method". This classical method made it possible for the first time to analyze flows past bodies of realistic geometry.

4.2.3. The method of Morino

Morino and Kuo (1974) proposed a new formulation of the problem of potential flow around complex configurations, which differs from the other PMs only in that the potential function on the submerged surface is generally unknown. Nevertheless, a program based on this concept is similar to the other PMs as to geometry discretization.

An advantage of Morino's formulation over the classical first-order methods seems to be the lower number of unknows needed to achieve a satisfactory degree of accuracy (cf Baston et al. (1986)).

4.2.4. Vortex lattice methods (VLMs)

It is essential to note that recently there are many versions of VLM available for solving of Eq (1.3): Quasi-VLM (cf Lan (1974)), Unsteady-VLM (cf Konstadinopoulus et al. (1985)), Subsonic Nonlinear VLM (cf Rom (1992)), and even ... Generalized VLM (cf Soviero and Bortolus (1992)). The proposed method is an extension of the classical VLM (cf Bertin and Smith (1989)) for calculation of the aerodynamic forces on lifting surfaces undergoing complex 3D unsteady motions. Hedman (1965) published the fundamental work, which became the base of further development of the Vortex Lattice Method.

4.2.5. Inverse methods

Most PMs for predicting potential flows over complex configuration are *analysis methods*, which predict the surface pressure distribution on a configuration with a specified geometry. Relatively few methods address the aerodynamic designer's task of designing such configuration as wing-body combination. Those fall into two categories.

The first category is the *optimization method*, which couples a conventional analysis method with an optimization algorithm to modify iteratively the geometry in order to minimize a certain "cost" function, such as the drag. The method of Vanderplaats (1979) for example represents this category.

The second category is the *inverse* or *design method*, in which one specifies the surface pressure distribution and the method calculates the corresponding airfoil geometry. This approach was pioneered by Lighthill (1945) for incompressible flow using a conformal mapping technique. The latest incompresible design methods are variants of Lighthill's basic approach.

Detailed review has been lately presented by Dulikravich (1992) – this study encompasses almost one hundered papers! However, among this large number there is only one (cf Kubryński (1991)), which deals with complete airplanes by means of PMs.

4.3. Classical VLM (CVLM)

From the mathematical point of view, the CVLM represents a particular case of the general PM with zero source distribution. On the other hand this method is an extension of the finite step lifting-line method originally described by Glauert (1948).

The VLM assumes the same model as the standard PM. The wing is divided into trapezoidal panels (also called *finite elements* or *lattices* – from which the name of this method comes). Each panel is replaced by a horseshoe vortex. Such vortices have a vortex filament across the 1/4-chord of the panel and two filaments stream wise: one on each side of the panel starting at the 1/4-chord and trailing downstream in the free-stream direction to infinity. Fig.6 shows a typical horseshoe vortex representation of a planform.

The velocities induced by each horseshoe vortex at a specified control point are calculated using the Biot-Savart law. A summation is performed over all control points on the wing to produce a set of linear algebraic equations for the horseshoe strength that satisfy the BC of no flow through the wing. The BC for each horseshoe is satisfied by requiring the inclination of the fluid stream lines to match the angle of attack at the 3/4-chord point of its elemental panel. The circulation required to satisfy this tangent flow BC is then determined by solving the matrix equation. Then, the Kutta-Joukowski theorem for lift from a vortex filament is used to determine the lift from each panel. These lifts are then summed up to obtain approximate aerodynamic characteristics.

It is worth noting that VLM is now usually included into basic academic courses. As a good example we can quote (cf Bertin and Smith (1989)) where the VLM is presented as the numerical-code-oriented procedure.

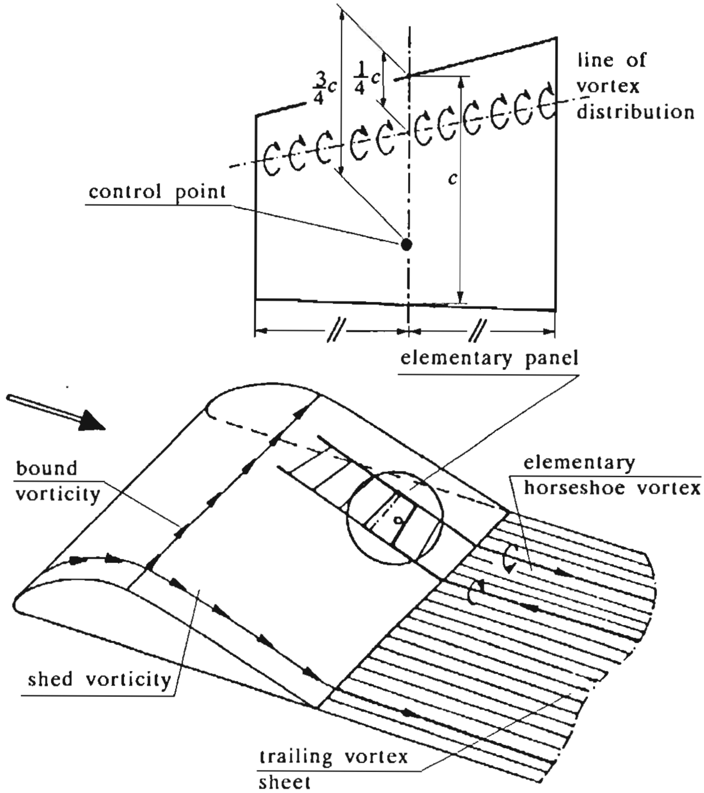


Fig. 6. Surface and trailing vorticity sheets

4.4. Presentation of various codes

Linear-potential PMs have been able to model arbitrary geometries for many years. One of these codes, PanAir, can be used to predict subsonic as well as supersonic flows about these general configurations. Some of those codes, with comments, are presented in Table 1.

Table 1

Year, name	Refer.	Boundary condition	Singularity (approxim.)	Surface approximat.
1964 DOUGLAS--NEUMANN	[15]	NEUMANN	S(C)	PLANE
1969 NLR	[28]	NEUMANN	S(C),D(C)	PLANE
1973 USSAERO	[49]	NEUMANN	S(L),V(L)	PLANE
1980 MCAIR	[4]	DIRICHLET	S(C),D(P)	PLANE
1980 SOUSSA	[38]	DIRICHLET	S(C),D(C)	PARABOLIC
1981 PAN AIR	[5,33]	MIXED	S(L),D(P)	PLANE (SUB)
1982 VSAERO	[36]	MIXED	S(C),D(C)	PLANE
1983 QUADPAN	[6]	DIRICHLET	S(C),D(C)	PLANE
1984 VORTR	[11]	NEUMANN	V(C),D(P)	PLANE
1984 HISSS	[9]	MIXED	S(L),D(P)	PLANE (SUB)
1988 PMARC	[1]	MIXED	S(C),D(C)	PLANE
1991 KKAERO	[27]	DIRICHLET	S(L),D(P)	PLANE

In Table 1 there are used following abbreviations:
S – sources, V – vortices, D – doublets, (C) – constant, (L) – linear, (P) – parabolic, (SUB) – subpanels.

PMARC – Panel Methods Ames Research Center
QUADPAN – QUADrilateral PANel Aerodynamic Programm
SOUSSA – Steady, Oscillatory and Unsteady, Subsonic and Supersonic Aerodynamics
VSAERO – Vortex Separation AEROdynamic Programm
HISSS – A HIgher-Order Subsonic Supersonic Singularity Method

(The remaining abbrreviations have not been unfortunately identificated).

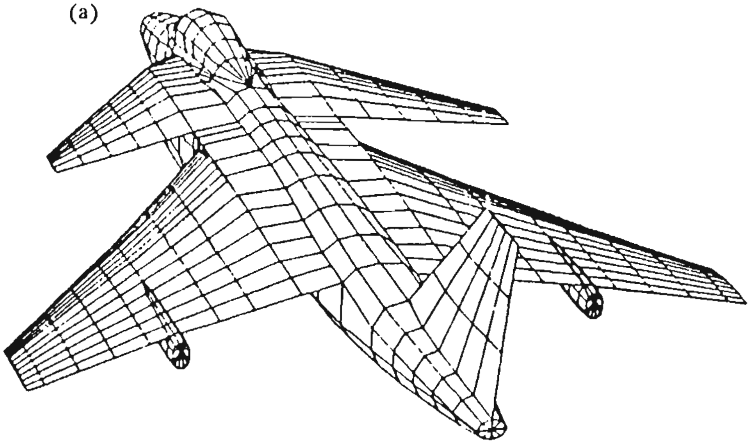
5. Sample solutions with panel codes

5.1. General remarks

In last decade new formulations and updates of existing methods were published (e.g. Woodward and Landrum (1980), Maskew (1982)), which combine some new features with the simplicity of low-order methods.

This chapter discusses the ability to routinely compute the aerodynamics of complex aircraft configurations. The field of computational fluid dynamics may be conveniently divided into two major areas:

(a)



(b)

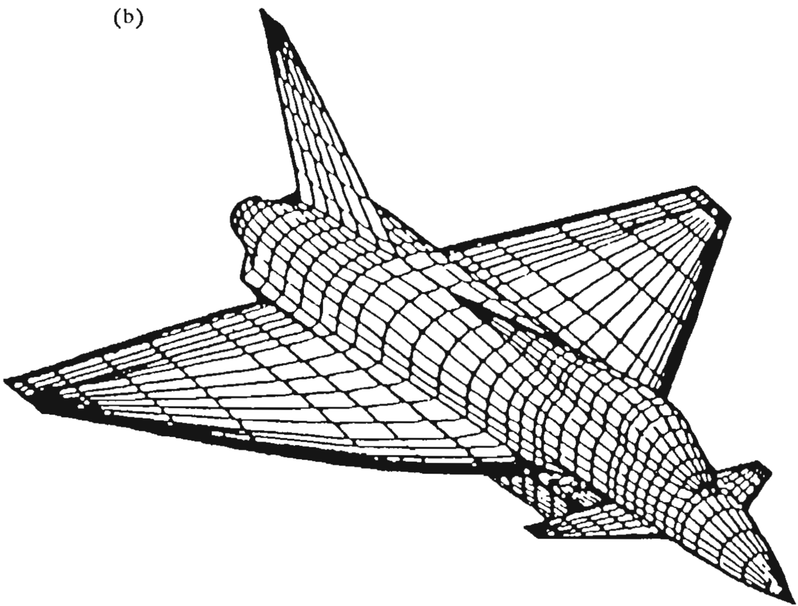


Fig. 7. (a) – PanAir paneling of V/STOL fighter/attack aircraft; (b) – HISSS paneling of an advanced supersonic fighter

1. Subsonic and supersonic regimes, where the nonlinear terms in the differential equation are negligible

2. The transonic regime, where the nonlinear terms are essential to describe the phenomenon.

Presented are only subsonic and supersonic results. Solutions are compared with experimental data as well as with the results from other computational fluid dynamics codes when available.

5.2. Aeroplane configurations in subsonic flow

In order to demonstrate the range of application of PMs, the PanAir surface-paneled scheme for V/STOL fighter/attack aircraft is shown in Fig.7a (cf Madson and Erickson (1990), p.727), whereas in Fig.7b is shown an advanced fighter configuration, for which the comparison with experimental data is available (cf Tinoco (1990), p.561). A total number of 990 surface panels were used to described the geometry of V/STOL.

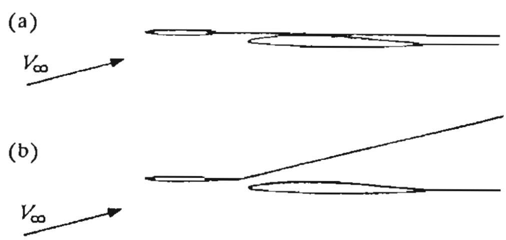


Fig. 8. Possible wing-wake location

Since PanAir requires wakes to be defined for any surfaces from which lift may be generated, two canard-wake cases were defined in order to evaluate the effect of wake position (not shape!) on the wing-lift distribution. Both cases are shown in Fig.8. If the case (a) is utilized through a wide range of angles of attack α, the inboard wing circulation becomes seriously affected (Fig.9a). By aligning the canard wake with α (case (b)), which is rather a simplistic modeling idea, the effect on the wing circulation can be readily observed (Fig.9a). An improved lift prediction at a higher α is also evident (cf Fig.9b).

After resolving modeling questions involving canard-wake positioning, the PanAir results were generated for $\mathrm{Ma}_\infty = 0.6$. The results are compared with

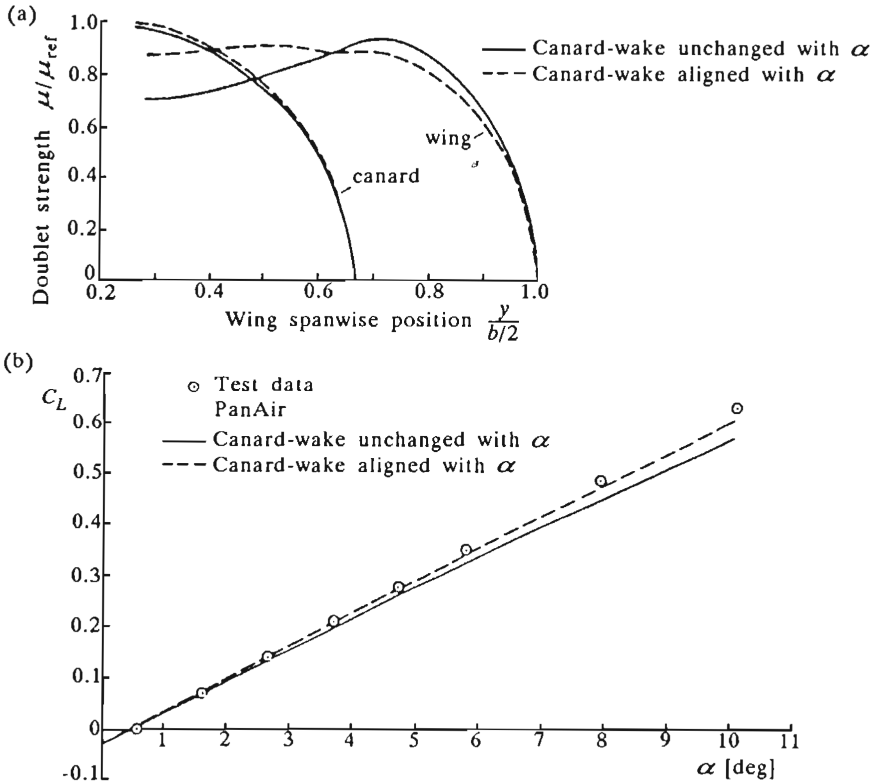


Fig. 9. Effect of canard-wake location on the PanAir result

experimental data (obtained in the Ames 11 × 11-ft Transonic Wind Tunnel) in Fig.10. The lift prediction is in generally good agreement with wind-tunnel data, even up to $\alpha = 10°$. Pitching-moment data were also in reasonable agreement with the linear portion of the wind-tunnel data (cf Madson and Erickson (1990), p.706).

In PanAir, pressure coefficients can be calculated from the isentropic equation for a flow of perfect gas, and from approximation based on small perturbation assumptions. The discrepancy in the isentropic and second-order results in Fig.10 indicate that there are regions of the configuration where the small-perturbation assumption inherent in linear-potential methods is being violated.

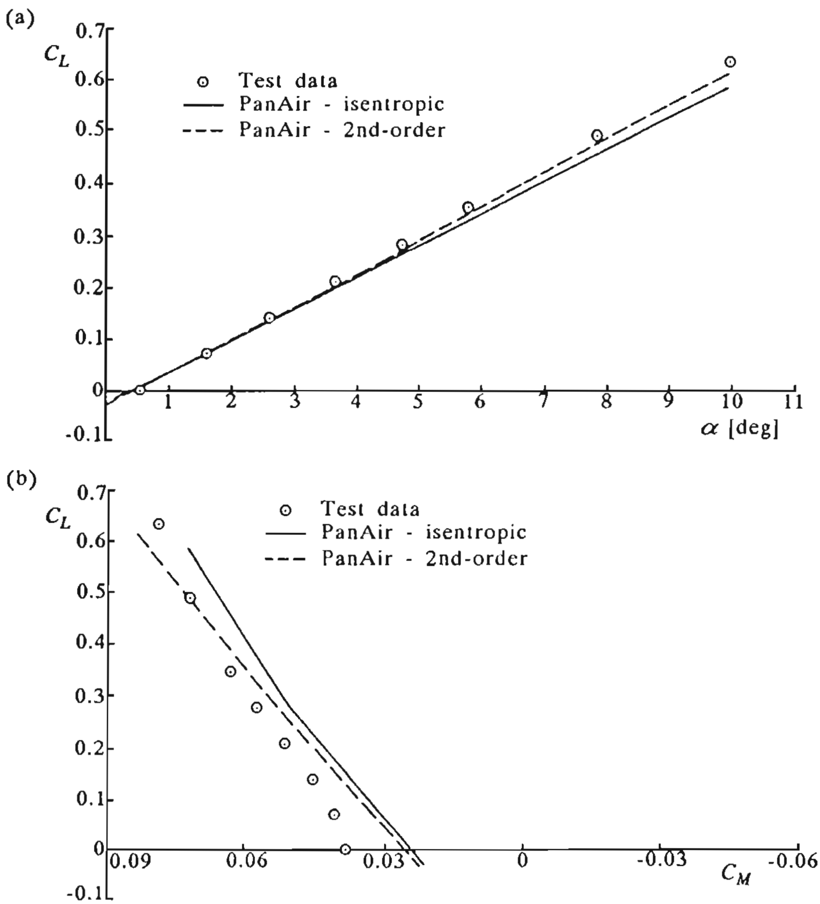


Fig. 10. Comparison of PanAir and Mach Box results with the wind-tunnel test results

5.3. Aeroplane configurations in supersonic flow

In Fig.11 is shown an advanced fighter configuration, for which comparison with experimental data are available (cf Tinoco (1990), p.561). Longitudinal characteristics comparison for $Ma_\infty = 0.4 \div 1.4$ are also presented in Fig.11. The computational results are from the HISSS code. The comparison, made at a low α before the onset of leading edge vortical flow, shows a good correlation between the computational results and experimental data.

Lateral-directional stability characteristics have always proved to be difficult to predict computationally. The NASA study in 1981 found that

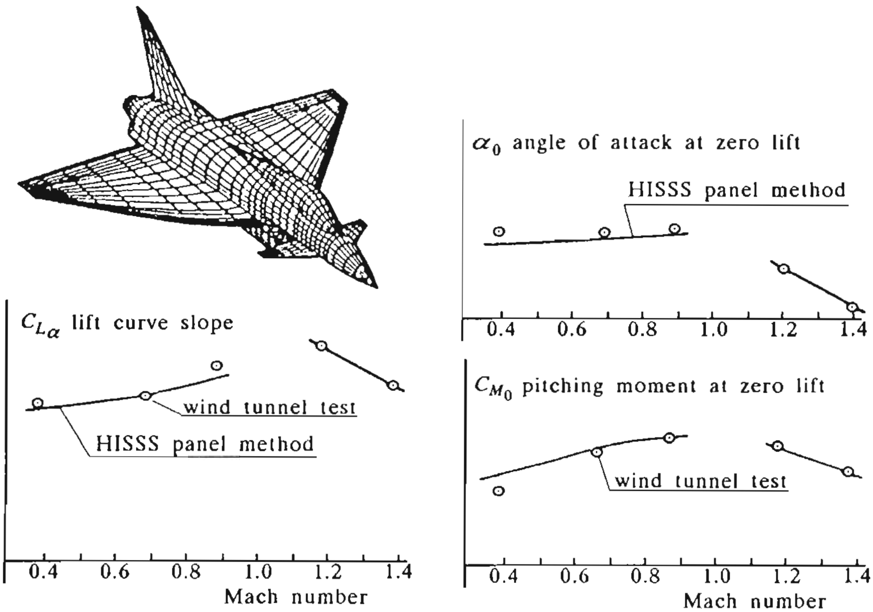


Fig. 11. Comparison of HISSS results with wind-tunnel test results for longitudinal characteristics

A502/PanAir generally provided accurate predictions at moderate α for a complete delta wing/body configuration with either single or twin vertical tails. Comparison with the experimental data for two of the vertical tail configurations tested is shown in Fig.12. Runs of yawing moment, rolling moment, and side force (each per unit sideslip) versus α are shown for $\mathrm{Ma}_\infty = 1.60$ and $\mathrm{Ma}_\infty = 2.86$. The agreement of the computed stability derivatives as a function of α with the experimental data is very good.

A comparison of computational results from two linear methods and a full potential marching code with wind-tunnel data for a supersonic cruise interceptor is shown in Fig.13. It can be seen that the results from the full potential code are in substantially better agreement with the test data than are the results from the linear methods, but nonlinear marching methods needed a Cray XMP.

5.4. Future directions

While a flowfield analysis of a complex 3D configuration is possible, the qu-

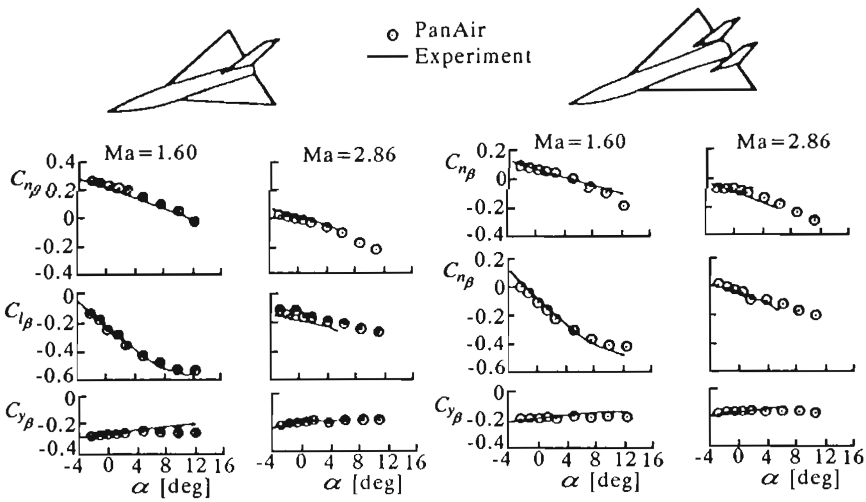


Fig. 12. Comparison of PanAir results with the experimental data for lateral-directional stability characteristics

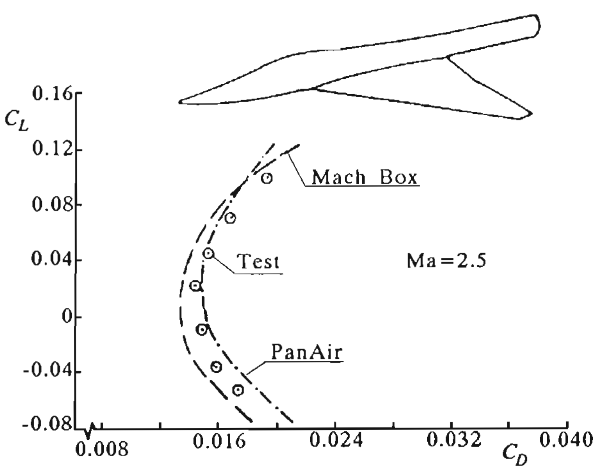


Fig. 13. Comparison of computational results with experimental data for a supersonic cruise interceptor

estion of the validation of the presented codes is still not settled, in particular when complex physical phenomena are present, e.g., trailing edge flows, shock-boundary layer interactions, separation, transition from laminar to turbulent flow, viscous wake interactions, etc.

Further efforts are desired for better understanding and modeling turbulence, transition prediction, and relaminarization. Engineers are working to perfect the combinations of physical models that enable them to deal properly with and resolve such phenomena as mentioned above. By reawakening interest in the fundamental fluid physics issues, and combining this interest with their computational skills, engineers are continually striving to improve PM methodology. However, this will be the objective of the next paper [13].

6. Concluding remarks

PMs have had a significant effect on the aircraft analysis and design procedures. A number of potential-flow solution methods were reviewed in order to establish an appropriate historical background as well as an appreciation of the current methods and thus provide a broad foundation of understanding in preparation for more detailed lectures on specific papers to follow in the daily life of an aeronautical engineer. Material has been selected to develop a perspective of the variety of methodology – past and present – that has been brought to bear on the problem of potential flow over complex aircraft surfaces.

We have attempted to establish the status of PMs for calculating aerodynamic loads to realistic aircraft configurations, with a focus on some essential features of the mathematical modeling.

Although PMs are used extensively in the aerodynamic calculations, researchers and engineers are becoming more aware of its limitations. To overcome these limitations, researchers are defining goals and directions for future research. They have concluded that not only are bigger computers and better algorithms required, but also that more research is needed into various fluid physics issues which are fundamental to the models to be solved.

Currently, numerical methods based on the potential model offer an excellent compromise between reliability, speed, and faithful representation of the field.

In summary, this article may be a useful guide to individuals who are starting work in this field as well as for those who are well versed in it. It may also, to a certain degree, be a useful grouping of current research papers.

Acknowledgements

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Klasyczne metody panelowe jako standardowa procedura obliczeń charakterystyk samolotu o złożonej konfiguracji geometrycznej – od teorii do algorytmów

Streszczenie

Spośród wszystkich metod numerycznej mechaniki płynów (CFD) prawdopodobnie najważniejsze z praktycznego punktu widzenia są metody panelowe. W niniejszej pracy nie tylko dokonano przeglądu istniejących metod panelowych (a więc technik obliczeniowych obciążeń aerodynamicznych dla rzeczywistych konfiguracji aerodynamicznych), ale również przedstawiono najważniejsze modele matematyczne potrzebne do ich zrozumienia. Zakres pracy obejmuje najbardziej aktualne zagadnienia analizy i syntezy konstrukcji lotniczych. Omówiono również przykladowe zastosowania metod panelowych wraz z wynikami obliczeń, otrzymanych dla szczególnie złożonych konfiguracji aerodynamicznych. Przedstawiono rezultaty zarówno dla prędkości pod- jak i naddźwiękowych.

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