MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
4, 33, 1995

ON THREE-DIMENSIONAL PROBLEMS OF MULTILAYERED PERIODIC ELASTIC COMPOSITES

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The purpose of the paper is to present some potential function method for solving three-dimensional static problems of multilayered periodic elastic composites. The periodically laminated bodies under consideration consist of two different elastic homogeneous materials. Perfect bonding between the layers is assumed. The analysis is carried out on the basis of the homogenized model with microlocal parameters. As an example of application of the presented method, the problem of multilayered periodic elastic half-space subjected to the pressure produced by a flat-ended rigid elliptical punch is solved.

1. Introduction

In view of the growing importance in geophysics, geotechnical and structural engineering, different homogenized models of periodic multilayered elastic bodies have engaged attention of investigators. The list of papers on this subject is very extensive and as main monographs we mention these by Achenbach (1975), Bakhvalov and Panasenko (1984), Bensoussan et al. (1978), Broutman and Krock edit. (1973-1976), Calcote (1969), Christensen (1980), Guz et al. (1982), Jones (1975), Pobedrja (1984), Vanin (1985). One of the methods of modelling of composite materials has been presented by Woźniak (1986) and (1987a,b), Matysiak and Woźniak (1987) and it was based on certain concepts of the nonstandard analysis and some postulated a priori physical assumptions. Equations of the homogenized model are formulated in terms of an unknown macrodisplacement vector and certain extra unknowns being referred to as microlocal parameters. The homogenized model with microlocal parameters enables mean and local values of strains and stresses in every material components of the stratified body to be evaluated.

The model has been applied to solutions of certain boundary value problems of periodic layered composites (mostly two-dimensional). The three-dimensional interface crack problems were solved by Kaczyński (1993) and (1995). The review of the papers on applications of homogenized models with microlocal parameters is given by Matysiak (1995).

In this paper the three-dimensional problems of periodic two-layered elastic composites are considered. The homogenized model with microlocal parameters is taken to the investigations. By using some potential functions the equations of homogenized model are reduced to a simpler form useful especially in the boundary value problems for a half-space or a layer. As an example the problem of flat-ended rigid elliptical punch pressed in the periodic multilayered elastic half-space is solved.

2. Basic equations

Consider a microperiodic stratified elastic body in which each repeated fundamental layer (unit) is composed of two different homogeneous isotropic layers. The scheme of the middle basic unit cross-section is given in Fig.1. Let λ_1, μ_1, λ_2, μ_2 be the Lamé constants, h_1, h_2 be thicknesses of the subsequent layers and $\delta = h_1 + h_2$ be the thickness of each basic unit of the body. Let (x_1, x_2, x_3) comprise the Cartesian coordinate system such that the x_1-axis is normal to the layers.

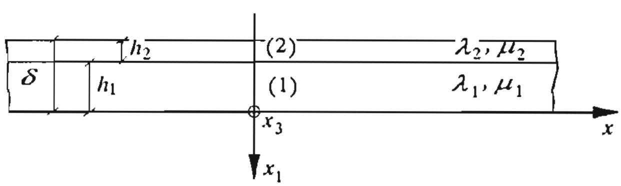

Fig. 1.

According to the results proposed by Woźniak (1986) and (1987a,b), Matysiak and Woźniak (1987) the displacement vector $\boldsymbol{u} = [u_1, u_2.u_3]$ for the static case is assumed in the form

$$u_i(x_1,x_2,x_3) = w_i(x_1,x_2,x_3) + l(x_1)q_i(x_1,x_2,x_3) \qquad i=1,2,3 \tag{2.1}$$

where $l(\cdot): \mathcal{R} \to \mathcal{R}$ is the known a priori continuous function, called the shape function given by

$$\begin{aligned} l(x_1) &= \begin{cases} x_1 - \frac{1}{2}h_1 & \text{for} \quad 0 \le x_1 \le h_1 \\ -\frac{\eta x_1}{1-\eta} - \frac{1}{2}h_1 + \frac{h_1}{1-\eta} & \text{for} \quad h_1 \le x_1 \le \delta \end{cases} \\ l(x_1+\delta) &= l(x_1) \end{aligned} \tag{2.2}$$

where $\eta = h_1/\delta$ and $w_i(\cdot, x_2, x_3)$, $q_i(\cdot, x_2, x_3)$ are unknown functions representing as the macrodisplacements and microlocal parameters, respectively, ($w_i(\cdot,x_2,x_3)$, $q_i(\cdot,x_2,x_3)$ together with their first and second order derivatives are macrofunctions related to the variable x_1).

The equations of homogenized model with microlocal parameters can be written in the form (the body forces are omitted), cf Matysiak and Woźniak (1987)

$$\begin{aligned} &(\widetilde{\lambda} + \widetilde{\mu})w_{i,i1} + \widetilde{\mu}w_{1,ii} + [\lambda + 2\mu]q_{1,1} + [\mu]q_{\alpha,\alpha} = 0 \\ &(\widetilde{\lambda} + \widetilde{\mu})w_{i,i\alpha} + \widetilde{\mu}w_{\alpha,ii} + [\lambda]q_{1,\alpha} + [\mu]q_{\alpha,1} = 0 \\ &(\hat{\lambda} + 2\hat{\mu})q_1 + [\lambda]w_{i,i} + 2[\mu]w_{1,1} = 0 \\ &\hat{\mu}q_\alpha + [\mu](w_{1,\alpha} + w_{\alpha,1}) = 0 \\ &\alpha = 2,3 \end{aligned} \tag{2.3}$$

where the comma indicates partial differentiation and summation convention holds over the repeated indices as well as where

$$\begin{aligned} (\widetilde{\lambda}, \widetilde{\mu}) &= (\eta\lambda_1 + (1-\eta)\lambda_2, \eta\mu_1 + (1-\eta)\mu_2) \\ ([\lambda],[\mu]) &= (\eta(\lambda_1 - \lambda_2), \eta(\mu_1 - \mu_2)) \\ (\hat{\lambda}, \hat{\mu}) &= \Big(\eta\lambda_1 + \frac{\eta^2}{1-\eta}\lambda_2, \eta\mu_1 + \frac{\eta^2}{1-\eta}\mu_2\Big) \end{aligned} \tag{2.4}$$

By using Eqs $(2.4)_{3,4}$ the microlocal parameters q_i, $i=1,2,3$, can be eliminated from Eqs $(2.4)_{1,2}$. Thus, one obtains

$$\begin{aligned} &A_1 w_{1,11} + C(w_{1,22} + w_{1,33}) + B(w_{2,21} + w_{3,31}) = 0 \\ &Bw_{1,12} + Cw_{2,11} + A_2 w_{2,22} + \widetilde{\mu}w_{2,33} + (A_2 - \widetilde{\mu})w_{3,23} = 0 \\ &Bw_{1,13} + (A_2 - \widetilde{\mu})w_{2,23} + Cw_{3,11} + \widetilde{\mu}w_{3,22} + A_2 w_{3,33} = 0 \end{aligned} \tag{2.5}$$

where

$$
\begin{aligned}
A_1 &= \widetilde{\lambda} + 2\widetilde{\mu} - \frac{([\lambda] + 2[\mu])^2}{\hat{\lambda} + 2\hat{\mu}} > 0 \\
A_2 &= \widetilde{\lambda} + 2\widetilde{\mu} - \frac{[\lambda]^2}{\hat{\lambda} + 2\hat{\mu}} > 0 \\
B &= \widetilde{\lambda} + \widetilde{\mu} - \frac{[\lambda]([\lambda] + 2[\mu])}{\hat{\lambda} + 2\hat{\mu}} - \frac{[\mu]^2}{\hat{\mu}} > 0 \\
C &= \widetilde{\mu} - \frac{[\mu]^2}{\hat{\mu}} > 0
\end{aligned}
\tag{2.6}
$$

To formulate the boundary value-problems for stresses we have to take into account the following relation (cf Kaczyński (1995))

$$
\begin{aligned}
\sigma_{11}^{(r)} &= A_1 w_{1,1} + (B - C)(w_{2,2} + w_{3,3}) \\
\sigma_{1\alpha}^{(r)} &= C(w_{1,\alpha} + w_{\alpha,1}) \\
\sigma_{22}^{(r)} &= D^{(r)} w_{1,1} + F^{(r)} w_{2,2} + E^{(r)} w_{3,3} \\
\sigma_{23}^{(r)} &= \mu^{(r)}(w_{2,3} + w_{3,2}) \\
\sigma_{33}^{(r)} &= D^{(r)} w_{1,1} + E^{(r)} w_{2,2} + F^{(r)} w_{3,3} \\
\alpha &= 2,3 \qquad\qquad r = 1,2
\end{aligned}
\tag{2.7}
$$

where for $r = 1,2$

$$
\begin{aligned}
D^{(r)} &= \frac{\lambda^{(r)}}{\lambda^{(r)} + 2\mu^{(r)}} A_1 \\
E^{(r)} &= \frac{2\mu^{(r)}\lambda^{(r)} + \lambda^{(r)}(B - C)}{\lambda^{(r)} + 2\mu^{(r)}} \\
F^{(r)} &= \frac{4\mu^{(r)}(\lambda^{(r)} + \mu^{(r)}) + \lambda^{(r)}(B - C)}{\lambda^{(r)} + 2\mu^{(r)}}
\end{aligned}
\tag{2.8}
$$

3. Representation of the solution to equations of homogenized model

This section aims at derivation of some representation of the solution to Eq (2.5) useful in boundary-value problems of a half-space, or a layer. Differentiating Eq $(2.5)_2$ with respect to x_3 and Eq $(2.5)_3$ with respect to x_2 one obtains

$$C\chi_{,11} + \widetilde{\mu}(\chi_{,22} + \chi_{,33}) = 0 \tag{3.1}$$

where

$$\chi = w_{2,3} - w_{3,2} \tag{3.2}$$

By differentiation of Eq $(2.5)_2$ with respect to x_2 and Eq $(2.5)_3$ with respect to x_3, respectively, it follows that

$$B(w_{1,122} + w_{1,133}) + C\theta_{,11} + A_2(\theta_{,22} + \theta_{,33}) = 0 \tag{3.3}$$

where

$$\theta = w_{2,2} + w_{3,3} \tag{3.4}$$

By virtue of Eq (3.4), Eq $(2.5)_1$ can be written in the form

$$A_1 w_{1,11} + C(w_{1,22} + w_{1,33}) + B\theta_{,1} = 0 \tag{3.5}$$

Thus, Eqs (2.5) of the homogenized model with microlocal parameters are given in the terms of χ, θ and w_1 by (3.1), (3.3) and (3.5). The function θ can be easily eliminated from Eq (3.3) by using Eq (3.5). The boundary conditions described on a plane parallel to the lamination by

$$w_i(x_1 = \text{const}, x_2, x_3) \equiv g_i(x_2, x_3) \qquad i = 1,2,3 \tag{3.6}$$

or

$$\begin{aligned} &\sigma_{11}^{(r)}(x_1 = \text{const}, x_2, x_3) \equiv -p(x_2, x_3) \\ &\sigma_{1\alpha}^{(r)}(x_1 = \text{const}, x_2, x_3) \equiv \tau_\alpha(x_2, x_3) \\ &\alpha = 2,3 \qquad r = 1,2 \end{aligned} \tag{3.7}$$

may be expressed in terms of χ, θ, w_1 as follows

$$\begin{aligned} w_1(x_1 = \text{const}, x_2, x_3) &= g_1(x_2, x_3) \\ \chi(x_1 = \text{const}, x_2, x_3) &= g_{2,3}(x_2, x_3) - g_{3,2}(x_2, x_3) \\ \theta(x_1 = \text{const}, x_2, x_3) &= g_{2,2}(x_2, x_3) - g_{3,3}(x_2, x_3) \end{aligned} \tag{3.8}$$

or

$$\begin{aligned} &\sigma_{11}^{(r)}(x_1 = \text{const}, x_2, x_3) = Aw_{1,1} + (B - C)\theta = -p(x_2, x_3) \\ &\sigma_{12,3}^{(r)} - \sigma_{13,2}^{(r)} = C\chi_{,1} = \tau_{2,3}(x_2, x_3) - \tau_{3,2}(x_2, x_3) \\ &\sigma_{12,2}^{(r)} + \sigma_{13,3}^{(r)} = C(w_{1,22} + w_{1,33} + \theta_{,1}) = \tau_{2,2}(x_2, x_3) + \tau_{3,3}(x_2, x_3) \end{aligned} \tag{3.9}$$

Thus, within the framework of the homogenized model with microlocal parameters the boundary-value problems of a microperiodic stratified layer or a half-space are determined by the functions w_1, χ, θ (see, Eqs (3.1), (3.3), (3.5) and the boundary conditions (3.8) or (3.9)). Taking the solution of Eqs (3.3) and (3.5) as

$$\theta = \varphi_{,1} \qquad\qquad w_1 = \alpha\varphi \tag{3.10}$$

where φ is an unknown function, α is an unknown parameter, then Eqs (3.3) and (3.5) can be written in the form

$$\varepsilon^2\varphi_{,11} + \varphi_{,22} + \varphi_{,33} = 0 \tag{3.11}$$

on the assumption that unknown parameters ε and α satisfy

$$\begin{aligned} (A_1 - C\varepsilon^2)\alpha + B &= 0 \\ B\varepsilon^2\alpha + (A_2\varepsilon^2 - C) &= 0 \end{aligned} \tag{3.12}$$

From Eqs (3.12) it follows that

$$A_2C\varepsilon^4 + (B^2 - A_1A_2 - C^2)\varepsilon^2 + CA_2 = 0 \tag{3.13}$$

and

$$\alpha = \frac{C - A\varepsilon^2}{B\varepsilon^2} \tag{3.14}$$

The solution of the characteristic equation (3.13), which depended on material constants of the layers was discussed by Kaczyński and Matysiak (1988). Following Kaczyński and Matysiak (1988) we can mark out the cases: $[\mu] \neq 0$ and $[\mu] = 0$. For the case of different shear modulae μ_1 and μ_2 of the subsequent layers, Eq (3.13) has two roots (the case of $\mu_1 = \mu_2$ has to be considered separately but here it will be omitted)

$$\begin{aligned} \varepsilon_1^2 &= \frac{A_1A_2 + C^2 - B^2 - \sqrt{\Delta}}{2A_2C} \\ \varepsilon_2^2 &= \frac{A_1A_2 + C^2 - B^2 + \sqrt{\Delta}}{2A_2C} \\ \Delta &= (A_1A_2 + C^2 - B^2)^2 - 4C^2A_1A_2 > 0 \end{aligned} \tag{3.15}$$

By using Eqs (3.10), (3.14) and (3.15) we obtain

$$\theta = \varphi_{1,1} + \varphi_{2,1} \qquad\qquad w_1 = \alpha_1\varphi_1 + \alpha_2\varphi_2 \tag{3.16}$$

where

$$\alpha_r = \frac{-A\varepsilon_r^2 + C}{B\varepsilon_r^2} \qquad r = 1,2 \tag{3.17}$$

and φ_1, φ_2 satisfy the equations

$$\varepsilon_r^2\varphi_{r,11} + \varphi_{r,22} + \varphi_{r,33} = 0 \qquad r = 1,2 \tag{3.18}$$

Thus, the three-dimensional static problems of periodic two-layered elastic composites are reduced to three equations of Laplace's type (Eqs (3.1) and (3.18)) with the boundary conditions (3.8) or (3.9).

4. Example

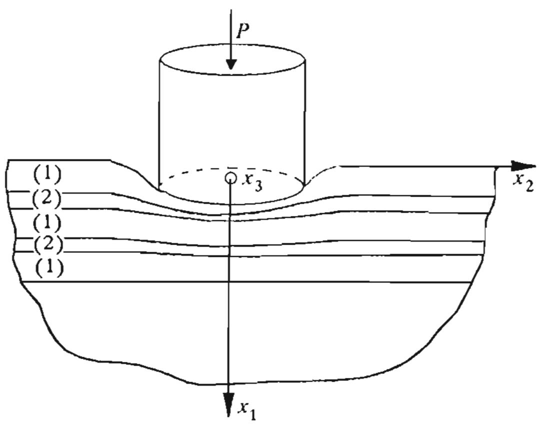

Fig. 2.

Consider now the problem of contact between a periodic two-layered half-space and a rigid flat-ended elliptical punch, see Fig.2. Let the boundary conditions be given in the form

$$\begin{aligned} w_1(x_1 = 0, x_2, x_3) &= \delta_0 && \text{for} \quad (x_2, x_3) \in \Omega \\ \sigma_{11}^{(1)}(x_1 = 0, x_2, x_3) &= 0 && \text{for} \quad (x_2, x_3) \notin \bar{\Omega} \\ \sigma_{12}^{(1)}(x_1 = 0, x_2, x_3) &= 0 && \text{for} \quad (x_2, x_3) \in R^2 \\ \sigma_{13}^{(1)}(x_1 = 0, x_2, x_3) &= 0 && \text{for} \quad (x_2, x_3) \in R^2 \end{aligned} \tag{4.1}$$

where $\Omega = \{(x_2, x_3): \frac{x_2^2}{a^2} + \frac{x_3^2}{b^2} \le 1\}$ and δ_0 is an identification parameter, unspecified yet.

Moreover, it is assumed that the punch is subject to a total load P directed along the axis normal to the lamination of the half-space

$$\begin{aligned} &\iint\limits_{\Omega} \sigma_{11}^{(1)}(x_2, x_3)\, d\Omega = -P && \text{with} \quad P > 0 \\ &\sigma_{11}^{(1)}(x_2, x_3) \le 0 && \text{for} \quad (x_2, x_3) \in \Omega \end{aligned} \tag{4.2}$$

and the vanishing of stresses at the infinity is assumed.

By using the notations given by Eqs (3.6) and (3.7) the problem is reduced to finding an unknown contact stresses $p(x_2, x_3)$, $(x_2, x_3) \in \Omega$ under conditions (see Eqs (3.6)÷(3.9) and (4.1))

$$\begin{aligned} &w_1(x_1 = 0, x_2, x_3) = \delta_0 && \text{for} \quad (x_2, x_3) \in \Omega \\ &p(x_1 = 0, x_2, x_3) = 0 && \text{for} \quad (x_2, x_3) \in R^2 - \bar{\Omega} \\ &\chi_{,1}(x_1 = 0, x_2, x_3) = 0 && \text{for} \quad (x_2, x_3) \in R^2 \\ &(w_{1,22} + w_{1,33} + \theta_{,1})(x_1 = 0, x_2, x_3) = 0 && \text{for} \quad (x_2, x_3) \in R^2 \end{aligned} \tag{4.3}$$

From Eq (3.1) and the boundary condition $(4.3)_3$ and conditions at the infinity it follows that

$$\chi(x_1, x_2, x_3) = 0 \qquad x_1 \ge 0 \qquad (x_2, x_3) \in R^2 \tag{4.4}$$

Taking into account the conditions $\sigma_{11}^{(1)}(x_1 = 0, x_2, x_3) = -p(x_2, x_3)$ and $\sigma_{1r}^{(1)}(x_1 = 0, x_2, x_3) = 0$, $r = 1, 2$, $(x_2, x_3) \in R^2$, and using Eqs (2.7), (3.2), (3.4), (3.16) we obtain for $x_1 = 0$

$$\begin{aligned} &(A_1\alpha_1 + B - C)\varphi_{1,1} + (A_1\alpha_2 + B - C)\varphi_{2,1} = -p(x_2, x_3) \\ &(1 - \alpha_1\varepsilon_1^2)\varphi_{1,11} + (1 - \alpha_2\varepsilon_2^2)\varphi_{2,11} = 0 \end{aligned} \tag{4.5}$$

By using the Fourier transforms with respect to x_2, x_3 from Eqs (3.18) and (4.5) one obtains

$$\begin{aligned} \varphi_1(x_1, x_2, x_3) &= \frac{1}{2\pi} \iint\limits_{R^2} \Phi_1(\xi, \eta) e^{-\gamma_1 x_1} e^{-i\xi x_2 - i\eta x_3}\, d\xi d\eta \\ \varphi_2(x_1, x_2, x_3) &= \frac{1}{2\pi} \iint\limits_{R^2} \Phi_2(\xi, \eta) e^{-\gamma_2 x_1} e^{-i\xi x_2 - i\eta x_3}\, d\xi d\eta \end{aligned} \tag{4.6}$$

where

$$\Phi_1(\xi,\eta) = \Big[\frac{\alpha_1 A_1 + B - C}{\varepsilon_1} - \frac{\alpha_2 A_1 + B - C}{\varepsilon_2}\frac{\varepsilon_2^2 - \alpha_2}{\varepsilon_1^2 - \alpha_1}\Big]^{-1} \frac{\widetilde{p}(\xi,\eta)}{\sqrt{\xi^2+\eta^2}}$$

$$\Phi_2(\xi,\eta) = \frac{\varepsilon_2^2 - \alpha_2}{\varepsilon_1^2 - \alpha_1}\Phi_1(\xi,\eta) \tag{4.7}$$

$$\gamma_r = \frac{\sqrt{\xi^2+\eta^2}}{\varepsilon_r} \qquad r = 1,2$$

and

$$\widetilde{p}(\xi,\eta) = \frac{1}{2\pi}\iint\limits_{\Omega} p(x_2,x_3)\mathrm{e}^{-\mathrm{i}\xi x_2 - \mathrm{i}\eta x_3}\,dx_2dx_3 \tag{4.8}$$

Substituting Eqs (4.6) into Eq $(3.16)_2$ and satisfying the boundary condition (4.1) the following integral equation is obtained in unknown function $p(x_2,x_3)$

$$\frac{W}{4\pi^2}\iint\limits_{\Omega} p(\xi_1,\eta_1)\,d\xi_1 d\eta_1 \iint\limits_{R^2} \frac{\mathrm{e}^{\mathrm{i}\xi(x_2-\xi_1)+\mathrm{i}\eta(x_3-\eta_1)}}{\sqrt{\xi^2+\eta^2}}\,d\xi d\eta = \delta_0 \tag{4.9}$$

where

$$W = \frac{(\varepsilon_1^2-\alpha_1)\alpha_1 - (\varepsilon_2^2-\alpha_2)\alpha_2}{(A_1\alpha_1+B-C)\varepsilon_1^{-1}(\varepsilon_1^2-\alpha_1) - (A_1\alpha_2+B-C)\varepsilon_2^{-1}(\varepsilon_2^2-\alpha_2)} \tag{4.10}$$

The solution of Eq (4.9) can be written in the form (cf Galin (1980))

$$p(x_2,x_3) = \frac{p_0}{\sqrt{1-\frac{x_2^2}{a^2}-\frac{x_3^2}{b^2}}} \tag{4.11}$$

$$p_0 = \frac{\delta_0}{WbK\Big(\frac{\sqrt{a^2-b^2}}{a}\Big)} \qquad \text{for } a \geq b \tag{4.12}$$

and where $K(\cdot)$ is the full elliptic integral of the first kind.

From Eqs (4.11), (4.12), $(3.9)_1$ and $(4.2)_1$ it follows that the identification parameter δ_0 is given in the form

$$\delta_0 = \frac{WPK\Big(\frac{\sqrt{a^2-b^2}}{b^2}\Big)}{2\pi a} \tag{4.13}$$

The knowledge of contact stresses $p(x_1,x_2)$ (see Eq (4.11)) together with Eqs (4.8), (4.7), (4.6) gives us functions φ_1 and φ_2. Thus, from Eqs (3.16), (4.4), (3.2), (3.4) and (2.7) the displacements and stresses can be obtained at every point of the two-layered periodic half-space (in terms of the Fourier integrals).

References

1. ACHENBACH J.D., 1975, *A Theory of Elasticity with Microstructure for Directionally Reinforced Composites,* CISM Courses and Lectures No. 167, Springer, New York
2. BAKHVALOV N.S., PANASENKO G.P., 1984, *Averaged Processes in Periodic Media,* Nauka, Moscow, (in Russian)
3. BENSOUSSAN A., LIONS J.L., PAPANICOLAOU G., 1978, *Asymptotic Analysis for Periodic Structure,* North Holland, Amsterdam
4. BROUTMAN L.J., KROCK R.H., EDIT., 1973-1976, *Composite Materials,* **1÷8**, Academic Press, New York
5. CALCOTE L.R., 1969, *The Analysis of Laminated Composite Structures,* Van Nostrand Reinhold Co., Amsterdam
6. CHRISTENSEN, 1980, *Mechanics of Composite Materials,* J.Wiley and Sons, New York
7. GALIN I.A., 1980, *Contact Problems of the Theory of Elasticity and Viscoelasticity,* Science, Moscow, (in Russian)
8. GUZ A.N. ET AL., 1982, *Mechanics of Composite Materials and Elements of Constructions,* Naukova Dumka, Kiev, (in Russian)
9. JONES R., 1975, *Mechanics of Composite Materials,* McGraw-Hill Book Co., New York
10. KACZYŃSKI A., 1993, On the Three-Dimensional Interface Crack Problems in Periodic Two-layered Composites, *Int. J. Fracture,* **62**, 283-306
11. KACZYŃSKI A., 1994, Three-Dimensional Thermoelatic Problems of Interface Cracks in Periodic Two-layered Composites, *Engng. Fracture Mech.,* **48**, 783-800
12. KACZYŃSKI A., 1995, Problems of Interface Crack in Microperiodic Thermoelastic Composites, *Prace Naukowe PW, Budownictwo,* (in Polish)
13. KACZYŃSKI A., MATYSIAK S. J., 1988, On the Complex Potentials of the Linear Thermoelasticity with Microlocal Parameters, *Acta Mechanica,* **72**, 245-259
14. MATYSIAK S.J., 1995, On the Microlocal Parameters Method in Modelling of Periodically Layered Thermoelastic Composites, *J. Theor. Appl. Mech.,* **2**, 481-487
15. MATYSIAK S. J., WOŹNIAK CZ., 1987, Micromorphic Effects in a Modelling of Periodic Multilayered Elastic Composites, *Int. J. Engng. Sci.,* **25**, 549-559
16. POBEDRYA B.J., 1984, *Mechanics of Composite Materials,* Izd. Mosk. Univ., Moscow, (in Russian)
17. VANIN G.A., 1985, *Micromechanics of Composite Materials,* Naukova Dumka, Kiev, (in Russian)
18. WOŹNIAK C., 1986, Nonstandard Analysis in Mechanics, *Advances in Mech.,* 3-36

19. WOŹNIAK CZ., 1987, A nonstandard Method of Modelling of Thermoelastic Periodic Composites, *Int. J. Engng. Sci.*, **25**, 483-499

20. WOŹNIAK CZ., 1987, On the Linearized Problems of Thermoelasticity with Microlocal Parameters, *Bull. Pol. Ac.; Techn.*, **35**, 143-153

O trójwymiarowych zagadnieniach wielowarstwowych periodycznych sprężystych kompozytów

Streszczenie

Celem pracy jest przedstawienie pewnej metody potencjałów, służącej do rozwiązywania trójwymiarowych statycznych zagadnień wielowarstwowych periodycznych kompozytów sprężystych. Przyjęto, że rozważany kompozyt zawiera dwie różne, jednorodne, sprężyste, powtarzające się periodycznie warstwy (idealnie ze sobą sklejone). Podstawę rozważań stanowi model homogenizowany z parametrami mikrolokalnymi. Jako przykład zastosowania wyprowadzonych w pracy potencjałów, rozwiązano zagadnienie periodycznie uwarstwionej sprężystej półprzestrzeni poddanej działaniu płasko zakończonego, eliptycznego, sztywnego stempla.

Manuscript received May 29, 1995; accepted for print Juny 12, 1995