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STOCHASTIC CONTACT EFFECTS IN PERIODIC FIBRE COMPOSITES[1]

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In this paper stochastic contact effects in the field of structural computational mechanics are introduced. The main ideas of this formulation are:
1. Approximation of unknown stochastic geometry of the contact surface by a sequence of assumed figures; volume and frequency of occurring on considered surface are Gaussian random variables with specified statistics
2. Replacing random contact region including all these figures with stochastically averaged material occupying deterministic contact region
3. Numerical solution to the stochastic static problem formulated in this way using the Stochastic Finite Element Method (SFEM).
Proposed model is intended for analysis of interface discontinuities in periodic fibre composites with random elastic properties, however there are many more complicated problems in which the presented method can be employed with a good result. In numerical analysis a quarter of square periodicity cell with round, centrally placed fibre is analysed using POLSAP and ABAQUS systems.

1. Introduction

Contemporary methods of solving contact effects based on numerical analysis (cf Schrefler and Zavarise (1993); Wriggers and Zavarise (1993); Zavarise et al. (1992a,b)) did not allow, until now, considering directly the randomness of contact zone geometry. Statistical parameters of this zone discontinuity were replaced with deterministic constants used for defining new, non-linear constitutive relations within the contact region. Definitely, in spite of random geometry of contact effect, obtained displacement and stress field was of

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deterministic character. Such a formulation of the problem was caused mainly by the lack of proper stochastic numerical tools. In the present paper a new approach will be formulated; it consists on mathematical and numerical considering of stochastic character of contact between two regions. It will be based on replacing random geometry of boundary with linear constitutive relation of probabilistic character (cf Kamiński (1993)), defined in the contact region with deterministic geometry. Obtained stochastic static formulation equivalent to the primary contact effect will be analysed numerically using the Stochastic Finite Element Method (SFEM) (cf Kamiński (1994); Kleiber and Hien (1992)). Alternative method of considering random geometry with application to composite materials is e.g. Delaunay networks method (cf Ostoja-Starzewski and Wang (1980)).

Such a mathematical-numerical model will be applied to static analysis of periodic fibre composites. Until now, in mechanics of these media, two geometric scales were distinguished: macro-scale connected with the whole composite structure and micro-scale connected with the periodicity cell (cf Bensoussan et al. (1978); Sanchez-Palencia (1980)). In this way, in analysis of fibre composites, all random local physical phenomena and dislocations of periodic geometry connected with the fibre-matrix boundary have always been omitted. That is why it seems useful to introduce an additional scale connected with the surface of contact between composite components (Fig.1), which would allow to consider random discontinuities occurring on the fibre-matrix boundary, possible on all its circumference. In this case we will obtain an equivalent stochastic constitutive relation by separating in the matrix, along all fibre circumference, interface zone including all discontinuities which will be then homogenized, i.e. replaced with effective material (cf Lené (1984); Milton and Kohn (1988); Suquet (1982)) having elastic properties weaker than the matrix does. To compute the effective properties we use the probability density function, analogically as Arminjon (1991) and Sobczyk (1982). Alternative methods of discontinuities modelling in periodic fibre composites with deterministic elastic properties (in their homogenization problems) based on variational inequalities may be found in the work of Gajl (1991) and Telega (1988).

It should be underlined that the presented model has been formulated in such a way that all data necessary for numerical analysis could be obtained in an experimental way, i.e. by laboratory strength tests and analysis of fibre composite sections in planes perpendicular to the fibres. In the future it would enable practical verification of stochastic formulation of fibre-matrix contact effects in fibre composites.

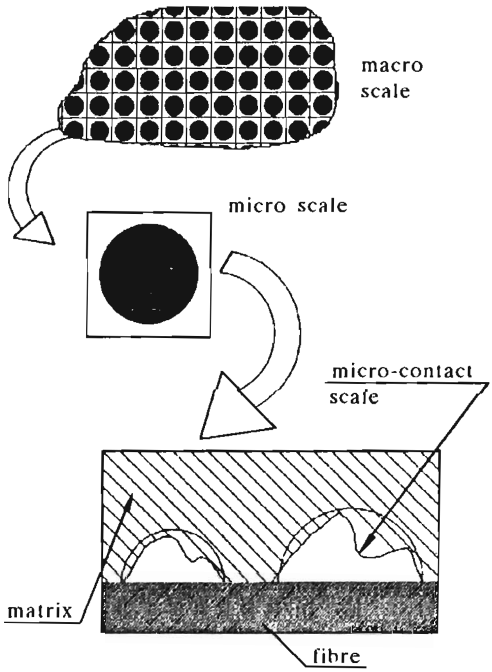

Fig. 1. Various scales in periodic fibre composite structures

2. Problem formulation

As it is known, the considered fibre composite consists of reinforcement made of a stronger material and of filling (matrix) (Christensen (1979)). As it is proved by various laboratory tests the surface of contact zone between two phases is characterized by discontinuities; their magnitude and position in space can be, for technological reasons, treated as random functions and they are concentrated usually around the filling material. To make a static analysis of such a structure it is necessary to approximate the character of curve (or surface), where the contact between fibre and matrix occurs. Let us assume that it is a band (Fig.1) geometrically located in matrix region, in which there are semicircles (or hemispheres in three-dimensional problem) with their diameters on the boundary line ("bubbles"). Below we will assume that sizes and frequencies of these "bubbles" occurring on the matrix boundary are Gaussian distributions, expected values and variances of which are known. This assumption is caused by frequent usage of this distribution in physical sciences and for mathematical and numerical reasons. Because of the fact that the assumed distribution is a limit distribution for many classes of other random distributions this assumption does not radically restrict the presented model. We will also assume that we know the first two moments of Young modulus of composite component material. Finally we will assume that random variables characterizing the geometry of considered boundary and random distributions of elasticity Young moduli are mutually stochastically independent. For so determined stochastic problem of elasticity theory we will look for the first two moments of a displacement function and for expected values of individual components of the stress state in a periodicity cell quarter subjected to uniform tension, considering the random character of the fibre-matrix connection. To obtain this we will first homogenize the layer containing random discontinuities on contact zone composite components, finding the first two moments of its elasticity modulus in an analytical way. The sought moments of displacements and stresses fields will be found numerically, using SFEM.

Because of the fact that the interface discontinuities averaging gives us statistical parameters of contact zone elastic properties which are constant along the circumference we will analyze only a quarter of the periodicity cell for simplicity. As we can see, it does not affect the general character of our considerations. Expanding the derived parameters on the whole cell can be obtained by changing the parameter characterizing the frequency of discontinuities occurrence for a four times bigger one. Moreover, analyzing a cell quarter makes the pictures illustrating used model more clear.

3. Mathematical model

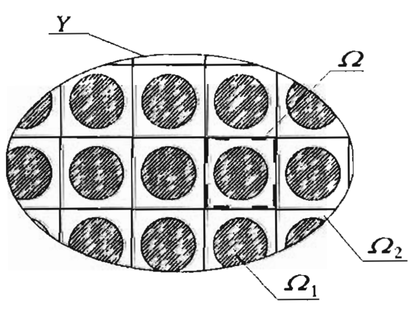

Fig. 2. Periodic fibre composite structure with the periodicity cell

Let $Y \subset \mathcal{R}^2$ be a linear-elastic continuum. Let then Y be a periodic random two-phase composite structure (cf Kamiński [15]; Kohn (1988)), in which Ω is a periodicity cell, cf Fig.2. Let us assume that Ω is a coherent bounded region uniplanar with $x_3 = 0$ plane and that it consists of two disjoint linear-elastic isotropic subdomains Ω_1 and Ω_2 called fibre and matrix, respectively, such that

$$\Omega_1 \cup \Omega_2 \neq \Omega$$

Let

$$\overline{\Omega} = \Omega - \{\Omega_1 \cup \Omega_2\}$$

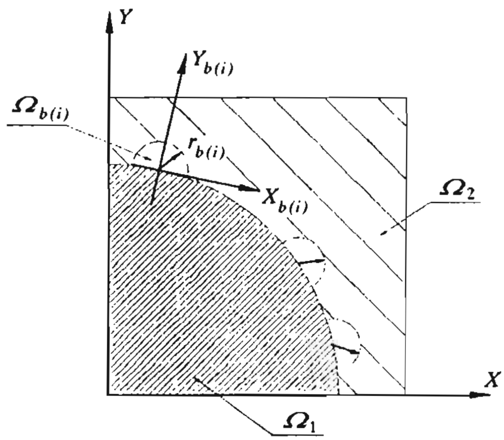

Fig. 3. "Bubbles" geometry on the fibre-matrix interface

Further let us assume that $\overline{\Omega}$ consists of a finite number of disjoint subsets $\overline{\Omega}_i$

$$\overline{\Omega} = \bigcup_{i=1}^{n} \overline{\Omega}_i$$

such that

$$\overline{\Omega}_i = \left\{(x_{(\overline{\Omega}_i)}, y_{(\overline{\Omega}_i)} \in \mathcal{R}^2 : \ [x_{(\overline{\Omega}_i)}]^2 + [y_{(\overline{\Omega}_i)}]^2 \leq r^2_{(\overline{\Omega}_i)}\right\}$$

where the coordinate system $0x_{(\overline{\Omega}_i)}y_{(\overline{\Omega}_i)}$ originates from the following transformation (a slip) of the coordinate system $0xy$ connected with the centre of symmetry of considered quarter (the figure below), and geometric loci $M(\overline{\Omega}_i)$ of local systems centres after the transformation are described in the following way

$$M(\overline{\Omega}_i) = \left\{(x_i, y_i) \in \mathcal{R}^2 : \ x_i^2 + y_i^2 = R^2\right\}$$

Let us finally denote by m_d the number of subsets $\overline{\Omega}_i$ per unit length of Ω_1 and Ω_2 regions boundaries. Let $r_{(\overline{\Omega}_i)}$ and m_d be Gaussian random fields describing volume and frequency of discontinuities on Ω_1 and Ω_2 boundaries, with known expected values $E[r_{(\overline{\Omega}_i)}]$ and $E[m_d]$ and variances $\mathrm{Var}[r_{(\overline{\Omega}_i)}]$ and $\mathrm{Var}[m_d]$, respectively.

For a given vector $\boldsymbol{Q}$ of external load we will look for random displacement and stresses fields $\boldsymbol{u}$ and $\boldsymbol{\sigma}$ respectively, satisfying the following system of boundary-differential equations

$$
\begin{aligned}
&\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} \\
&\varepsilon_{kl} = \frac{1}{2}\Big(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_k}{\partial x_l}\Big) \\
&\mathrm{div}\boldsymbol{\sigma} = \boldsymbol{Q} \\
&\boldsymbol{u} = \hat{\boldsymbol{u}} \qquad\qquad x \in \partial\Omega_{\hat{u}} \\
&\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}} \qquad\qquad x \in \partial\Omega_{\hat{\sigma}}
\end{aligned}
\tag{3.1}
$$

where

$$
C_{ijkl} = \Big[\delta_{ij}\delta_{kl}\frac{\nu(x)}{(1+\nu(x))(1-2\nu(x))} + (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\frac{1}{2(1+\nu(x))}\Big]e(x) \tag{3.2}
$$

and

$e(x)$ – Young modulus

$\nu(x)$ – Poisson ratio.

We will finally assume that $e(x) = e(x,w)$ is also a Gaussian random field with the known function of expected values for Ω_1 and Ω_2 regions

$$
E(e(x)) = \begin{cases} E(e_1) & x \in \Omega_1 \\ E(e_2) & x \in \Omega_2 \end{cases} \tag{3.3}
$$

and covariance matrix

$$
\mathrm{Cov}(e_i, e_j) = \begin{bmatrix} \mathrm{Var}(e_1) & \mathrm{Cov}(e_1, e_2) \\ \mathrm{Cov}(e_2, e_1) & \mathrm{Var}(e_2) \end{bmatrix} \tag{3.4}
$$

The random field $e(x,w)$ defined on coherent subsets of the plane with random boundary geometry makes it impossible to carry out a stochastic numerical analysis. Therefore to find random displacements and stresses fields in Ω we will additionally define a subset $\Omega_k \subset \mathcal{R}^2$. Let $\Omega_k \subset \Omega_2$ and let it contain all subsets $\overline{\Omega}_i \in \overline{\Omega}$ with probability density tending to 1. Such conditions are fulfilled by the subset Ω_k shown in Fig.4., where thickness estimation Δ_k of Ω_k region (cf Morrison (1976)) is assumed to be constant on the $\Omega_1 \cap \Omega_k$ boundary length

$$\Delta_k = E[r_{(\overline{\Omega}_i)}] + 3\sqrt{\mathrm{Var}[r_{(\overline{\Omega}_i)}]} \tag{3.5}$$

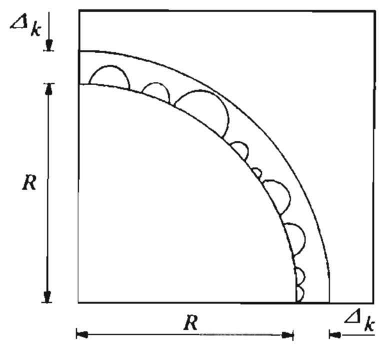

Fig. 4. Contact zone geometry

In order to define random Young modulus parameters for $x \in \Omega_k$ we use the homogenization method, according to which a given effective property Y^{eff} characterizing homogenized area f is a weighted average of Y_i properties defined on f_i areas

$$Y^{\mathrm{eff}} = \frac{\sum\limits_i f_i Y_i}{f} \tag{3.6}$$

According to this formulation we obtain

$$e_k = \frac{S_{\Omega_k} - S_{\overline{\Omega}}}{S_{\Omega_k}} e_2 \tag{3.7}$$

where S_A is the area of the region A.

As we may easily notice from the above formula we have

$$S_{\Omega_k} = \frac{\pi}{4}\Big\{(R + \Delta_k)^2 - R^2\Big\} \tag{3.8}$$

Using Eq (3.7) we will find the expected value and the variance of effective Young modulus e_k, terms missing in the covariance matrix extended by one rank as well as the averaged value of Poisson ratio.

We calculate the sought expected value

$$E[e_k] = E\Big[\frac{S_{\Omega_k} - S_{\overline{\Omega}}}{S_{\Omega_k}} e_2\Big] = \Big[e_2 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}} e_2\Big] = E[e_2]\Big(1 - \frac{1}{S_{\Omega_k}} E[S_{\overline{\Omega}}]\Big) \tag{3.9}$$

Analogously we obtain the variation

$$\mathrm{Var}[e_k] = \mathrm{Var}\Big[\Big(1 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big)e_2\Big] \tag{3.10}$$

Using the property (A.8) enclosed in Appendix A we obtain

$$\begin{aligned}\mathrm{Var}[e_k] &= E^2\Big[1 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big]\mathrm{Var}[e_2] + \mathrm{Var}\Big[1 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big]\mathrm{Var}[e_2] + \\ &+ E^2[e_2]\mathrm{Var}\Big[1 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big]\end{aligned}$$

using lemmas (A.3), (A.1) and (A.4) successively, it is

$$E^2\Big[1 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big] = \Big\{E\Big[1 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big]\Big\}^2 = \Big\{E[1] - E\Big[1 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big]\Big\}^2 = \Big\{1 - \frac{1}{S_{\Omega_k}}E[S_{\overline{\Omega}}]\Big\}^2$$

and due to (A.7), (A.2) and (A.5) we have

$$\mathrm{Var}\Big[1 - \frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big] = \mathrm{Var}[1] + \mathrm{Var}\Big[\frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big] = \mathrm{Var}\Big[\frac{S_{\overline{\Omega}}}{S_{\Omega_k}}\Big] = \frac{1}{S^2_{\Omega_k}}\mathrm{Var}[S_{\overline{\Omega}}]$$

Finally we obtain

$$\begin{aligned}\mathrm{Var}[e_k] &= \Big\{1 - \frac{1}{S_{\Omega_k}}E[S_{\overline{\Omega}}]\Big\}^2\mathrm{Var}[e_2] + \frac{1}{S^2_{\Omega_k}}\mathrm{Var}[S_{\overline{\Omega}}]\mathrm{Var}[e_2] + \\ &+ \frac{1}{S^2_{\Omega_k}}\mathrm{Var}[S_{\overline{\Omega}}]E^2[e_2]\end{aligned} \tag{3.11}$$

Now we have to find $S_{\overline{\Omega}}$ distribution parameters. As we may notice we have

$$S_{\overline{\Omega}} = \frac{\pi}{2}r^2_{\overline{\Omega}}m_b \tag{3.12}$$

where m_b is the number of $\overline{\Omega}_i$ areas found in Ω_k and amounts to

$$m_b = \frac{\pi}{2}Rm_d \tag{3.13}$$

Therefore, using properties (A.4) and (A.5), we have

$$E[m_b] = \frac{\pi R}{2}E[m_d] \tag{3.14}$$

and

$$\mathrm{Var}[m_b] = \frac{\pi^2 R^2}{4}\mathrm{Var}[m_d] \tag{3.15}$$

We derive next

$$E[S_{\overline{\Omega}}] = \frac{\pi}{2}E[r_{\overline{\Omega}}^2 m_b] = \frac{\pi}{2}E[r_{\overline{\Omega}}^2]E[m_b]$$

what follows assumed independence of $r_{\overline{\Omega}}$ and m_b. Using Eq (B.2) obtained in Appendix B we have

$$E[S_{\overline{\Omega}}] = \frac{\pi}{2}\Big\{E^2[r_{\overline{\Omega}}] + \mathrm{Var}[r_{\overline{\Omega}}]\Big\}E[m_b] \tag{3.16}$$

Finally we find the variance of $S_{\overline{\Omega}}$ variable

$$\mathrm{Var}[S_{\overline{\Omega}}] = \mathrm{Var}\Big[\frac{\pi}{2}r_{\overline{\Omega}}^2 m_b\Big] = \frac{\pi^2}{4}\mathrm{Var}[r_{\overline{\Omega}}^2 m_b]$$

which, using the theorem (A.8) mentioned before, is equal to

$$\mathrm{Var}[S_{\overline{\Omega}}] = \frac{\pi^2}{4}\Big\{E^2[r_{\overline{\Omega}}^2]\mathrm{Var}[m_b] + \mathrm{Var}[r_{\overline{\Omega}}^2]\mathrm{Var}[m_b] + E^2[m_b]\mathrm{Var}[r_{\overline{\Omega}}^2]\Big\}$$

Using formulas (B.2) and (B.3) we finally obtain

$$\begin{aligned}\mathrm{Var}[S_{\overline{\Omega}}] &= \frac{\pi^2}{4}\Big(E^2[r_{\overline{\Omega}}] + \mathrm{Var}[r_{\overline{\Omega}}]\Big)^2\mathrm{Var}[m_b] + \\ &+ \frac{\pi^2}{2}\mathrm{Var}[r_{\overline{\Omega}}]\Big(E^2[m_b] + \mathrm{Var}[m_b]\Big)\Big(2E^2[r_{\overline{\Omega}}] + \mathrm{Var}[r_{\overline{\Omega}}]\Big)\end{aligned} \tag{3.17}$$

Substituting equations describing $S_{\overline{\Omega}}$ distribution parameters into equations describing expected values and the variance of e_k modulus distribution we can derive analytically the data necessary for numerical analysis.

Note that the covariance matrix of Young modulus in Ω will finally have the following form

$$\mathrm{Cov}\Big(e^{(i)}, e^{(j)}\Big) = \begin{bmatrix} \mathrm{Var}[e_1] & 0 & 0 \\ 0 & \mathrm{Var}[e_2] & \sqrt{\mathrm{Var}[e_k]\mathrm{Var}[e_2]} \\ 0 & \sqrt{\mathrm{Var}[e_k]\mathrm{Var}[e_2]} & \mathrm{Var}[e_k] \end{bmatrix} \tag{3.18}$$

At the end it should be noted that by homogenization of the Poisson ratio in Ω_k region we obtain

$$\nu_k = \Big(1 - \frac{E(S_{\overline{\Omega}})}{S_{\Omega_k}}\Big)\nu_2 \tag{3.19}$$

4. Numerical implementation and analysis

As we know (cf Kleiber and Hien (1992)) variational formulation of the equation system (3.1) may be written as follows

$$\int_{\Omega} \partial u_{i,j} C_{ijkl} u_{k,l}\, d\Omega = \int_{\Omega} \partial u_i \rho f_i\, d\Omega + \int_{\Omega} \partial u_i \hat{t}_i\, d(\partial\Omega^{t_i}) \tag{4.1}$$

Discretizing the set Ω into E coherent regions Ω_e such that

$$\Omega = \bigcup_{e=1}^{E} \Omega_e$$

we can define, in Ω region, the vector of random variables with the Gaussian distribution $b(x) = \{b^r(x)\}$, $r = 1,...,R$, $R \leq E$ using its first two moments. Denoting by $g(b^r)$ the probability density of $b^r(x)$ and analogically $g(b^r, b^s)$ for variables $b^r(x)$ and $b^s(x)$ we have the expected value of this variable

$$E(b^r) = \int_{-\infty}^{+\infty} b^r g(b^r)\, db^r \tag{4.2}$$

and the covariance matrix

$$\mathrm{Cov}(b^r, b^s) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \Big(b^r - E(b^r)\Big)\Big(b^s - E(b^s)\Big) g(b^r, b^s)\, db^r db^s \tag{4.3}$$

It can be shown that the variational principle after minimalization of potential energy functional and after expansion of all random functions into Taylor series consisting of zero-th, first and second order terms, respectively, is equivalent to the following system of equations

$$\begin{aligned}
&\mathsf{K}^0 \boldsymbol{q}^0 = \boldsymbol{Q}^0 \\
&\mathsf{K}^0 \boldsymbol{q}^{,r} = \boldsymbol{Q}^{,r} - \mathsf{K}^{,r} \boldsymbol{q}^0 \\
&\mathsf{K}^0 \boldsymbol{q}^{(2)} = \frac{1}{2}\Big(\boldsymbol{Q}^{,rs} - 2\mathsf{K}^{,r}\boldsymbol{q}^{,s} - \mathsf{K}^{,rs}\boldsymbol{q}^0\Big)\mathrm{Cov}(b^r, b^s)
\end{aligned} \tag{4.4}$$

where

$$\boldsymbol{q}^{(2)} = \frac{1}{2}\boldsymbol{q}^{,rs}\mathrm{Cov}(b^r, b^s)$$

while $\mathbf{K}$, $\boldsymbol{q}$ and $\boldsymbol{Q}$ are stiffness matrix, displacement and external load vectors, respectively, and terms $(\cdot)^{,r}$, $(\cdot)^{,rs}$ denote the first and the second partial derivatives with respect to random variable. The stiffness matrix and its corresponding derivatives may be defined as

$$K^{0}_{\alpha\beta} = \sum_{e=1}^{E} \int_{\Omega_e} C^{0}_{ijkl} B_{ij\alpha} B_{kl\beta} \, d\Omega \tag{4.5}$$

$$K^{,r}_{\alpha\beta} = \sum_{e=1}^{E} \int_{\Omega_e} C^{,r}_{ijkl} B_{ij\alpha} B_{kl\beta} \, d\Omega \tag{4.6}$$

$$K^{,rs}_{\alpha\beta} = \sum_{e=1}^{E} \int_{\Omega_e} C^{,rs}_{ijkl} 0 B_{ij\alpha} B_{kl\beta} \, d\Omega \tag{4.7}$$

In the system of equations (4.4), after considering formulas (4.5) $\div$ (4.7), we have

– the first partial derivatives of elasticity tensor

$$\begin{aligned} C^{,r}_{ijkl} &= \frac{\partial C_{ijkl}}{\partial e} = \delta_{ij}\delta_{kl} \frac{\nu(x)}{(1+\nu(x))(1-2\nu(x))} + \\ &+ (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \frac{1}{2(1+\nu(x))} \end{aligned} \tag{4.8}$$

– the second partial derivatives of elasticity tensor

$$\forall r,s = 1,...,E \quad C^{rs}_{ijkl} = 0 \Leftrightarrow K^{,rs}_{\alpha\beta} = 0 \tag{4.9}$$

– the first and the second partial derivatives of the external load vector

$$\forall r,s = 1,...,E \ (\boldsymbol{Q}^{,r} = \mathbf{0} \ \wedge \ \boldsymbol{Q}^{,rs} = \mathbf{0}) \tag{4.10}$$

Finally the system of equations to be solved (4.4) is of the following form

$$\mathbf{K}^{0}\boldsymbol{q}^{0} = \boldsymbol{Q}^{0} \tag{4.11}$$

$$\mathbf{K}^{0}\boldsymbol{q}^{,r} = -\mathbf{K}^{,r}\boldsymbol{q}^{0} \tag{4.12}$$

$$\mathbf{K}^{0}\boldsymbol{q}^{(2)} = -\mathbf{K}^{,r}\boldsymbol{q}^{,s}\mathrm{Cov}(b^{r}, b^{s}) \tag{4.13}$$

Obtaining from these equations successively $\boldsymbol{q}^{0}$ from the first system, $\boldsymbol{q}^{,r}$ from the second one and $\boldsymbol{q}^{,rs}$ from the last one, we derive expected values and the covariance matrix of displacement field as well as expected values of stress tensor components from the following equations

$$E[q_\beta] = q^0_\beta + \frac{1}{2} q^{,rs}_\beta \mathrm{Cov}(b^r, b^s) = q^0_\beta + q^{(2)}_\beta \tag{4.14}$$

$$\mathrm{Cov}[q^r_\alpha, q^s_\beta] = q^{,r}_\alpha q^{,s}_\beta \mathrm{Cov}(b^r, b^s) \tag{4.15}$$

$$\begin{aligned} E[\sigma^{(e)}_{ij}] &= C^{0(e)}_{ijkl} B^{(e)}_{kl\alpha} q^0_{\alpha(e)} + \\ &+ \frac{1}{2}\Big[2C^{,r(e)}_{ijkl} q^{,s}_{\alpha(e)} + C^{0(e)}_{ijkl} q^{,rs}_{\alpha(e)}\Big] B^{(e)}_{kl\alpha} \mathrm{Cov}(b^r, b^s) \end{aligned} \tag{4.16}$$

Numerical calculations have been done using a plane stochastic 4-node element (plane stress/strain analysis) of POLSAP system code (cf Bathe et al. (1973); Hien (1990)) written in FORTRAN 77 [26] and compiled in DOS. Composite quarter was discretized by 60 finite elements. Comparative tests were done thanks to the ABAQUS [1] working in UNIX system on a mesh condensed to 224 elements. Both discretizations are shown in Fig.5.

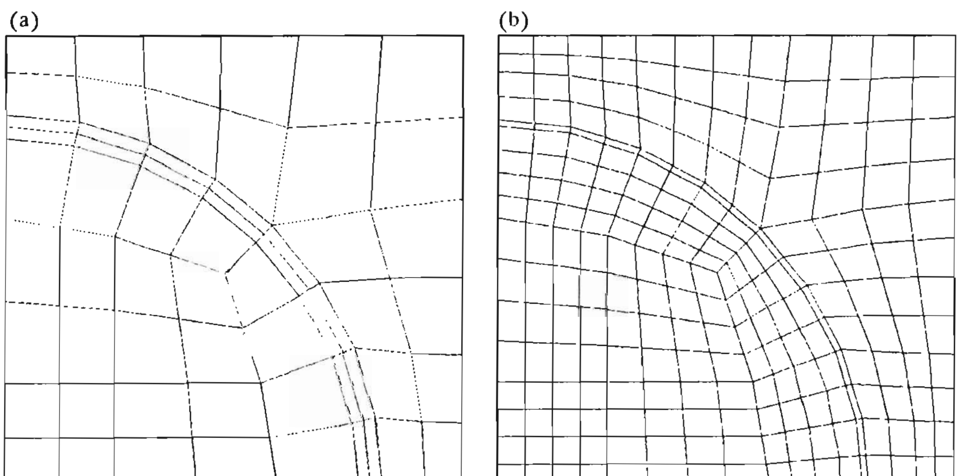

Fig. 5. Periodicity cell quarter mesh for: (a) – POLSAP, (b) – ABAQUS

Discretization of a periodicity cell for the POLSAP computations was done in such a way, that to change the contact zone area it is enough to transform the nodal points settings the boundary between this area and matrix through homothety with respect to the centre of coordinate system connected with the periodicity cell (Fig.3).

In all tests the analyzed quarter is subjected to uniform tension on the right vertical edge. On the remaining edges orthogonal displacements and rotations are fixed (symmetry conditions).

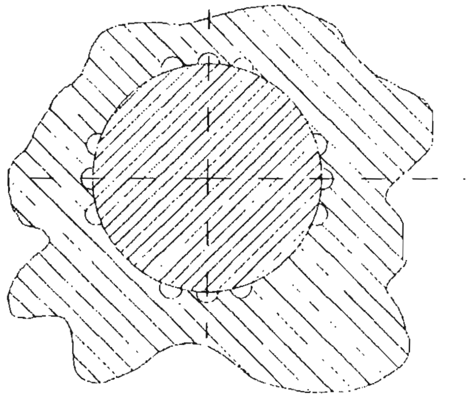

Fig. 6. Tested "bubbles" arrangement

Data concerned with random geometry of the boundary between composite materials were assumed so that they correspond with the arrangement of "bubbles" shown in Fig.6.

Above discontinuity distribution is equivalent to the following parameters of distribution of their frequency in a quarter of composite $E[n] = 3$ with assumed standard deviation $\sigma[n] = 0.05E[n] = 0.15$. The volume of these discontinuities is described by the following moments: $E[r] = 0.02R = 8.0e-3$ and $\sigma[r] = 0.10E[r] = 8.0e-4$. Assuming fibre material with the Young modulus $E[e_1] = 84.0$ GPa, $\sigma[e_1] = 8.4$ GPa and the Poisson ratio $\nu_1 = 0.22$, whereas matrix material with $E[e_2] = 4.0$ GPa, $\sigma[e_2] = 0.40$ GPa and $\nu_2 = 0.34$ a result of homogenization was the interface layer described by the following parameters: $\Delta_k = 1.04e-2$, $E[e_k] = 3.82$ GPa, $\mathrm{Var}[e_k] = 1.48$ GPa, $\nu_k = 0.324$.

5. Analysis of results

The results of tests carried out are collected in Fig.7 ÷ Fig.16 (Fig.7 ÷ Fig.10 and Fig.13 ÷ Fig.16 - ABAQUS system, Fig.11 and Fig.12 - POLSAP system), respectively. Fig.7 ÷ Fig.10 represent horizontal displacement field obtained using a plane 4-node deterministic element of the ABAQUS system,

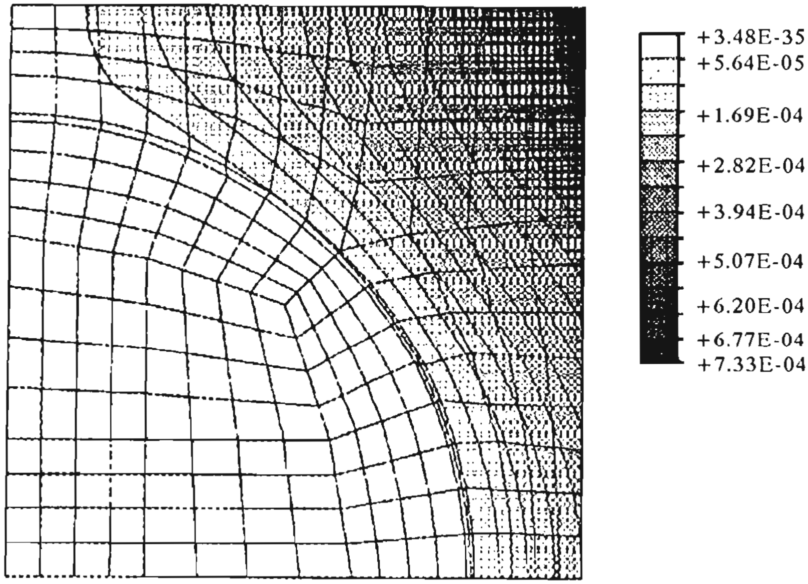

Fig. 7. Horizontal displacements in the model without "bubbles"

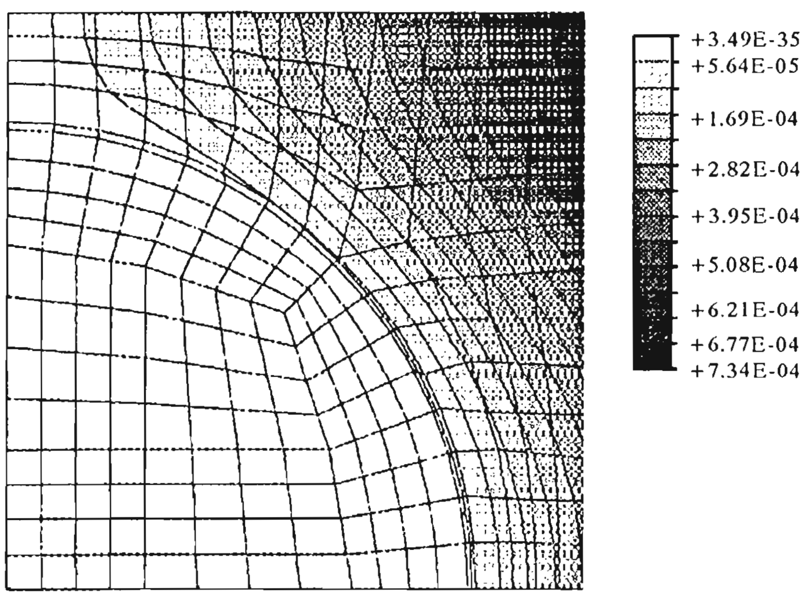

Fig. 8. Horizontal displacements in model with the Young modulus expected value in the contact zone

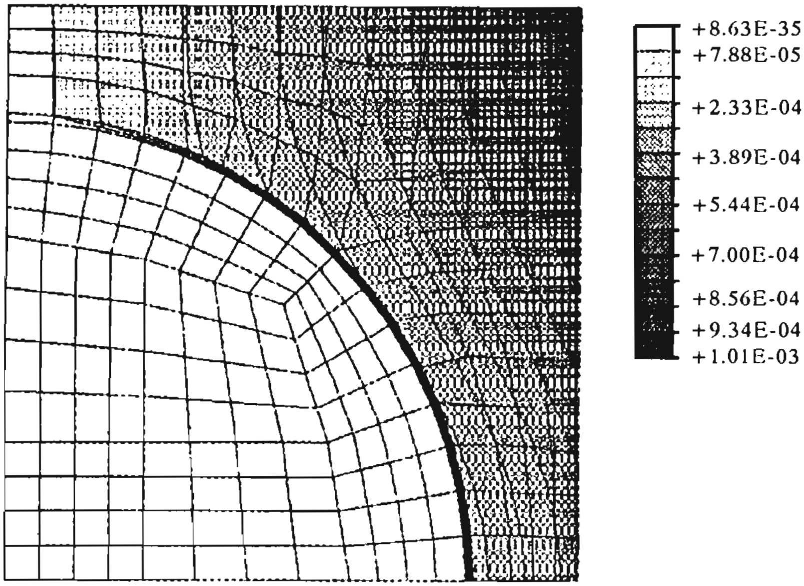

Fig. 9. Horizontal displacements in model with the Young modulus lower bound in the contact zone

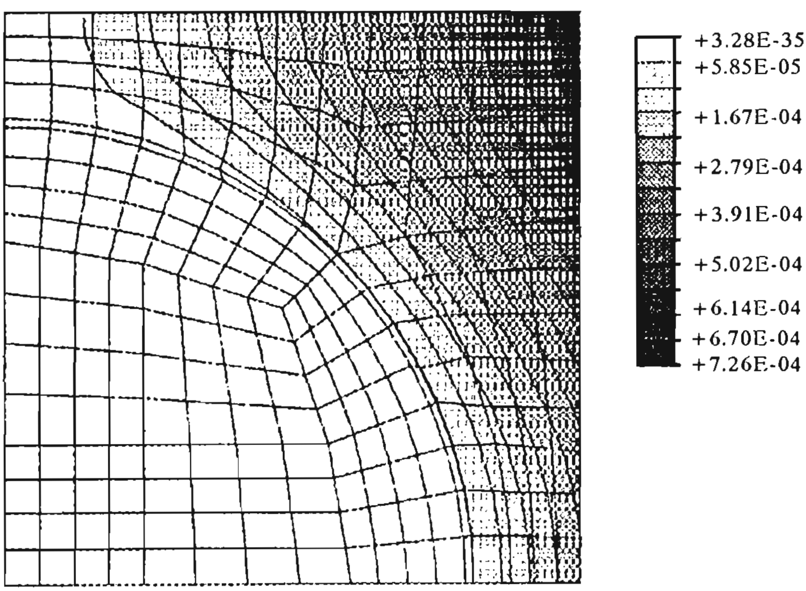

Fig. 10. Horizontal displacements in model with the Young modulus upper bound in the contact zone

in which successively we consider the problem of composite tension without introducing discontinuities (Fig.7), the composite is assumed according to proposed model (Fig.8), where Young modulus is taken as the expected value calculated from the formula (3.9). In Fig.9 and Fig.10 the Young modulus is calculated as lower and upper bounds of random distribution of this modulus (cf Fisher (1971); Larson (1974)), which is taken analogically to thickness of contact region being homogenized, cf (3.5)

$$e_k = E[e_k] \pm 3\sigma[e_k] \tag{5.1}$$

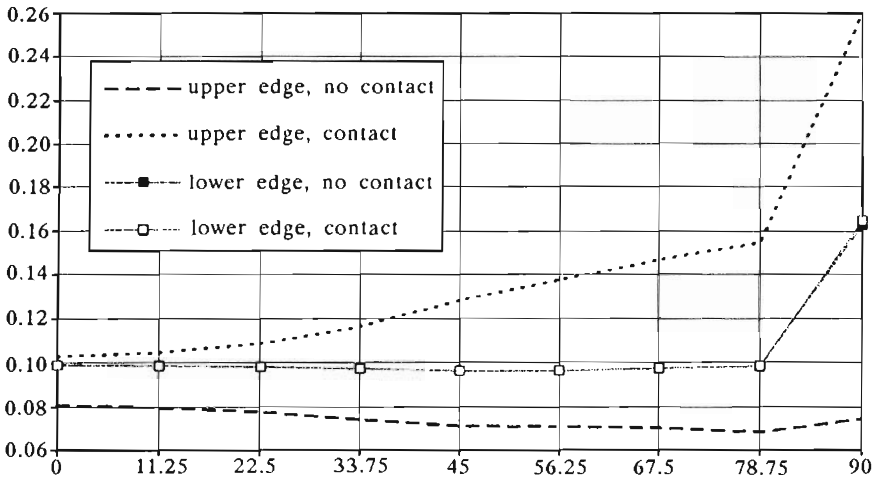

Fig. 11. Displacements coefficients of variation on interface edges

Fig.13 ÷ Fig.16 represent the shear stress σ_{12} in the same tests. Fig.11 and Fig.12 represent the results obtained using the POLSAP processed by EXCEL 5.0 – Microsoft Windows spreadsheet [27]. Fig.11 represents α as a function of β angle determining the location of the point belonging to fibre-matrix boundary. This angle is measured in the anticlockwise direction from the positive part of x-axis. Let us define for this purpose the displacement coefficient of variation α

$$\alpha^2 = \frac{\mathrm{Var}[q]}{E^2[q]} \tag{5.2}$$

Obtained dependencies are compared with analogous results obtained for a composite model regardless of heterogeneity on the fibre-matrix boundary (cf Kamiński [18]).

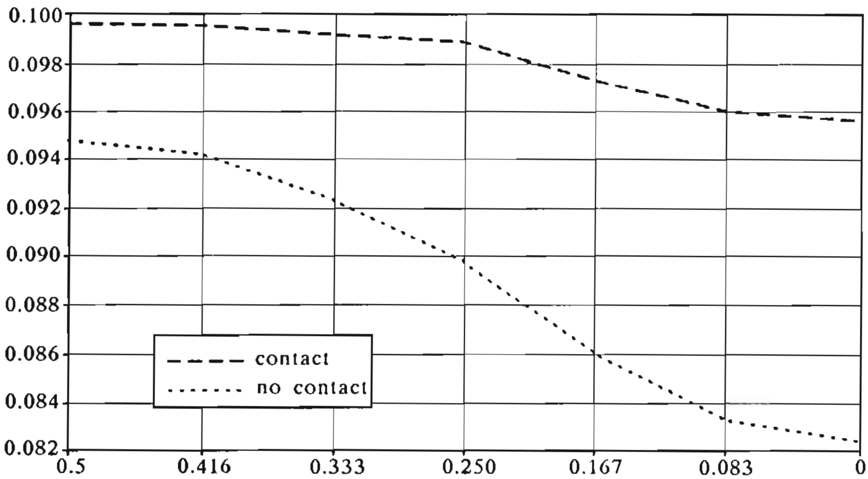

Fig. 12. Displacements coefficients of variation on tensioned edge

Fig.12 represents coefficients of displacements variation of points on tensioned periodicity cell quarter edge (relation of α to height h of point on the edge). Analogously to tests shown in Fig.11 the model being analyzed was compared with a simplified model – without heterogeneities.

The expected values of displacement fields obtained as a result of numerical analysis with the use of ABAQUS prove that a tensioned quarter of fibre composite shows small sensitivity to existence of statistically homogenized heterogeneities on the fibre-matrix interface (Fig.7 and Fig.8). Analyzing two next figures, cf Fig.9 and Fig.10, it can be seen that only weakening of interface layer (decreasing of Young modulus) in relation to its expected value essentially changes the form of expected values of the horizontal displacement field. For lower bound of the Young modulus distribution of homogenized zone the expected values of horizontal displacements in this zone increase rapidly, the contrary to the remaining tests. This jump (from minimum to maximum of the scale) can be interpreted as desizing of fibre from surrounding matrix.

Data presented in Fig.11 prove that displacement coefficients of variation of nodes on the upper edge of homogenized interface zone in the stochastic model with discontinuities are much bigger ("upper edge, contact") than in an analogous model without discontinuities ("upper edge, no contact"). It results from the fact that Young modulus coefficient variation of material in this zone is greater than in matrix, which seems to be justified from the physical point of view.

Results obtained for the lower edge, i.e. for the boundary between fibre and interface zone are identical for models with and without discontinuities because coefficients of variation of points on this edge depend mainly on the random character of fibre material (cf Kamiński [18]), which is the same in both cases.

Displacement coefficients of variation on tensioned edge are shown in Fig.12. Differences between models ("contact","no contact") obtained in this case are not so big as in the previous case. Inverse proportionality between a distance of point on the boundary from the interface boundary and the difference between displacement coefficients of variation in both tests can be easily observed. This dependence proves that the random character of interface zone influences the random character of displacement state of tensioned edge. It is the result of dependence of random displacements on tensioned edge of a cell on the random character of the matrix Young modulus, observed also in a numerical simulation (cf Lawrence (1986)). In relation to the conventional model without heterogeneity on the edge, in the composite with homogenized voids dependence between the point height on the tensioned edge and the displacement coefficient of variation does not change so rapidly for different values of h (14% and 3.5%, respectively). It results from the fact that a narrow zone of material with the coefficient of variation three times higher than for component materials was found between fibre and matrix. The interface region makes the discussed dependence smooth thanks to a short distance between points with h close to 0 (15% increase of α) and a longer distance from the upper end of tensioned edge (4% increase of α).

While analyzing the shear stress σ_{12} compared in Fig.13 $\div$ Fig.16 it can be easily noticed that in all cases there is an evident jump in values of these stresses on the fibre-matrix boundary of homogenized contact zone; sometimes it is even a difference in sign. In case of models without interface zone and also models with the expected value and the upper limit of Young modulus in contact region values of shear stresses and their positions are similar, cf Fig.13 $\div$ Fig.16; differences can be observed for angles $\gamma \approx 0°$ and $\gamma \approx 90°$. The stress state σ_{12} for the model with the lower bound of Young modulus in interface zone is definitely distinguished here (Fig.15). On the bigger part of length of upper and lower bound of zone with heterogeneities, particularly for $\gamma \leq 45°$ there are differences in shear stresses between this zone and adjacent regions of both fibre and matrix. It seems to prove the argument, stated on the base of horizontal displacement field of this test, for the possibility of fibre and matrix desizing in the region containing heterogeneities being modelled.

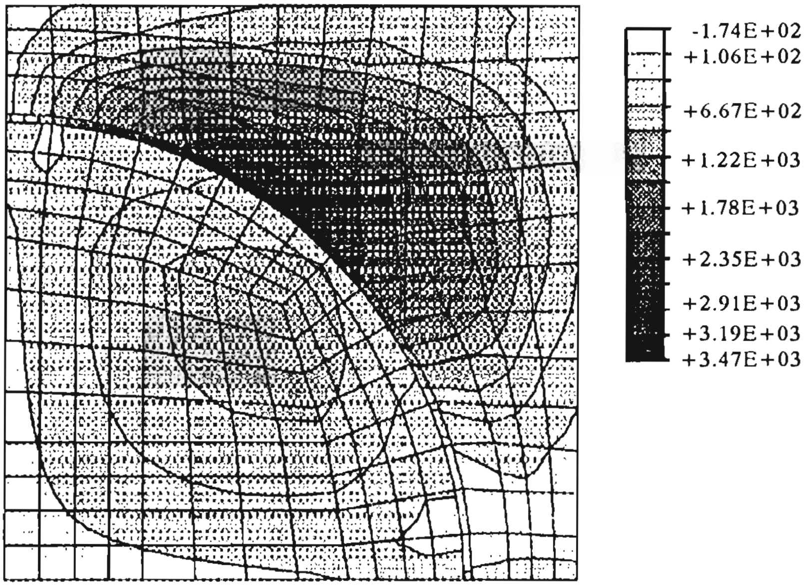

Fig. 13. Shear stresses in the model without "bubbles"

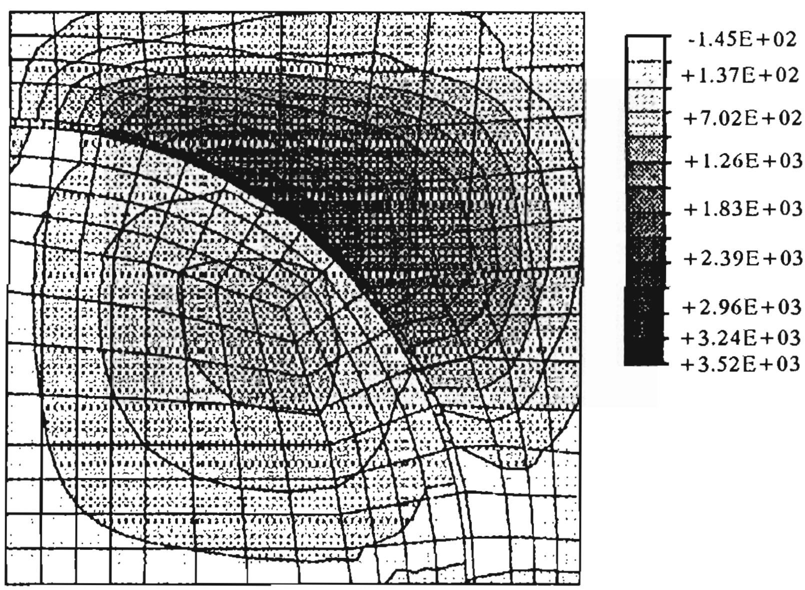

Fig. 14. Shear stresses in the model with the Young modulus expected value in the contact zone

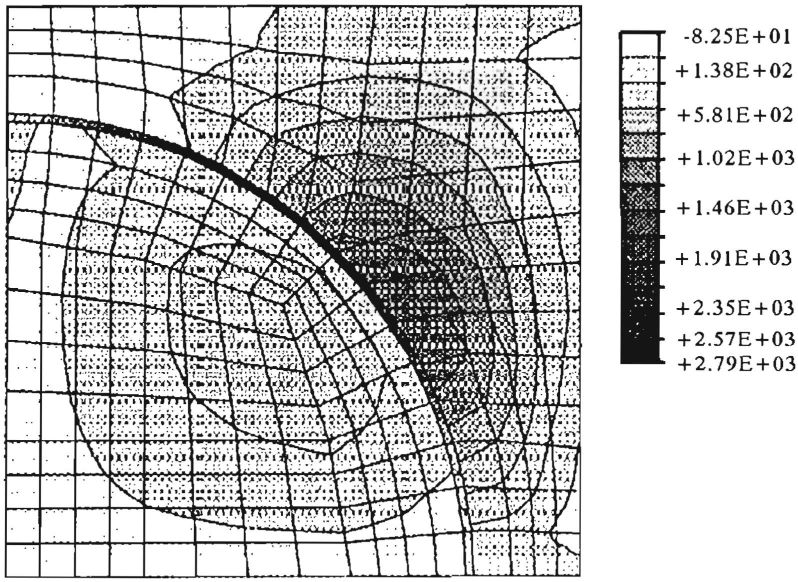

Fig. 15. Shear stresses in model with the Young modulus lower bound in the contact zone

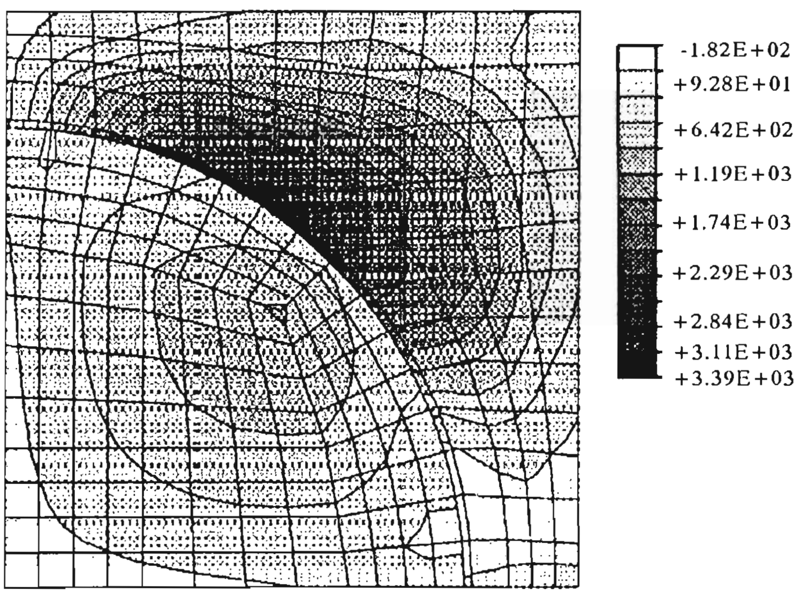

Fig. 16. Shear stresses in the model with the Young modulus upper bound in the contact zone

6. Concluding remarks

- *Presented idea of replacing the stochastic contact effect in fibre composite* with the stochastic problem of theory of elasticity with homogenized fibre-matrix interface let us observe the phenomena, which can be interpreted as desizing of fibre from matrix (jump of horizontal displacements in observed zone). It confirms the possibility of using this kind of model in discontinuity analysis on component boundaries in composite materials.

- Invented method of interface homogenization affects big standard deviation of homogenized material even for small deviations of "bubble" sizes and low frequency of their occurrence. Because of this fact obtaining more precise results in probabilistic sense for bigger standard deviations of the Young modulus of homogenized material would be guaranteed by using the Monte-Carlo method (cf Kamiński [15]; Tocher (1968)).

- Existence of random heterogeneities on the fibre-matrix boundary in the way presented above generally caused the increase of displacement coefficients of variation in these regions of periodicity cell, in which, in the stochastic model without heterogeneities (cf Kamiński and Gajl [18]), the dependence of these coefficients on the random character of matrix material was found. The upper edge of zone containing 'bubbles' and the vertical edge of cell being uniformly tensioned are these regions. Observed dependence results from the probabilistic correlation of the Young modulus of the matrix and of the homogenized interface region.

- Because of the fact that this is one of the first papers on the use of Stochastic Finite Element Method in modelling of contact phenomena in numerical mechanics of composites (cf Gajl (1991)) it is necessary to notice that simulation of a crack can be carried out directly without introducing the interface layer. Properly defined covariance matrix can be used to obtain this (cf Hien (1990); Lawrence (1986)).

7. Propositions

- It seems to be interesting to continue the analysis of modelling stochastic contact phenomena in fibre composites using the SFEM. It would be important to examine, on the basis of proposed model, the influence of particular parameters of this model (size of contact zone and random parameters of its material) on the random displacement and stress state, and also deriving a non-linear stochastic model of contact zone (cf Kleiber and Hien (1991); Liu et al. (1986)).

- Unquestionably, sensitivity analysis is a more precise tool for modelling random contact phenomena, and particularly in this case of stochastic sensitivity (cf Hien (1990); Kleiber and Hien (1991)). Using it for this kind of problems would enable:
  - examination of influence of the local increase in contact zone thickness (cf Dems and Haftka (1988-89); Dems and Mróz (1987) and (1993)) on the stress state in periodicity cell, instead of global estimation of static character used here,
  - using, in mathematical analysis, continuous distributions of individual random variables (cf Arminjon (1991); Sobczyk (1982)), thanks to which continuous functions of the first two moments for homogenized contact zone would be obtained,
  - application of the invented model to some problems of shape optimization in fibre composites (cf Haftka and Gürdal (1990)).
- To make the presented mathematic-numerical model corresponding with the results of experimental tests (cf Grayson (1983)) it seems appropriate to implement a two-parametric stochastic problem with the random Young modulus and Poisson ratio.
- From the point of view of real discontinuities between matrix and fibre it would be advisable to use the basic theory of curves (or surfaces) of the second order in the model of micro-contact geometry. It would enable precise modelling of deep 'craters' or large 'spots', which very often form on these interfaces for technological reasons (cf Grayson (1983)).
- It would be interesting to expand this kind of analysis on three-dimensional case (on the basis of cubic finite element), which would let us analyse the influence of interface randomness and elastic properties of composite components in direction parallel to fibres.

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Appendix A

Properties of the expected value and variance (cf Morrison (1976); Rosenblatt (1962))

Lemma 1.

$$\forall c \in \mathcal{R} \quad \Big(E[c] = c \ \wedge \ \mathrm{Var}(c) = 0\Big) \tag{A.1,2}$$

Lemma 2. For any two random variables X and Y we have

$$E[X \pm Y] = E[X] \pm E[Y] \tag{A.3}$$

Lemma 3.

$$\forall c \in \mathcal{R} \quad \Big(E[cX] = cE[X] \ \wedge \ \mathrm{Var}[cX] = c^2\mathrm{Var}[X]\Big) \tag{A.4,5}$$

Lemma 4. For two independent random variables X and Y we have

$$E[XY] = E[X]E[Y] \tag{A.6}$$

$$\mathrm{Var}[X \pm Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] \tag{A.7}$$

$$\mathrm{Var}[XY] = E^2[X]\mathrm{Var}[Y] + \mathrm{Var}[X]\mathrm{Var}[Y] + E^2[Y]\mathrm{Var}[X] \tag{A.8}$$

Appendix B

Problem: Determine the value of $\mathrm{Var}(X^2)$ using the parameters of X variable distribution, i.e. $E(X)$ and $\mathrm{Var}(X)$.

Solution: As it is known from the definition of random variable variance with discrete distribution we have

$$\mathrm{Var}(Y) = E(Y^2) - E^2(Y)$$

Let $Y = X^2$. We have

$$\mathrm{Var}(X^2) = E\Big((X^2)^2\Big) - E^2(X^2) = E(X^4) - E^2(X^2)$$

$E(X^4)$ is to be determined by integration of characteristic function using the fact that it is the Gaussian distribution

$$E(X^4) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} x^4 \exp\Big(-\frac{(x-m)^2}{2\sigma^2}\Big)\, dx$$

where m and σ are the expected value and the standard deviation of examined distribution, respectively.

Introducing a standarized variable $t = \frac{x-m}{\sigma}$, $x = t\sigma + m$, $dx = \sigma dt$ we obtain

$$E(X^4) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (t\sigma + m)^4 e^{-\frac{t^2}{2}}\, dt$$

After some algebraic transformations on the integrand function we have

$$E(X^4) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (\sigma^4 t^4 + 4\sigma^3 m t^3 + 6\sigma^2 m^2 t^2 + 4\sigma m^3 t + m^4) e^{-\frac{t^2}{2}}\, dt$$

and splitting it into particular component integrals

$$E(X^4) = \frac{1}{\sqrt{2\pi}} (\sigma^4 I_1 + 4\sigma^3 m I_2 + 6\sigma^2 m^2 I_3 + 4\sigma m^3 I_4 + m^4 I_5) e^{-\frac{t^2}{2}}$$

where there is

$$I_1 = \int_{-\infty}^{+\infty} t^4 e^{-\frac{t^2}{2}}\, dt \qquad I_4 = \int_{-\infty}^{+\infty} t e^{-\frac{t^2}{2}}\, dt$$

$$I_2 = \int_{-\infty}^{+\infty} t^3 e^{-\frac{t^2}{2}}\, dt \qquad I_5 = \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}}\, dt$$

$$I_3 = \int_{-\infty}^{+\infty} t^2 e^{-\frac{t^2}{2}}\, dt$$

Because of integrands being odd, integrals I_2 and I_2 are equal to 0 for integrating on the whole set of real numbers. We then determine integrals with odd indices. We have

$$I_5 = \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}}\, dt = \sqrt{2\pi}$$

$$I_3 = \int_{-\infty}^{+\infty} t^2 e^{-\frac{t^2}{2}}\, dt = -\int_{-\infty}^{+\infty} t\left(te^{-\frac{t^2}{2}}\right) dt = -\int_{-\infty}^{+\infty} t\, d\left(e^{-\frac{t^2}{2}}\right) =$$

$$= -te^{-\frac{t^2}{2}} \Big|_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}}\, dt = \sqrt{2\pi}$$

$$I_1 = \int_{-\infty}^{+\infty} t^4 e^{-\frac{t^2}{2}}\, dt = -\int_{-\infty}^{+\infty} t^3\, de^{-\frac{t^2}{2}} = -\left[t^3 e^{-\frac{t^2}{2}} \Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}}\, dt^3 \right] =$$

$$= 3\int_{-\infty}^{+\infty} t^2 e^{-\frac{t^2}{2}}\, dt = -3\int_{-\infty}^{+\infty} t\, de^{-\frac{t^2}{2}} = -3\left[t e^{-\frac{t^2}{2}} \Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}}\, dt \right] = 3\sqrt{2\pi}$$

Finally, we obtain

$$E(X^4) = 3\sigma^4 + 6\sigma^2 m^2 + m^4 = E^4(X) + 6\mathrm{Var}(X)E^2(X) + 3\mathrm{Var}^2(X) \quad \text{(B.1)}$$

$$E(X^2) = \sigma^2 + m^2 = E^2(X) + \mathrm{Var}(X) \quad \text{(B.2)}$$

$$\mathrm{Var}(X^2) = E(X^4) - E^2(X^2) = 2\sigma^2(\sigma^2 + 2m^2) = \quad \text{(B.3)}$$

$$= 2\mathrm{Var}(X)\Big(\mathrm{Var}(X) + 2E^2(X)\Big)$$

Stochastyczne efekty kontaktowe w periodycznych kompozytach włóknistych

Streszczenie

W niniejszej pracy zostało zaprezentowane nowe, analityczno-numeryczne podejście do zagadnień kontaktu w mechanice.

Najważniejszymi elementami tego sformułowania są:

1. aproksymacja nieznanej stochastycznej powierzchni kontaktu przez ciąg figur o założonej geometrii; jej wielkość i częstość występowania na rozważanym brzegu są gaussowskimi zmiennymi losowymi o znanych parametrach statystycznych;
2. zastąpienie obszaru kontaktu zawierającego z dobrym przybliżeniem wszystkie te figury przez stochastycznie uśredniony materiał, zajmujący po uśrednieniu deterministycznie określony obszar kontaktu;
3. numeryczne rozwiązanie stochastycznego problemu statycznego za pomocą Stochastycznej Metody Elementów Skończonych (SFEM).

Zaproponowany model został sformułowany specjalnie do analizy nieciągłości występujących na granicach składników w periodycznych kompozytach włóknistych o losowych własnościach sprężystych. Analizie numerycznej poddano ćwiartkę kwadratowej komórki periodyczności z centralnie usytuowanym włóknem o przekroju kołowym. Analiza ta została wykonana za pomocą programów POLSAP i ABAQUS.

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