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STRESS INTENSITY FACTORS FOR AN INTERFACE PENNY-SHAPED CRACK IN LAMINATED ELASTIC LAYER

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This paper concerns the crack-border stress singularities in a microperiodically stratified elastic layer weakened by an interface central penny-shaped crack. By using the homogenized model with microlocal parameters (cf Woźniak, 1987a) the stress field with an inverse square-root singularity is obtained. In the context of linear fracture mechanics, the stress intensity factors are defined as the local crack response parameters which can be obtained in terms of the solutions to the corresponding Fredholm integral equations. Some examples are solved numerically and the results are presented in the diagram form.

1. Introduction

As the result of increasing use of advanced composites in various branches of modern technology, the study of fracture toughness of these materials has received wide attention. The recent literature on the crack problems in composites is very extensive and will not be wholly discussed here. We can mention only some basic monographs by Sih and Chen (1981), Cherepanov (1983), Sih and Tamuzs (1979), Sih and Skudra (1985).

A large set of problems is associated with interface cracks which are considered as one of the most commonly encountered types of damage in the failure of composites. Many investigators have reported on this subject (cf Williams (1959), Cherepanov (1962), England (1965), Erdogan (1965), Kassir and Bregman (1972), Willis (1972)). In these studies, it was pointed out that the conventional solutions for interface cracks possess a controversial oscillatory stress singularity leading to the physically inadmissible phenomenon of crack surface interpenetration. This result indicates that the classical approach is not suitable for the crack lying on the interface of two different media. The unreasonable behavior mentioned above was discussed by many researches and several models have been proposed to correct the unsatisfactory features of the oscillatory crack-tip characteristics, e.g., by Dundurs and Comninou (1979), Atkinson (1977), Itou (1986), Hills and Barber (1993).

Lately, an efficient attempt has been made at solving some elasticity problems of interface cracks in microperiodic two-layered space (see Kaczyński and Matysiak (1988), (1989a,b), (1993), (1994); Matysiak (1989), Kaczyński (1993a,b), (1994); Kaczyński, Matysiak and Pauk (1994)). The closed-form solutions with the standard inverse square-root singularities have been achieved using the homogenized model of microperiodic composites given by Woźniak (1986), (1987a,b); Matysiak and Woźniak (1987). The advantage of this model is a relatively simple form of the governing equations and a possibility of evaluating not only mean but also the local values of strains and stresses in every material components of the stratified body. Moreover, such an approach has made it possible to determine the stress intensity factors for the above crack problems which are the counterpart of those well-known for homogeneous bodies (cf Panasyuk, Savruk and Datsyshyn (1970); Kassir and Sih (1975), Andreĭkiv (1982); Murakami (1987)) used to predict crack propagation in linear fracture mechanics.

It is the aim of this paper to discuss the singular crack-border stress field in a periodically stratified elastic layer containing an interface central penny-shaped crack on the basis of the homogenized model mentioned above. We pay attention to the particular fundamental solution to tension problems (so-called Mode I – see, e.g. Sneddon and Lowengrub (1969)) associated with the action of concentrated, axial symmetric and circumferential forces.

In Section 2, based on the results of papers given by Woźniak (1987a), Matysiak and Woźniak (1987), the governing equations of the linear elasticity with microlocal parameters in the axisymmetric case of periodic two-layered composites are presented. Moreover, the penny-shaped crack problem in the stratified layer is formulated.

In Section 3 the solution is obtained by using the method of Hankel transforms and then reducing the resulting system of dual integral equations to the Fredholm integral equations of the second kind. Utmost attention is payed to determine the singular stress field in a region close to the crack periphery. Physically meaningful fracture mechanics parameters such as the stress intensity factor and the energy release rate are defined and extracted from the solution to the Fredholm integral equations.

Section 4 contains some numerical calculations of the stress intensity factors. The obtained results are examined and presented graphically.

2. Problem description

Let us consider a microperiodic laminated layer, the middle cross section of which is given in Fig.1. A repeated fundamental layer of thickness l is composed of two homogeneous isotropic elastic layers denoted by 1 and 2 and characterized by the Lame constants λ_i, μ_i; i = 1, 2. Let l_1 and l_2 be the thicknesses of the subsequent layers, so $l = l_1 + l_2$. The cylindrical coordinate system (r, ϕ, z), such that the z-axis is normal to the layering and to the interface with a penny shaped crack situated on the plane $z = 0$, will be applied.

For the static axisymmetric case the equations of the homogenized model (after eliminating the microlocal parameters) take the form (see Pusz (1988))

$$\begin{aligned} &CD_0[w_z(r,z)] + B\frac{1}{r}\frac{\partial^2[rw_r(r,z)]}{\partial z \partial r} + A_1\frac{\partial^2 w_z(r,z)}{\partial z^2} = 0 \\ &A_2 D_1[w_r(r,z)] + B\frac{\partial^2 w_z(r,z)]}{\partial r \partial z} + C\frac{\partial^2 w_r(r,z)}{\partial z^2} = 0 \end{aligned} \tag{2.1}$$

where D_0, D_1 are the differential operators

$$\begin{aligned} D_0[f] &\equiv \frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial f}{\partial r}\Big) \\ D_1[f] &\equiv \frac{\partial}{\partial r}\Big(\frac{1}{r}\frac{\partial (rf)}{\partial r}\Big) \end{aligned} \tag{2.2}$$

and A_1, A_2, B, C denote

$$A_1 = \frac{(\lambda_1 + 2\mu_1)(\lambda_2 + 2\mu_2)}{(1-\eta)(\lambda_1 + 2\mu_1) + \eta(\lambda_2 + 2\mu_2)} > 0$$

$$
\begin{aligned}
A_2 &= \frac{4\eta(\eta-1)(\mu_1-\mu_2)(\lambda_1-\lambda_2+2\mu_1-\mu_2)}{(1-\eta)(\lambda_1+2\mu_1)+\eta(\lambda_2+2\mu_2)} + A_1 > 0 \\
B &= \frac{(1-\eta)\lambda_2(\lambda_1+2\mu_1)+\eta\lambda_1(\lambda_2+2\mu_2)}{(1-\eta)(\lambda_1+2\mu_1)+\eta(\lambda_2+2\mu_2)} > 0 \\
C &= \frac{\mu_1\mu_2}{(1-\eta)\mu_1+\eta\mu_2} > 0 \\
\eta &= \frac{l_1}{l}
\end{aligned}
\tag{2.3}
$$

Unknown functions $w_r(r,z)$ and $w_z(r,z)$ are the components of the macrodisplacement vector.

The stresses in the layer of jth kind are expressed by the macrodisplacement vector in the form

$$
\begin{aligned}
\sigma_{zz}^{(j)} &= A_1\frac{\partial w_z}{\partial r} + B\frac{1}{r}\frac{\partial(rw_r)}{\partial r} \\
\sigma_{zr}^{(j)} &= C\Big(\frac{\partial w_z}{\partial r} + \frac{\partial w_r}{\partial r}\Big) \\
\sigma_{rr}^{(j)} &= D_j\frac{\partial w_z}{\partial z} + E_j\frac{1}{r}\frac{\partial(rw_r)}{\partial r}
\end{aligned}
\tag{2.4}
$$

where

$$
\begin{aligned}
D_j &= \frac{\lambda_j}{\lambda_j+2\mu_j}A_1 \\
E_j &= \frac{4\mu_j(\lambda_j+\mu_j)}{\lambda_j+2\mu_j} + \frac{\lambda_j}{\lambda_j+2\mu_j}B \qquad j=1,2
\end{aligned}
\tag{2.5}
$$

The considered penny-shaped crack problem is described by the following boundary conditions (see Fig.1)

$$
\begin{aligned}
\sigma_{zz}^{(1)}(r,z=0^+) &= \sigma_{zz}^{(2)}(r,z=0^-) = 0 \\
\sigma_{zr}^{(1)}(r,z=0^+) &= \sigma_{zr}^{(2)}(r,z=0^-) = 0 \qquad r<a
\end{aligned}
\tag{2.6}
$$

and

$$
\begin{aligned}
\sigma_{zz}^{(2)}(r,z=H) &= \frac{P}{2\pi r}\delta(r-b) \qquad \sigma_{zr}^{(2)}(r,z=H)=0 \\
\sigma_{zz}^{(1)}(r,z=-H) &= -\frac{P}{2\pi r}\delta(r-b) \qquad \sigma_{zr}^{(1)}(r,z=-H)=0 \qquad r\geq 0
\end{aligned}
\tag{2.7}
$$

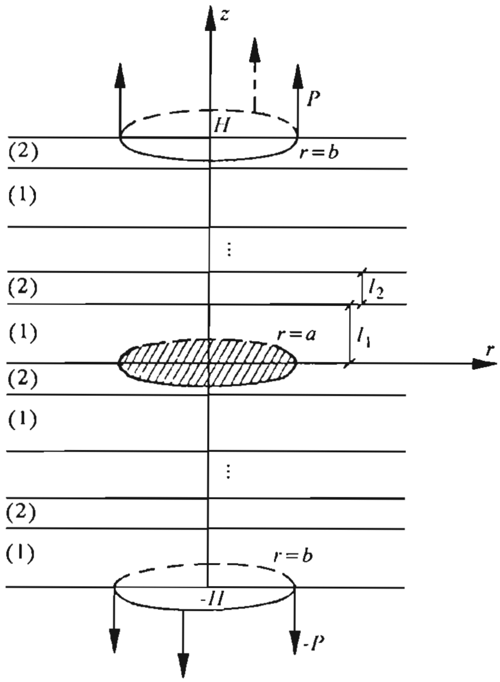

Fig. 1.

where $\delta(\cdot)$ is the Dirac delta-function and P is a given constant, $P > 0$.

The solution to the formulated above problem can be interpreted as a Green function for the symetrically loaded laminated layer weakened by a penny-shaped crack.

3. Solution to the boundary-value problem

Making use of the superposition principle, the boundary-value problem stated in Section 2 is separated into two parts. For the first part, the uncracked stratified layer is assumed to be loaded by constant forces distributed on the circumference of radius bacting symmetrically with respect to the middle plane. For the second part the negative tractions generated at the crack area from the first part are applied to the crack faces.

By using the method of Hankel transforms the solution to Eqs (2.1) with boundary conditions (2.7) can be written for the case of different shear modulae μ_1 and μ_2 of the subsequent layers in the form (the case of $\mu_1 = \mu_2$ has to be considered separately but here it will be omitted)

$$w_r(r,z) = \int_0^\infty \xi\Phi(\xi)\Phi_0(\xi,z)J_1(\xi r)\,d\xi \equiv \mathcal{H}_1\{\Phi_0(\xi,z)\Phi(\xi);\ \xi \to r\}$$
$$w_z(r,z) = \int_0^\infty \xi\Phi(\xi)\Phi_1(\xi,z)J_0(\xi r)\,d\xi \equiv \mathcal{H}_0\{\Phi_1(\xi,z)\Phi(\xi);\ \xi \to r\} \tag{3.1}$$

where

$$\Phi_0(\xi,z) = e^{k_1\xi z} + \beta(\xi)e^{-k_1\xi z} + \beta_1\Big[e^{k_2\xi z} + \beta(\xi)e^{-k_2\xi z}\Big] +$$
$$+\beta_2\Big[e^{-k_2\xi z} + \beta(\xi)e^{k_2\xi z}\Big]$$
$$\Phi_1(\xi,z) = G_1\Big[e^{k_1\xi z} - \beta(\xi)e^{-k_1\xi z}\Big] + G_2\Big\{\beta_1\Big[e^{k_2\xi z} - \beta(\xi)e^{-k_2\xi z}\Big] +$$
$$-\beta_2\Big[e^{-k_2\xi z} - \beta(\xi)e^{k_2\xi z}\Big]\Big\}$$
$$\beta_{1,2} = -\frac{1}{2}\Big(\frac{B + A_1G_1k_1}{B + A_1G_2k_2} \pm \frac{k_1 - G_1}{k_2 - G_2}\Big) \tag{3.2}$$
$$G_j = \frac{Ck_j^2 - A_2}{B + C}k_j \qquad j = 1,2$$
$$k_{1,2} = \sqrt{\frac{A_1A_2 - B^2 - 2BC \pm \sqrt{D}}{2A_1C}}$$
$$D = (A_1A_2 - B^2 - 2BC)^2 - 4A_1A_2C^2$$
$$\beta(\xi) = \frac{(k_1 - G_1)e^{-2\xi k_1 H} + (k_2 - G_2)\Big[\beta_1 e^{-(k_1+k_2)H} - \beta_2 e^{-\xi(k_1-k_2)H}\Big]}{k_1 - G_1 + (k_2 - G_2)\Big[\beta_1 e^{-\xi(k_1-k_2)H} - \beta_2 e^{-\xi(k_1+k_2)H}\Big]}$$

Unknown function $\Phi(\xi)$ has to be determined from boundary condition $(2.7)_1$. Thus, using Eqs (3.1), (2.4) and boundary condition $(2.7)_1$ we obtain

$$\Phi(\xi) = \frac{P}{2\pi}L(\xi)J_0(\xi b)$$
$$L(\xi) = \frac{k_1(B^2 + G_1A_1) - k_2(B^2 + G_2A_1)\gamma(\xi)}{k_1(B^2 + G_1A_1)\cosh(\xi k_1 H) - k_2(B^2 + G_2A_1)\cosh(\xi k_2 H)} \tag{3.3}$$

By using Eqs (3.3), (3.1) and (2.4), the first part of the solution can be written in the form of Hankel integrals (the formulae are too lengthy and here will be omitted).

Applying the superposition principle to ensure a traction-free crack described by the boundary conditions (2.6) we take the displacement w_z and the stress $\sigma_{zz}^{(j)}$, $j = 1,2$ on the plane $z = 0$ in the form

$$\begin{aligned} w_z(r, z=0) &= \mathcal{H}_0\{\xi^{-1}\psi(\xi)\Phi_1(\xi,0);\ \xi \to r\} \\ \sigma_{zz}^{(j)}(r, z=0) &= \mathcal{H}_0\{\psi(\xi)\phi_2(\xi);\ \xi \to r\} \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} \phi_2(\xi) = (B + A_1G_1k_1)\Big[e^{-k_1\xi H} + \beta(\xi)e^{k_1\xi H}\Big] + \\ +(B + A_1G_2k_2)\Big\{\beta_1\Big[e^{-k_2\xi H} + \beta(\xi)e^{k_2\xi H}\Big] + \beta_2\Big[e^{k_2\xi H} + \beta(\xi)e^{-k_2\xi H}\Big]\Big\} \end{aligned} \tag{3.5}$$

and the unknown function $\psi(\xi)$ has to be determined from the following dual intergal equations

$$\begin{aligned} \mathcal{H}_0\{\psi(\xi)[1 - g(\xi)];\ \xi \to r\} &= \frac{s(r)}{X_0} \qquad & 0 < r < a \\ \mathcal{H}_0\{\xi^{-1}\psi(\xi);\ \xi \to r\} &= 0 \qquad & r > a \end{aligned} \tag{3.6}$$

where

$$\begin{aligned} g(\xi) &= 1 - \frac{X(\xi)}{X_0} \qquad X(\xi) = \frac{\Phi_2(xi)}{\Phi_1(\xi,0)} \\ X_0 &= \lim_{\xi\to\infty} X(\xi) = \frac{(B + A_1G_1k_1)(k_2 - G_2) + (B - A_1G_2k_2)(k_1 - G_1)}{k_1G_2 - k_2G_1} \\ S(r) &= \frac{P}{2\pi}\mathcal{H}_0\{L(\xi)J_0(\xi b);\ r \to \xi\} \\ \gamma(\xi) &= \frac{(Bk_2^2 - G_2)\sinh(\xi k_2 H)}{(Bk_1^2 - G_1)\sinh(\xi k_2 H)} \end{aligned} \tag{3.7}$$

By introduction the following dimensionless notations

$$\begin{aligned} \zeta &= a\xi \qquad \rho = \frac{r}{a} \qquad \varepsilon = \frac{b}{a} \\ h &= \frac{H}{a} \qquad \psi^*(\zeta) = \frac{2B + C}{P}\psi(a\xi) \end{aligned} \tag{3.8}$$

the dual integral equations (3.6) can be written in the form

$$
\begin{aligned}
&\mathcal{H}_0\{\psi^*(\zeta)[1-g(\zeta)];\ \zeta\to\rho\} = s^*(\rho) \qquad && 0<\rho<1 \\
&\mathcal{H}_0\{\zeta^{-1}\psi^*(\zeta);\ \zeta\to\rho\} = 0 && \rho>1
\end{aligned}
\tag{3.9}
$$

where

$$
\begin{aligned}
s^*(\rho) &= \frac{1}{2\pi X_0^*}\int\limits_0^\infty \zeta L(\zeta)J_0(\zeta\xi)J_0(\zeta\xi)\,d\zeta \\
X_0^* &= \frac{X_0}{2B+C}
\end{aligned}
\tag{3.10}
$$

Let the solution to the dual integral equations (3.9) have the form (Sneddon (1966))

$$
\psi^*(\zeta) = \int\limits_0^1 h(\tau)\sin\zeta\tau\,d\tau \tag{3.11}
$$

where $h(\cdot)$ is an unknown function.

By substituting Eq (3.11) into Eq (3.9) we arrive at the integral Fredholm equation of the second kind

$$
h(t) - \frac{1}{\pi}\int\limits_0^1 K(t,\tau)h(\tau)\,d\tau = F(t) \qquad 0<t<1 \tag{3.12}
$$

with the kernel and the free term given by

$$
\begin{aligned}
K(t,\tau) &= \int\limits_0^\infty g(\xi)[\cos(t-\tau)\xi - \cos(t+\tau)\xi]\,d\xi \\
F(t) &= \frac{1}{\pi^2 X_0^*}\int\limits_0^\infty L(\xi)J_0(\xi\varepsilon)\sin(\xi t)\,d\xi
\end{aligned}
\tag{3.13}
$$

From the standpoint of fracture mechanics a quantity of primary importance is the field of stresses in the vicinity of the crack tip. The local stress field reveals the typical inverse square root singularity and is characterized by the stress intensity factor (SIF) playing the main object in the linear elastic fracture mechanics.

The SIF is defined in the conventional manner as

$$K_I = \lim_{r\to a^+} \sqrt{2(r-a)}\, \sigma_{zz}^{(j)}(r,0) \tag{3.14}$$

By using Eqs (3.11), (3.8) and (3.3) we obtain

$$\begin{aligned} \sigma_{zz}^{(1)}(\rho,0^+) = \sigma_{zz}^{(2)}(\rho,0^-) &\sim \frac{Ph(1)X_0}{a^2(2B+C)\sqrt{\rho-1}} \\ \rho = \frac{r}{a} &\to 1^+ \end{aligned} \tag{3.15}$$

so the stress intensity factor K_I is given by

$$K_I = \frac{Ph(1)X_0}{a^2(2B+C)} \tag{3.16}$$

where $h(1)$ is the solution to the Fredholm integral equation (3.12) at the point $t=1$.

The obtained results can be used together with the appropriate fracture criterion for the prediction of fracture initiation. Employing the classical concept of fracture toughness and assuming that the crack extends along the interface, the fracture initiation can be controlled by the critical value of the strain energy release rate (see Sih (ed.) (1973-1981)).

After some calculations for the considered axisymmetric case of the microperiodic layered body with penny-shaped crack, the strain energy release rate G_I is expressed by

$$G_I = \frac{\pi}{2(\sqrt{A_1A_2}+B)}\sqrt{\frac{A_2(\sqrt{A_1A_2}+B+2C)}{C(\sqrt{A_1A_2}-B)}}\, K_I^2 \tag{3.17}$$

4. Numerical example

The stress intensity factor K_I given in Eq (3.15) depends on the material properties of the subsequent laminae, geometrical parameters $\eta = l_1/\sigma$, $\varepsilon = b/a$, $h = H/a$ and the intensity of external force P. As an example we consider the stress intensity factor K_I under assumptions that the Lame constants λ_1, μ_1 and λ_2, μ_2 satisfy the conditions

$$\lambda_1 = \mu_1 \qquad\qquad \lambda_2 = \mu_2 \tag{4.1}$$

On introducing the notations

$$\gamma = \frac{\mu_1}{\mu_2} \qquad K_I^* \equiv \frac{K_I(2B+C)a^2}{PX_0} \tag{4.2}$$

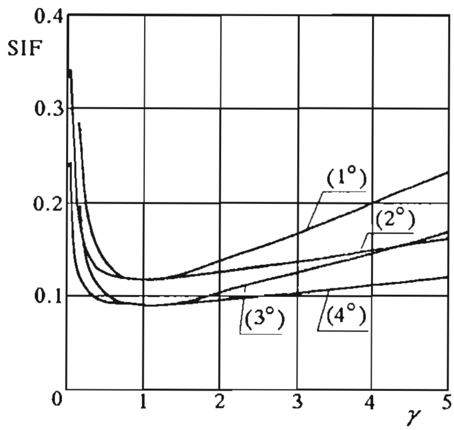

Fig. 2.

The dimensionless intensity stress factor $K_I^* = h(1)$ is plotted in Fig.2 as a function of γ for $h/a = 5$ and

(1°)	$b/a = 0$	$\eta = 0.5$
(2°)	$b/a = 0$	$\eta = 0.1$
(3°)	$b/a = 2$	$\eta = 0.5$
(4°)	$b/a = 2$	$\eta = 0.1$.

The curves (1°) and (2°) determine K_I^* for the case of concentrated loadings at the points $(r = 0, z = \pm H)$.

Fig.3 shows the SIF K_I^* as a function of the ratio b/a for $\eta = 0.1, \gamma = 0.5$ and

(1°)	$h/a = 2$
(2°)	$h/a = 3$
(3°)	$h/a = 5$
(4°)	$h/a = 10$.

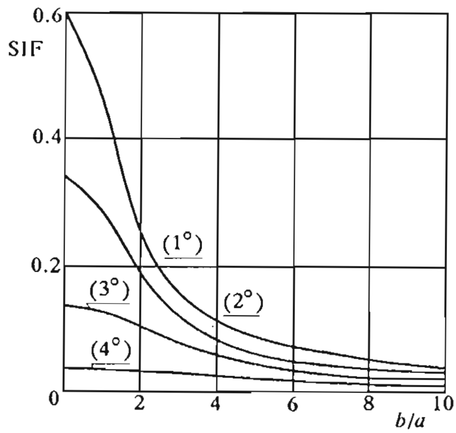

Fig. 3.

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Współczynniki intensywności naprężeń dla kołowej szczeliny międzywarstwowej w laminatowej warstwie sprężystej

Streszczenie

Niniejsza praca dotyczy osobliwości naprężeń wokół szczeliny kołowej, położonej centralnie na złączeniu warstw w mikroperiodycznie uwarstwionej warstwie sprężystej. Stosując model homogenizowany z parametrami mikrolokalnymi (por. Woźniak (1987a)) otrzymuje się pole naprężeń z osobliwością odwrotnie proporcjonalną do pierwiastka kwadratowego z odległości od punktu brzegowego szczeliny. Z punktu widzenia liniowej mechaniki pękania, lokalnymi parametrami odpowiedzialnymi za rozwój *szczeliny są współczynniki intensywności naprężeń*, które otrzymuje się z rozwiązań odpowiednich równań całkowych typu Fredholma. Przeprowadzono ich analizę w pewnych przypadkach i wyniki przedstawiono w postaci wykresów.

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