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COMPARISON OF SOLUTIONS IN A FRAME-WORK OF DIFFERENT PLATE THEORIES

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A system of 2D-model equations characterizing elastostatic states, based on a non-classical continuum theory with internal constrains is given. This model is compared with lower-order and higher-order plate theories through application to a particular problem involving an infinite strip acted upon by a sinusoidal surface loads. The solution is compared also with the exact solution to this problem.

1. Introduction

In the two-dimensional theory of a thin plate, a great number of investigations have been developed on the well-known Kirchhoff-Love hypothesis. Although the classical plate theory is refined and well established, the applicability range of such a simplified theory would naturally limited to a thin plate. In order to analyze a thick plate, various two-dimensional theories have also been investigated by taking into account three-dimensional characteristics of stress and displacement fields.

On the same assumption of displacement distributions in the classical theory, Reissner (1945), Hencky (1947) have developed a generalized theory of the bending of elastic plates by taking into account the effects of transverse shear deformation. Reissner postulated that the displacements have the form

$$\begin{aligned} u_1 &= u_1^0 + x_3\theta_1 \\ u_2 &= u_2^0 + x_3\theta_2 \\ u_3 &= u_3^0 \end{aligned} \tag{1.1}$$

where x_3 is the coordinate normal to the midplane, and u_1^0, u_2^0, θ_1, θ_2 and u_3^0 depend upon the in-plane coordinates x_1 and x_2, and θ_1, θ_2 are the rotations of normals to midplane about the x_2 and x_1 axes, respectively. The governing equilibrium equations have been derived through the principle of stationary potential energy for a 3D elastic body by introducing a set of stress distributions which satisfy the boundary conditions on the surfaces of the plate.

A set of governing equations,on the same assumption of displacement distributions, has been also derived by Mindlin (1951). In Mindlin's theory, distributions of stress components are not specified in the thickness direction of the plate and, therefore, stress boundary conditions on the surfaces of the plate cannot be satisfied.

A mathematical model of elastodynamics of finite thickness layers with kinematical constraints has been proposed by Baczyński (1985). In this paper we simplified this model equations to deal with the elastostatic state of an isotropic plate. This model is tested in the case of an infinite strip loaded by sinusoidal surface pressure. A comparison is made between the solution obtained using this model and the other solutions obtained by the other theorems.

The aim of this paper is to show some consequences of the pure kinematical constraints to solutions to the boundary value problems within the frame of six-parameter plate theory (consistent with 3D theory of elasticity).

The number of cited literature is limited to the items which have been used for comparison of the solutions, so that, the paper does not contain the analysis of different plate theories, based on kinematical and kinetic constraints, which have been wide by discussed, e.g. in the papers by Reissner (1985), Rychter (1986) and Jemielita (1993).

2. Model assumptions

We assume that the region of plate B in the natural state configuration will be parameterized by rectangular coordinates $\boldsymbol{x} = (x_i)$, $i = 1,2,3$; such that $B = \Pi \times (0,h)$, where Π is the regular region in the plane $0x_1x_2$ bounded by $\partial\Pi$ and $h > 0$ is the thickness of the plate.

The displacement field will be approximated in the form

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{u}_{(1)}(\boldsymbol{z}) + \frac{x_3}{h}[\boldsymbol{u}_{(2)}(\boldsymbol{z}) - \boldsymbol{u}_{(1)}(\boldsymbol{z})] \tag{2.1}$$

where $\boldsymbol{x} = (\boldsymbol{z}, x_3) \in B$, $\boldsymbol{z} = (x_1, x_2) \in \Pi$, $0 \leq x_3 \leq h$ and the functions $\boldsymbol{u}_{(1)}$,

$\boldsymbol{u}_{(2)}$ are the 2D-displacement fields of the lower and upper surfaces $\Pi_{(1)}$, $\Pi_{(2)}$ bounding the plate.

The equilibrium equations in terms of the 3D-field quantities are postulated in the form

$$
\begin{aligned}
& T_{ij,j}(\boldsymbol{x}) + \rho b_i(\boldsymbol{x}) + r_i(\boldsymbol{x}) = 0 \\
& T_{ij}(\boldsymbol{x}) = T_{ji}(\boldsymbol{x}) \qquad\qquad i,j = 1,2,3
\end{aligned}
\tag{2.2}
$$

where $T_{ij}(\boldsymbol{x})$ denote the stress tensor, $r_i(\boldsymbol{x})$ are the body reaction forces of internal constraints and $b_i(\boldsymbol{x})$ are the external body forces acting on the plate.

The kinetic boundary conditions in terms of the 3D-fields are postulated in the form

$$
T_{ij}(\boldsymbol{x})n_j(\boldsymbol{x}) = p_i(\boldsymbol{x}) + s_i(\boldsymbol{x}) \qquad\qquad i = 1,2,3 \tag{2.3}
$$

where $\boldsymbol{x} \in \partial B$; n_j *denote components of the outward normal unit vector on* ∂B, s_i are components of the surface reaction forces of the internal constraints and p_i are components of the surface loads.

The constitutive equations for the stress tensor components are in the form

$$
T_{ij}(\boldsymbol{x}) = \mu[u_{i,j}(\boldsymbol{x}) + u_{j,i}(\boldsymbol{x})] + \lambda u_{k,k}(\boldsymbol{x})\delta_{ij} \qquad\qquad i,j,k = 1,2,3 \tag{2.4}
$$

where μ and λ are Lame constants.

The above assumptions together with the postulated appropriate ideality principle of constrains (cf Woźniak (1973)) let us to derive the 2D-model equations of elastostatic for plates of arbitrary thickness.

3. Elastostatic model of finite thickness plate

In the static case, the 2D-formulated equations consist of

— *The equilibrium equations*

$$
\begin{aligned}
& N_{(1)KL,K}(z) - \frac{1}{h}M_{(1)KL,K}(z) + \frac{1}{h}N_{(1)3L}(z) + f_{(1)L}(z) = 0 \\
& N_{(1))K3,K}(z) - \frac{1}{h}M_{(1)K3,K}(z) + \frac{1}{h}N_{(1)33}(z) + f_{(1)3}(z) = 0 \\
& \frac{1}{h}M_{(1)KL,K}(z) - \frac{1}{h}N_{(1)3L}(z) + f_{(2)L}(z) = 0 \\
& \frac{1}{h}M_{(1)K3,K}(z) - \frac{1}{h}N_{(1)33}(z) + f_{(2)3}(z) = 0
\end{aligned}
\tag{3.1}
$$

where $N_{(1)ij}$, $M_{(1)ij}$ stand for the generalized internal stress resultants and stress couples and $f_{(1)i}$, $f_{(2)i}$ are the generalized external forces. These all quantities are related to unit length of parametric lines on the fundamental surfaces $\Pi_{(1)}$, $\Pi_{(2)}$ and are defined as follows

$$
\begin{aligned}
N_{(1)ij}(z) &= \int_0^h T_{ij}(x)\,dx_3 \\
M_{(1)ij}(z) &= \int_0^h T_{ij}(x)x_3\,dx_3 \\
M_{(1)33}(z) &\equiv 0 \\
f_{(1)i}(z) &= \frac{\rho}{h}\int_0^h b_i(x)(h-x_3)\,dx_3 + \Big[p_i(x)\Big]_{x_3=0} \\
f_{(2)i}(z) &= \frac{\rho}{h}\int_0^h b_i(x)x_3\,dx_3 + \Big[p_i(x)\Big]_{x_3=h}
\end{aligned}
\tag{3.2}
$$

— *The kinetic boundary equations*

The kinetic boundary equations in terms of generalized stress resultants, stress couples and loads take the form

$$
\begin{aligned}
\Big[N_{(1)KL}(z) - \frac{1}{h}M_{(1)KL}(z)\Big]n_{(1)K}(z) &= p_{(1)L}(z) \\
\Big[N_{(1)K3}(z) - \frac{1}{h}M_{(1)K3}(z)\Big]n_{(1))K}(z) &= p_{(1)3}(z) \\
\frac{1}{h}M_{(1)KL}(z)n_{(1)K}(z) &= p_{(2)L}(z) \\
\frac{1}{h}M_{(1)K3}(z)n_{(1)K}(z) &= p_{(2)3}(z)
\end{aligned}
\tag{3.3}
$$

where $\boldsymbol{p}_{(1)}$, $\boldsymbol{p}_{(2)}$ stand for the generalized external forces. These quantities are related to unit length of the curves $\partial\Pi_{(1)}$, $\partial\Pi_{(2)}$ and have the following definitions

$$
\begin{aligned}
p_{(1)i}(z) &= \frac{1}{h}\int_0^h p_i(x)(h-x_3)\,dx_3 \\
p_{(2)i}(z) &= \frac{1}{h}\int_0^h p_i(x)x_3\,dx_3
\end{aligned}
$$

— *The constitutive equations*

The constitutive equations for the generalized stress resultants and stress couples in terms of the 2D-field quantities are in the form

$$\begin{aligned}
N_{(1)KL}(z) &= 2\mu h e_{(1)KL}(z) + \mu h^2 e'_{(1)KL}(z) + \lambda h e_{(1)MM}(z)\delta_{KL} + \\
&+ \frac{1}{2}\lambda h^2 e'_{(1)MM}(z)\delta_{KL} + \lambda h e_{(1)33}(z)\delta_{KL} \\
N_{(1)K3}(z) &= 2\mu h e_{(1)K3}(z) + \mu h^2 e'_{(1)K3}(z) \\
N_{(1)33}(z) &= (\lambda + 2\mu) h e_{(1)33}(z) + \lambda h e_{(1)MM}(z) + \frac{1}{2}\lambda h^2 e'_{(1)MM}(z) \\
M_{(1)KL}(z) &= \mu h^2 e_{(1)KL}(z) + \frac{2\mu h^3}{3} e'_{(1)KL}(z) + \frac{1}{2}\lambda h^2 e_{(1)MM}(z)\delta_{KL} + \\
&+ \frac{1}{3}\lambda h^3 e'_{(1)MM}(z)\delta_{KL} + \frac{1}{2}\lambda h^2 e_{(1)33}(z)\delta_{KL} \\
M_{(1)K3}(z) &= \mu h^2 e_{(1)K3}(z) + \frac{2}{3}\mu h^3 e'_{(1)K3}(z) \\
M_{(1)33}(z) &\equiv 0
\end{aligned} \tag{3.4}$$

— *The geometrical relations*

The geometrical relations for the 2D-strain measures in terms of displacements are defined in the form

$$\begin{aligned}
e_{(1)KL}(z) &= u_{(1)(K,L)}(z) \\
e'_{(1)KL}(z) &= \frac{1}{h}[u_{(2)(K,L)}(z) - u_{(1)(K,L)}(z)] \\
e_{(1)K3}(z) &= \frac{1}{2h}[u_{(2)K}(z) - u_{(1)K}(z)] + \frac{1}{2}u_{(1)3,K}(z) \\
e'_{(1)K3}(z) &= \frac{1}{2h}[u_{(2)3,K}(z) - u_{(1)3,K}(z)] \\
e_{(1)33}(z) &= \frac{1}{h}[u_{(2)3}(z) - u_{(1)3}(z)] \\
e'_{(1)33}(z) &\equiv 0
\end{aligned} \tag{3.5}$$

4. Model equations in displacements

For the isotropic plate and the displacement field postulated in Eqs (2.1), the generalized equilibrium Eqs (3.1) take the form

$$
\begin{aligned}
&(\lambda+\mu)h[u_{(2)K}+2u_{(1))K}]_{,KL}+\mu h[u_{(2)L}+2u_{(1)L}]_{,KK}+\\
&+3[(\lambda+\mu)u_{(2)3}-(\lambda-\mu)u_{(1)3}]_{,L}+\frac{6\mu}{h}[u_{(2)L}-u_{(1)L}]+6f_{(1)L}=0\\
&\mu h[u_{(2)3}+2u_{(1)3}]_{,KK}+3[(\lambda+\mu)u_{(2)K}+(\lambda-\mu)u_{(1)K}]_{,K}+\\
&+\frac{6(\lambda+2\mu)}{h}[u_{(2)3}-u_{(1)3}]+6f_{(1)3}=0\\
&(\lambda+\mu)h[2u_{(2)K}+u_{(1)K}]_{,KL}+\mu h[2u_{(2)L}+u_{(1)L}]_{,KK}+\\
&+3[(\lambda-\mu)u_{(2)3}-(\lambda+\mu)u_{(1)3}]_{,L}-\frac{6\mu}{h}[u_{(2)L}-u_{(1)L}]+6f_{(2)L}=0\\
&\mu h[2u_{(2)3}+u_{(1)3}]_{,KK}-3[(\lambda-\mu)u_{(2)K}+(\lambda+\mu)u_{(1)K}]_{,K}+\\
&-\frac{6(\lambda+2\mu)}{h}[u_{(2)3}-u_{(1)3}]+6f_{(2)3}=0
\end{aligned}
\tag{4.1}
$$

Let us introduce the auxiliary vector fields $\boldsymbol{v}=(v_L,v_3)$, $\boldsymbol{w}=(w_L,w_3)$ characterizing the general symmetric and antisymmetric states such that

$$
\begin{aligned}
v_L&=\frac{1}{2}[u_{(2)L}+u_{(1)L}] \qquad & v_3&=\frac{1}{h}[u_{(2)3}-u_{(1)3}]\\
w_L&=\frac{1}{h}[u_{(2)L}-u_{(1)L}] \qquad & w_3&=\frac{1}{2}[u_{(2)3}+u_{(1)3}]
\end{aligned}
\tag{4.2}
$$

and also introduce the symmetric and antisymmetric external forces $\boldsymbol{g}=(g_L,g_3)$, $\boldsymbol{q}=(q_L,q_3)$ such that

$$
\begin{aligned}
g_L&=\frac{1}{2}[f_{(2)L}+f_{(1)L}] \qquad & g_3&=\frac{1}{h}[f_{(2)3}-f_{(1)3}]\\
q_L&=\frac{1}{h}[f_{(2)L}-f_{(1)L}] \qquad & q_3&=\frac{1}{2}[f_{(2)3}+f_{(1)3}]
\end{aligned}
\tag{4.3}
$$

by these definitions, Eqs (4.1) can be decomposed into two independent systems of equations characterizing the symmetric and antisymmetric states, respectively. The first system of equations has the form

$$
\begin{aligned}
&(\lambda+\mu)v_{K,KL} + \mu v_{L,KK} + \lambda v_{3,L} + \frac{2}{h} g_L = 0 \\
&\mu v_{3,KK} - \frac{12\lambda}{h^2} v_{K,K} - \frac{12(\lambda+2\mu)}{h^2} v_3 + \frac{6}{h} g_3 = 0
\end{aligned}
\tag{4.4}
$$

and the second system of equations has the form

$$
\begin{aligned}
&(\lambda+\mu)w_{K,KL} + \mu w_{L,KK} - \frac{12\mu}{h^2} w_{3,L} - \frac{12\mu}{h^2} w_L + \frac{6}{h} q_L = 0 \\
&\mu w_{3,KK} + \mu w_{K,K} + \frac{2}{h} q_3 = 0
\end{aligned}
\tag{4.5}
$$

Similarly, the boundary conditions (2.3) can be written in terms of displacements, taking the form

$$
\begin{aligned}
&h\lambda v_3 n_L + 2h\mu v_{(K,L)} n_K + h\lambda v_{K,K} n_L = 2\widehat{g}_L \\
&h\mu v_{3,K} n_K = 6\widehat{g}_3 \\
&2h\mu w_{(K,L)} n_K + h\lambda w_{K,K} n_L = 6\widehat{q}_L \\
&h\mu w_K n_K + h\mu w_{3,K} n_K = 2\widehat{q}_3
\end{aligned}
\tag{4.6}
$$

where $\widehat{g}_L$, $\widehat{q}_L$ and $\widehat{g}_3$, $\widehat{q}_3$ represent the plane and anti-plane components of the boundary loads, acting on the contour $\partial\Pi$ of the midplane Π, with the relations

$$
\begin{aligned}
&\widehat{g}_L = \frac{1}{2}[p_{(2)L} + p_{(1)L}] \qquad && \widehat{g}_3 = \frac{1}{h}[p_{(2)3} - p_{(1)3}] \\
&\widehat{q}_L = \frac{1}{h}[p_{(2)L} - p_{(1)L}] \qquad && \widehat{q}_3 = \frac{1}{2}[p_{(2)3} + p_{(1)3}]
\end{aligned}
\tag{4.7}
$$

A solution to the displacement equations (4.4) and (4.5) which satisfies the boundary condition (4.6) together with the definition (4.1) enables us to determine the displacement field $\boldsymbol{u}_{(1)}$ and $\boldsymbol{u}_{(2)}$ of the lower and upper surfaces, respectively.

5. Formulation of the problem

Consider an infinite strip of thickness h subjected to a loading on the upper surface $x_3 = h$ of the form

$$
q = q_0 \sin \frac{\pi x_1}{L}
$$

where q_0 is constant and L is the half wave length.

The solutions to Eqs (4.4) and (4.5) with the boundary condition (4.6) for this example have the following form

$$
\begin{aligned}
v_1 &= \frac{6\lambda L^3}{\mu(\lambda+2\mu)\pi^3h^2 + 48\mu\pi L^2(\lambda+\mu)} q_0 \cos\frac{\pi x_1}{L} \\
v_3 &= \frac{6(\lambda+\mu)L^2}{\mu(\lambda+2\mu)\pi^2h^2 + 48\mu L^2(\lambda+\mu)} q_0 \sin\frac{\pi x_1}{L} \\
w_1 &= -\frac{12L^3}{(\lambda+2\mu)\pi^3h^3} q_0 \cos\frac{\pi x_1}{L} \\
w_3 &= \Big[\frac{L^2}{\mu\pi^2h} + \frac{12L^4}{(\lambda+2\mu)\pi^4h^3}\Big] q_0 \sin\frac{\pi x_1}{L}
\end{aligned}
\tag{5.1}
$$

From Eqs (4.2) and Eqs (5.1) we find out that

$$
\begin{aligned}
u_{(1)1} &= \Big[\frac{6\lambda L^3}{\mu(\lambda+2\mu)h^2\pi^3 + 48\mu\pi(\lambda+\mu)L^2} + \frac{6L^3}{(\lambda+2\mu)h^2\pi^3}\Big] q_0 \cos\frac{\pi x_1}{L} \\
u_{(2)1} &= \Big[\frac{6\lambda L^3}{\mu(\lambda+2\mu)h^2\pi^3 + 48\mu\pi(\lambda+\mu)L^2} - \frac{6L^3}{(\lambda+2\mu)h^2\pi^3}\Big] q_0 \cos\frac{\pi x_1}{L} \\
u_{(1)3} &= \Big[\frac{L^2}{\mu h\pi^2} + \frac{12L^4}{(\lambda+2\mu)h^3\pi^4} - \frac{3(\lambda+2\mu)L^2h}{\mu(\lambda+2\mu)\pi^2h^2 + 48\mu(\lambda+\mu)L^2}\Big] q_0 \sin\frac{\pi x_1}{L} \\
u_{(2)3} &= \Big[\frac{L^2}{\mu h\pi^2} + \frac{12L^4}{(\lambda+2\mu)h^3\pi^4} + \frac{3(\lambda+2\mu)L^2h}{\mu(\lambda+2\mu)\pi^2h^2 + 48\mu(\lambda+\mu)L^2}\Big] q_0 \sin\frac{\pi x_1}{L}
\end{aligned}
\tag{5.2}
$$

From Eqs (2.1) and Eqs (5.2) we find that the displacement field has the following form

$$
\begin{aligned}
u_1(\boldsymbol{x}) &= \Big[\frac{6\lambda L^3}{\mu(\lambda+2\mu)h^2\pi^3 + 48\mu\pi(\lambda+\mu)L^2} + \frac{6L^3}{(\lambda+2\mu)h^2\pi^3} + \\
&- \frac{12L^3x_3}{(\lambda+2\mu)\pi^3h^3}\Big] q_0 \cos\frac{\pi x_1}{L} \\
u_3(\boldsymbol{x}) &= \Big[\frac{L^2}{h\mu\pi^2} - \frac{3(\lambda+2\mu)L^2h}{\mu(\lambda+2\mu)\pi^2h^2 + 48\mu(\lambda+\mu)L^2} + \\
&+ \frac{6(\lambda+2\mu)L^2x_3}{\mu(\lambda+2\mu)\pi^2h^2 + 48\mu(\lambda+\mu)L^2} + \frac{12L^4}{(\lambda+2\mu)h^3\pi^4}\Big] q_0 \sin\frac{\pi x_1}{L}
\end{aligned}
$$

6. Comparison with the other theories

The solution which we have obtained can be used to evaluate the displacement of the midsurface and stress for comparison purposes. The displacement of midsurface of the plate is given by

$$u_3 = \frac{q_0 L^4}{D\pi^4}\Big[\frac{1-2\nu}{(1-\nu)^2} + \frac{K^2}{6(1+\nu)}\Big]\sin\frac{\pi x_1}{L}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)} \qquad K = \frac{\pi h}{L}$$

and ν is Poisson ratio, and the stress component T_{11} is

$$T_{11} = \Big(\frac{12}{K^2}\frac{x_3}{h} - \frac{6}{K^2}\Big)q_0 \sin\frac{\pi x_1}{L}$$

These results are to be compared with the other solutions which have obtained by using other theories. First, the solution obtained by using the classical plate theory for the midplane displacement is

$$u_3 = \frac{q_0 L^4}{D\pi^4}\sin\frac{\pi x_1}{L}$$

The Reissner plate theory yields the result

$$u_3 = \frac{q_0 L^4}{D\pi^4}\Big[1 + \frac{K^2(2-\nu)}{10(1-\nu)}\Big]\sin\frac{\pi x_1}{L}$$

For higher-order theories, Essenburg's theory (cf Essenburgb(1975)) gives

$$u_3 = \frac{q_0 L^4}{D\pi^4}\Big[1 + \frac{K^2(\nu^2-\nu+2)}{10(1-\nu)} + \frac{\nu K^2}{40} - \frac{3K^4}{1120}\Big]\sin\frac{\pi x_1}{L}$$

For the stress component T_{11}, the classical, the shear deformation Reissner and the Essenburg theories all yield the same result

$$T_{11} = \frac{12 q_0}{K^2}\frac{x_3}{h}\sin\frac{\pi x_1}{L}$$

Also this problem is solved by Lo et al. (1977) by using a higher-order theory which gives for the midplane displacement the result

$$u_3 = \frac{q_0 L^4}{D\pi^4}\frac{1}{4(1-\nu)[8400(1-2\nu) + 120(1-\nu)K^2 + (1-\nu)^2K^4]}\cdot$$

$$\cdot[33600(1-\nu)(1-2\nu) + (7200 - 16920\nu + 5520\nu^2)K^2 +$$

$$+140\nu(1-\nu)K^4 - (1-\nu)^2K^6]\sin\frac{\pi x_1}{L}$$

and for the stress component T_{11} gives

$$T_{11} = \frac{q_0 \sin \frac{\pi x_1}{L}}{16800(1-2\nu) + 240(1-\nu)K^2 + 2(1-\nu)^2 K^4} \cdot$$

$$\cdot \Big\{ \frac{15(1-\nu)^2 K^2 \left(12\frac{x_3}{h} - 1\right)}{720(1-2\nu) + 24(1-\nu)K^2 + (1-\nu)^2 K^4} + \frac{6}{K^2}\frac{x_3}{h} \cdot$$

$$\cdot [33600(1-2\nu) - 120(1-\nu)(10-7\nu)K^2 - 80(1-\nu)^2 K^4] +$$

$$+12(1-\nu)\left(\frac{x_3}{h}\right)^3 [2800(2-\nu) + 280(1-\nu)K^2] \Big\}$$

For the exact solution (cf Little (1973)), setting $\kappa = K/2$ the midplane displacement is given by

$$u_3 = \frac{q_0 L^4}{D\pi^4} \frac{K^3 \cosh\kappa [2 + (1+\nu)\kappa \tanh\kappa] \sin\frac{\pi x_1}{L}}{24(1-\nu^2)[\sinh\kappa\cosh\kappa - \kappa]}$$

and the stress component T_{11} is given by

$$T_{11} = \frac{q_0}{2}\Big[\left(\frac{\cosh\kappa - \kappa\sinh\kappa}{\sinh\kappa\cosh\kappa - \kappa} + \frac{\frac{\pi x_3}{L}\sinh\kappa}{\sinh\kappa\cosh\kappa + \kappa} \right) \sinh\frac{\pi x_3}{L} +$$

$$+ \left(\frac{\sinh\kappa - \kappa\cosh\kappa}{\sinh\kappa\cosh\kappa + \kappa} + \frac{\frac{\pi x_3}{L}\cosh\kappa}{\sinh\kappa\cosh\kappa - \kappa} \right) \cosh\frac{\pi x_3}{L} \Big] \sin\frac{\pi x_1}{L}$$

7. Discussion and conclusion

In Fig.1 the variation of the coefficient of the midplane displacement versus the ratio h/L is shown, where the displacement of midplane u_3 is written in the form

$$u_3 = (\text{displacement coefficient}) \frac{q_0 L^4}{D\pi^4} \sin\frac{\pi x_1}{L}$$

We see from this figure that for $h/L = 1.5$ the differences between the approximate theories and the exact solution are substantial, and continue to increase with increasing h/L. Also we see that the higher-order theory and the Essenburg's theory yield close results while the present theory and Reissner's theory give near results. This is clear because the latest are of the order one of the out-plane coordinate while the others are of high order of the out-plane coordinate.

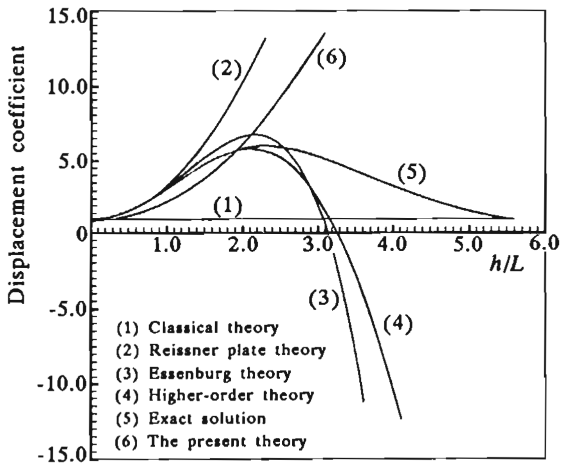

Fig. 1. Midplane displacement solution for $\nu = 0.25$

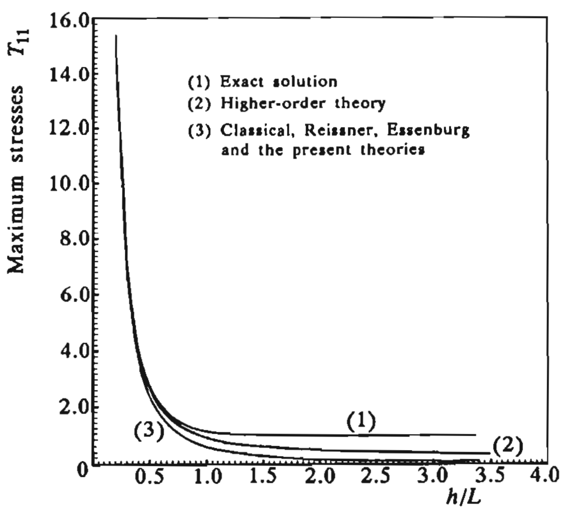

Fig. 2. Maximum flexural stress distribution for $\nu = 0.25$

In Fig.2 the maximum value of the flexural stress T_{11} is plotted against the ratio h/L. It is seen from this figure that if is smaller then one, results of all theories are close each other the ratio between the thickness and the characteristic length of the load.

8. References

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Porównanie wyników rozwiązań w ramach różnych teorii płyt

Streszczenie

W artykule przedstawiono uklad równań 2-wymiarowego modelu elastostatyki plyt, który bazuje na teorii nieklasycznego kontinuum z więzami wewnętrznymi. Przytoczony model został przeanalizowany na przykładzie prostego problemu nieskończonego pasma plytowego z periodycznym obciążeniem. Wyniki rozwiązań zostały porównane ze znanymi rozwiązaniami w ramach innych modeli dwuwymiarowych i w

ramach teorii ścisłej. Z porównania wynika, iż model oparty na założeniu więzów czysto kinematycznych, dla pewnych parametrów problemu brzegowego może dawać zadawalające wyniki rozwiązań.

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