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COMPARATIVE ANALYSIS OF CONSTRUCTIONAL PARAMETERS AND THREE-LAYERED PLATE SUPPORT INFLUENCE ON FREE VIBRATIONS FREQUENCIES

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The analytical and numerical research into a three-layered plate with light filler free vibrations was presented in this paper. The following assumptions were made: the linings displayed the isotropic characteristic while the filler the orthotropic one. Influence of the flexural rigidity on a vibrations frequency in three variants of the plate support was detected. The influence of some geometrical parameters on the free vibrations frequency was traced as well. Research results were verified on comparing the value of vibration frequency obtained for a plate with joint support with the experimental (Raville and Ueng (1967)) and the FEM results (Cheung and Khatua (1973)).

1. Introduction

The necessity for the stiff and light load-bearing structures stimulated the development of three-layered plate systems and their theory. The earlier three-layered plates theory was presented by Habip (1965), then there were other theories by Yu (1960), Dowell and Yan (1972) and (1974), Durocher and Solecki (1976). This paper was written on the basis of theory applied to a three-layered plate with a thick and light plastic core glued between two thin, stiff layers. In this theory the hypothesis of broken line is assumed. Its basic assumptions were presented by Volmir (1967).

Research into the three-layered plate with light filler free vibrations frequency and mode was proposed in this paper. For cognitive and practical purposes it is important to estimate possibility of effective influence on the values of plate free vibrations first frequencies by changing material and geometrical parameters also boundary conditions. Because of necessity for resonances avoidance, such research may be useful at designing load-bearing structures with three-layered plates. Method, presented in this paper, works for various support conditions of the rectangular sandwich plate. Such plates have usually complicated shapes in practical applications. The free vibrations of monolithic plates with shapeless edge line were investigated by Nagaya (1980), Waberski (1978), whereas Półtorak and Nagaya (1985) traced sandwich plates. The investigations of the sandwich plates with arbitrary shaped edge line were continued by Półtorak (1990a,b).

2. Assumptions and the differential equations of motion

Let us assume that the plate has thin linings and light, but relatively thick filler. Then let us treat linings as thin plates and assume, that the filler bears stresses only the transverse, tangent. If we neglect deformability of the core in transverse direction, all layer deflections are the same. The plate is considered as the linear-elastic system. The internal and external damping is neglected in the analysis of free vibrations.

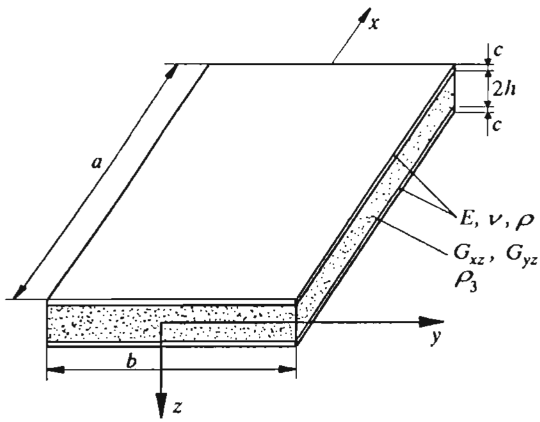

Fig. 1. Three-layered plate in 3D coordinate system

Let us take that the straight line perpendicular to the core mid-plane remains straight after strain but is not perpendicular to the deformed plate mid-plane. The assumption of the linear strains distribution along the core thickness and the normal line theory concerning linings lead to the so called the broken line hypothesis (cf Volmir (1967)). Let us assume that the plate mid-surface is its symmetry plane so the lining thicknesses are the same. The anisotropic core has different shape elasticity modules along the xz and yz planes, respectively. Let us neglect the inertial forces of rotation and the inertial forces resulting from displacement in the lengthwise direction in the transverse vibrations analysis. Considering only the inertial forces resulting from displacements of particular layers in the transverse direction, we get mathematical model of the plate free vibrations as the system of three partial differential equations (Kaźmir (1992) in the unknown functions $u_\beta(x,y,t)$, $v_\beta(x,y,t)$, $w(x,y,t)$.

$$\begin{aligned}
&\frac{Bh}{G_{xz}}\Big(\frac{\partial^2 u_\beta}{\partial x^2}+\frac{1-\nu}{2}\frac{\partial^2 u_\beta}{\partial y^2}+\frac{1+\nu}{2}\frac{\partial^2 v_\beta}{\partial x\partial y}\Big)-u_\beta+\Big(h+\frac{c}{2}\Big)\frac{\partial w}{\partial x}=0\\
&\frac{Bh}{G_{yz}}\Big(\frac{\partial^2 v_\beta}{\partial y^2}+\frac{1-\nu}{2}\frac{\partial^2 v_\beta}{\partial x^2}+\frac{1+\nu}{2}\frac{\partial^2 u_\beta}{\partial x\partial y}\Big)-v_\beta+\Big(h+\frac{c}{2}\Big)\frac{\partial w}{\partial y}=0\\
&2B\Big(h+\frac{c}{2}\Big)\nabla^2\Big(\frac{\partial u_\beta}{\partial x}+\frac{\partial v_\beta}{\partial y}\Big)+2D\nabla^4 w+(2\rho c+2\rho_3 h)\frac{\partial^2 w}{\partial t^2}=0
\end{aligned} \tag{2.1}$$

where

$$u_\beta=\frac{1}{2}(\bar{u}_1-\bar{u}_2) \qquad v_\beta=\frac{1}{2}(\bar{v}_1-\bar{v}_2) \qquad B=\frac{Ec}{1-\nu^2} \tag{2.2}$$

and

E	–	Young modulus of linings
G_{xz}, G_{yz}	–	Kirchhoff's modulus of core
ν	–	Poisson ratio of linings
$2h$	–	core thickness
c	–	linings thickness
ρ	–	material density of linings
ρ_3	–	material density of core
$\bar{u}_1, \bar{v}_1$	–	displacements in directions x and y of the points lying on the mid-surface of upper lining
$\bar{u}_2, \bar{v}_2$	–	displacements in directions x and y of the points lying on the mid-surface of lower lining
D	–	flexural rigidity of linings.

3. Boundary conditions

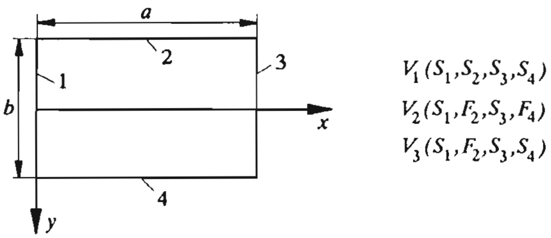

Fig. 2. Support variants; S – simple support, F – clamped support

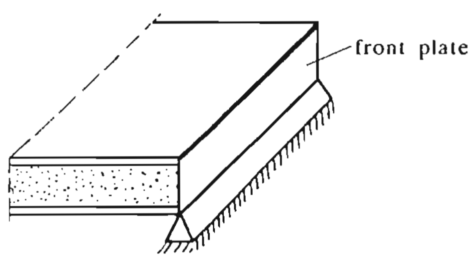

Fig. 3. Joint support – S

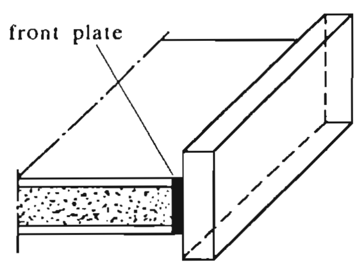

Fig. 4. Clamped support – F

Research of the free vibrations frequencies was calculated in three variants of the plate support denoted: V_1, V_2, V_3. In all cases parallel to the y axis, the edges 1 and 3 of plate were jointly supported. Every edge was connected with the front plate (Fig.3 and Fig.4) stiff on shearing which prevented from the relative displacement of linings along the edges. Two kinds of support of the edges number 2 and 4 were introduced (Fig.2): simple support (Fig.3) and clamped (Fig.4). The V_1 variant (Fig.2) means the simple support of all edges, V_2 – clamped edges number 2 and 4 and simple support of remaining

edges, V_3 clamped edge 2 and simple support of the three remaining edges.
Boundary conditions imposed on edges $x = 0$ and $x = a$ take the form

$$w(x,y,t) = 0 \qquad \Big[\frac{\partial^2 w(x,y,t)}{\partial x^2} + \nu\frac{\partial^2 w(x,y,t)}{\partial y^2}\Big] = 0 \tag{3.1}$$

$$v_\beta(x,y,t) = 0 \qquad \Big[\frac{\partial u_\beta(x,y,t)}{\partial x} + \nu\frac{\partial v_\beta(x,y,t)}{\partial y}\Big] = 0 \tag{3.2}$$

Neglecting the influence of linings flexural rigidity, we can omit the second condition in Eq (3.1) as well. Instead of eight conditions we obtain six ones. In the case of simple support S of the edge $y = -b/2$ or $y = b/2$

$$w(x,y,t) = 0 \qquad \frac{\partial v_\beta(x,y,t)}{\partial y} + \nu\frac{\partial u_\beta(x,y,t)}{\partial x} = 0 \tag{3.3}$$

$$u_\beta(x,y,t) = 0 \qquad \frac{\partial^2 w(x,y,t)}{\partial y^2} + \nu\frac{\partial^2 w(x,y,t)}{\partial x^2} = 0 \tag{3.4}$$

If we omit influence of linings flexural ridigity, the second depedence in Eq (3.4) should be neglected. In the case F of the edge $y = -b/2$ or $y = b/2$, the deflection and the rotation angle have to be zero. The relative linings displacements in both x and y axis directions, respectively, are not possible. The boundary conditions are as follows

$$w(x,y,t) = 0 \qquad \frac{\partial w(x,y,t)}{\partial y} = 0 \tag{3.5}$$

$$u_\beta(x,y,t) = 0 \qquad v_\beta(x,y,t) = 0 \tag{3.6}$$

Neglecting the linings flexural rigidity of the three-layered plate, the second dependence in Eq (3.5) should be omitted as well.

4. Solution to the differential equations system

We present particular integrals of Eq (2.1) as the functions of separable variables

$$\begin{aligned} w(x,y,t) &= \bar{W}(x,y)\widetilde{T}(t) \\ u_\beta(x,y,t) &= \bar{U}_\beta(x,y)\widetilde{T}(t) \\ v_\beta(x,y,t) &= \bar{V}_\beta(x,y)\widetilde{T}(t) \end{aligned} \tag{4.1}$$

where

$$\widetilde{T}(t) = A\sin(\omega t + \varphi)$$

Putting Eqs (4.1) into equations system (2.1) we get

$$\begin{aligned}
&\frac{Bh}{G_{xz}}\Big(\frac{\partial^2 \bar{U}_\beta}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 \bar{U}_\beta}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 \bar{V}_\beta}{\partial x \partial y}\Big) - \bar{U}_\beta + \Big(h + \frac{c}{2}\Big)\frac{\partial \bar{W}}{\partial x} = 0\\
&\frac{Bh}{G_{yz}}\Big(\frac{\partial^2 \bar{V}_\beta}{\partial y^2} + \frac{1-\nu}{2}\frac{\partial^2 \bar{V}_\beta}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 \bar{U}_\beta}{\partial x \partial y}\Big) - \bar{V}_\beta + \Big(h + \frac{c}{2}\Big)\frac{\partial \bar{W}}{\partial y} = 0 \qquad (4.2)\\
&2B\Big(h + \frac{c}{2}\Big)\nabla^2\Big(\frac{\partial \bar{U}_\beta}{\partial x} + \frac{\partial \bar{V}_\beta}{\partial y}\Big) + 2D\nabla^4 \bar{W}(x,y) - \mu\omega^2 \bar{W} = 0
\end{aligned}$$

where

$$\mu = 2\rho c + 2\rho_3 h$$

and

$$\frac{d^2\widetilde{T}}{dt^2} + \omega^2 \widetilde{T} = 0 \qquad (4.3)$$

In the differential equations (4.2), functions $\bar{U}_\beta(x,y)$, $\bar{V}_\beta(x,y)$, $\bar{W}(x,y)$ and the frequency ω of plate free vibrations are unknown. For accepted boundary conditions, we can present particular fuctions as the products

$$\begin{aligned}
\bar{W}(x,y) &= W(y)\sin\frac{m\pi}{a}x\\
\bar{U}_\beta(x,y) &= U_\beta(y)\cos\frac{m\pi}{a}x \qquad (4.4)\\
\bar{V}_\beta(x,y) &= V_\beta(y)\sin\frac{m\pi}{a}x
\end{aligned}$$

On the basis of Eqs (4.4), introducing independent variable $\eta = y/b$, from Eqs (4.2) we get the system of homogeneous differential equations

$$\begin{aligned}
&\beta\Big[-\gamma^2 U_\beta + \frac{1-\nu}{2}\ddot{U}_\beta + \frac{1+\nu}{2}\gamma\dot{V}_\beta\Big] - U_\beta + \alpha\gamma W = 0\\
&\frac{\beta}{\psi}\Big[\ddot{V}_\beta - \frac{1-\nu}{2}\gamma^2 V_\beta - \frac{1+\nu}{2}\gamma\dot{U}_\beta\Big] - V_\beta + \alpha\dot{W} = 0 \qquad (4.5)\\
&\gamma^3 U_\beta - \gamma\ddot{U}_\beta - \gamma^2\dot{V}_\beta + \dddot{V}_\beta + \bar{D}\,\ddddot{W} - 2\gamma^2\bar{D}\ddot{W} + (\bar{D}\gamma^4 - k\omega^2)W = 0
\end{aligned}$$

where

$$\alpha = \frac{h + \frac{c}{2}}{b} \qquad \beta = \frac{Bh}{G_{xz}b^2} \qquad \lambda = \frac{a}{b}$$

$$\gamma = \frac{m\pi}{\lambda} \qquad \psi = \frac{G_{yz}}{G_{xz}} \qquad k = \frac{\mu b^3}{2B\left(h + \frac{c}{2}\right)}$$

$$\bar{D} = \frac{c^2}{12b\left(h + \frac{c}{2}\right)} \qquad \frac{d}{d\eta} = (\dot{\ })$$

$\bar{D}$ – reduced flexural rigidity of linings.

The solution to Eqs (4.5) is assumed as

$$W(\eta) = f_1 e^{r\eta} \qquad U_\beta(\eta) = f_2 e^{r\eta} \qquad V_\beta(\eta) = f_3 e^{r\eta} \tag{4.6}$$

Putting Eqs (4.6) into Eqs (4.5) we get the algebraic equations system in the unknowns f_1, f_2, f_3

$$\begin{aligned}
&\alpha\gamma f_1 + \Big(\frac{1-\nu}{2}\beta r^2 - \gamma^2\beta - 1\Big) f_2 + \frac{1+\nu}{2}\gamma\beta r f_3 = 0\\
&\alpha r f_1 - \frac{1+\nu}{2}\gamma\frac{\beta}{\psi} r f_2 + \Big(\frac{\beta}{\psi}r^2 - \frac{1-\nu}{2}\frac{\beta}{\psi}\gamma^2 - 1\Big) f_3 = 0\\
&\Big(\bar{D}\gamma^4 - k\omega^2 - 2\gamma^2\bar{D}r^2 + \bar{D}r^4\Big) f_1 + \Big(-\gamma r^2 + \gamma^3\Big) f_2 + \Big(r^3 - \gamma^2 r\Big) f_3 = 0
\end{aligned} \tag{4.7}$$

On the basis of Eqs (4.7), the characteristic equation of the eight order was obtained. Calculating the $f_1^{(i)}$, $f_2^{(i)}$ to $f_3^{(i)}$ ratios, corresponding to each r_i $(i = 1, ..., 8)$ root of the characteristic equation

$$K_i = \frac{f_1^{(i)}}{f_3^{(i)}} \qquad M_i = \frac{f_2^{(i)}}{f_3^{(i)}}$$

we obtain a general solution to the system of equation

$$\begin{aligned}
W(\eta) &= f_3^{(1)} K_1 e^{\eta r_1} + f_3^{(2)} K_2 e^{\eta r_2} + ... + f_3^{(8)} K_8 e^{\eta r_8}\\
U_\beta(\eta) &= f_3^{(1)} M_1 e^{\eta r_1} + f_3^{(2)} M_2 e^{\eta r_2} + ... + f_3^{(8)} M_8 e^{\eta r_8}\\
V_\beta(\eta) &= f_3^{(1)} e^{\eta r_1} + f_3^{(2)} e^{\eta r_2} + ... + f_3^{(8)} e^{\eta r_8}
\end{aligned} \tag{4.8}$$

The constants $f_3^{(i)}$, K_i, M_i contain searched parameter ω – which represents the frequency of plate free vibrations. If we put the solutions (4.8) into eight boundary conditions, corresponding to one of the plate supporting versions, we get the following system of algebraic, linear, homogeneous equations

$$
\begin{aligned}
&f_3^{(1)}X_{11} + f_3^{(2)}X_{12} + \ldots + f_3^{(8)}X_{18} = 0\\
&f_3^{(1)}X_{21} + f_3^{(2)}X_{22} + \ldots + f_3^{(8)}X_{28} = 0\\
&\vdots\\
&f_3^{(1)}X_{81} + f_3^{(2)}X_{82} + \ldots + f_3^{(8)}X_{88} = 0
\end{aligned}
\tag{4.9}
$$

The X_{ij} terms are functions of parameter ω, depend on the root kinds of the characteristric equation and boundary conditions existing on the two edges parallel to x axis. Calculations of the free vibrations frequency were done out applying the method of successive approximations. From Eqs (4.9) we can determine forms of the free vibrations corresponding to the particular $\widehat{\omega}_{mn}$ frequencies.

5. Numerical examples, analysis of the research results

The linings were made of dural with Young modulus $E = 0.72 \cdot 10^5$ MPa, density $\rho = 2800$ kg/m^3, Poisson ratio $\nu = 0.3$, Kirchhoff's modulus of filler $G_{xz} = 10$ MPa, G_{yz} takes three various values: $G_{yz} = 15$ MPa ($\psi = 1.5$), $G_{yz} = 10$ MPa ($\psi = 1$), $G_{yz} = 7.5$ MPa ($\psi = 0.75$). The density of filler $\rho_3 = 75$ kg/m^3. Calculations of frequencies for the plate free vibrations frequencies were done for three support variants V_1, V_2, V_3, taking flexural rigidity of linings into account $(D \neq 0)$ and then neglecting this rigidity $(D = 0)$. Omitting the rigidity influence results in significant simplification of calculation. In that case the equation system (4.9) is reduced to 6 equations.

In Table 1 four of the first free vibrations frequencies $\widehat{\omega}_{11}$, $\widehat{\omega}_{12}$, $\widehat{\omega}_{21}$, $\widehat{\omega}_{22}$ for $D \neq 0$ and various ratios of the edges lengths $\lambda = a/b$ for three support variants were compiled. Changes in the λ parameter value were made for the constant edge length $b = 0.5$ m. Analysis was carried out for the orthotropic plate $\psi = 1.5$ at the ratio $2h/c = 20$, where $2h$ – thickness of the middle layer, c – thickness of linings. For the same material data, the frequencies of vibrations ω_{11}, ω_{12}, ω_{21}, ω_{22} were calculated without taking into account the flexural rigidity of linings $(D = 0)$. In Table 1, besides the frequencies of vibrations for $D \neq 0$, the relative scatter of research results as a measure of the influence of neglecting linings flexural rigidity were presented

$$
\delta = \Big(\frac{\omega_{mn} - \widehat{\omega}_{mn}}{\widehat{\omega}_{mn}}\Big) \cdot 100\% \tag{5.1}
$$

Table 1

	λ	$D \neq 0$ $\hat{\omega}_{11}$ [rad/s]	δ [%]	$\psi = 1.5$ $\hat{\omega}_{12}$ [rad/s]	δ [%]	$2h/c = 20$ $\hat{\omega}_{21}$ [rad/s]	δ [%]	$\hat{\omega}_{22}$ [rad/s]	δ [%]
V_1	0.8	1181.73	−0.06	2215.61	−0.08	2488.64	−0.11	3237.43	−0.13
	1.0	970.33	−0.06	2065.74	−0.08	1936.04	−0.09	2787.79	−0.11
	1.2	847.02	−0.05	1980.70	−0.07	1584.71	−0.07	2514.55	−0.10
	1.4	769.34	−0.05	1928.09	−0.07	1348.07	−0.06	2336.99	−0.09
	1.6	717.46	−0.05	1893.36	−0.07	1181.73	−0.06	2215.61	−0.08
V_2	0.8	1357.95	−0.33	2441.83	−0.63	2537.71	−0.23	3343.87	−0.47
	1.0	1199.82	−0.37	2322.47	−0.66	2015.28	−0.25	2932.95	−0.52
	1.2	1117.24	−0.39	2257.12	−0.68	1696.88	−0.28	2692.64	−0.57
	1.4	1069.89	−0.40	2217.64	−0.69	1493.19	−0.31	2541.59	−0.59
	1.6	1040.60	−0.40	2192.02	−0.70	1357.95	−0.33	2441.83	−0.63
V_3	0.8	1264.13	−0.18	2330.99	−0.35	2781.47	−0.06	3290.17	−0.29
	1.0	1078.15	−0.19	2198.08	−0.37	1973.42	−0.16	2860.22	−0.31
	1.2	974.87	−0.20	2124.27	−0.38	1637.29	−0.16	2604.11	−0.32
	1.4	912.50	−0.20	2079.29	−0.38	1415.92	−0.17	2440.78	−0.34
	1.6	872.27	−0.21	2049.92	−0.39	1264.13	−0.18	2330.99	−0.35

The calculations of the ω_{mn} $(D = 0)$ were shown in the form of charts.

In Table 2 the $\hat{\omega}_{mn}$ frequencies of plate vibrations for $\lambda = 1.4$ when changing the thickness of filler are shown. The plate core has the orthotropic properties $\psi = 1.5$. Linings thickness is constant $c = 0.3$ mm. The relative percentage scatter δ of the vibrations frequencies were calculated using (5.1). Research results of the vibrations frequencies for $D \neq 0$ and $D = 0$ when $\psi = 1.5$ and $2h/c = 20$ prove that the modulus of maximal percentage relative deviation of frequency δ is 0.7% and occurs for V_2 support conditions and $\lambda = 1.6$, then $\hat{\omega}_{12} = 2192.02$ [rad/s]. Decreasing $2h/c$ parameter at constant ratio $\lambda = 1.4$ for the particular frequencies and boundary conditions (Table 2), modulus of the δ parameter increases insignificantly. The maximal value of the deviation modulus amounts 0.87% for V_2 boundary conditions, $2h/c = 10$ and $\hat{\omega}_{12} = 1509.72$ [rad/s]. The above results prove that for the sufficiently thin linings, in the three-layered plates with light filler for $2h/c > 10$, the influence of their flexual rigidity can be neglected. Simplified calculations can be employed $(D = 0)$ without the risk of significant error appearance. The differences between calculated frequencies for $D \neq 0$ and $D = 0$ are insignificant. The negative values of deviation mean, that the linings produce the increase in free vibrations frequencies. Results of frequency investigations presented below are obtained for $D = 0$. The influence of parameter λ changes on the $\omega_{11}, \omega_{12}, \omega_{21}, \omega_{22}$ frequencies under the boundary conditions V_1, V_2, V_3 and $\psi = 1.5$ are shown in Fig.5a,b,c. Changes in the parameter $\lambda = a/b$ value have been obtained for the constant value of $b = 0.5$ m. The linings thickness $c = 0.3$ mm, the filler thickness $2h = 6$ mm. Other data are the same as in the calculations described ealier. The results of investgations point out that for the plate with orthotropic core under particular boundary conditions, frequency of free vibrations decreases as the length of the plate increases. The lowest values of frequency occur for the V_1 support, higher ones for the V_3 and the highest for the V_2. It is due to the stiff character of support. Moreover, as the λ parameter increases, the quicker decrease of free vibration frequencies ω_{21}, ω_{22} compared with ω_{11}, ω_{12}, can be observed.

Table 2

	$2h/c$	$D \neq 0$ $\psi = 1.5$ $\lambda = 1.4$							
		$\widehat{\omega}_{11}$ [rad/s]	δ [%]	$\widehat{\omega}_{12}$ [rad/s]	δ [%]	$\widehat{\omega}_{21}$ [rad/s]	δ [%]	$\widehat{\omega}_{22}$ [rad/s]	δ [%]
V_1	10	455.70	−0.16	1222.38	−0.20	843.57	−0.19	1529.60	−0.23
	15	621.99	−0.08	1607.19	−0.11	1117.29	−0.10	1974.43	−0.13
	20	769.34	−0.05	1928.09	−0.07	1348.07	−0.07	2336.99	−0.09
V_2	10	684.32	−0.50	1509.72	−0.87	967.17	−0.45	1740.70	−0.81
	15	895.00	−0.43	1903.35	−0.76	1255.60	−0.36	2186.62	−0.68
	20	1069.89	−0.40	2217.64	−0.69	1493.19	−0.31	2541.59	−0.59
V_3	10	561.08	−0.30	1369.15	−0.52	899.72	−0.30	1634.72	−0.50
	15	750.29	−0.23	1760.63	−0.43	1181.19	−0.21	2081.28	−0.40
	20	912.50	−0.20	2079.29	−0.38	1415.92	−0.17	2410.78	−0.34

Fig.5a,b,c show the influence of changes in length ratio of plate sizes λ on the $\omega_{11}, \omega_{12}, \omega_{21}, \omega_{22}$ frequencies under boundary condition V_1 also $\psi = 0.75$ and $\psi = 1$. Courses of values of vibration frequency versus edges length ratio λ at support variant V_1 and different values ψ (Fig.5a,d,e) show that character of curves of frequency changes is similar. Frequencies take the lower values for $\psi = 0.75$ and greater for $\psi = 1.5$ irrespective of λ values. The maximal increase of vibrations frequency for plate amounts 9.6% at ω_{12} for the $\psi = 1.5$ and $\lambda = 1.4$ in relation to the $\psi = 1$. At $\psi = 0.75$ the maximal decrease of vibration frequencies appears also for ω_{12} and amounts 7.3%. In Fig.5 curves of frequencies ω_{12} and ω_{21} intersect. The intersection point determines the value of frequency at which two vibration forms are possible. For the V_1 support variant and the isotropic filler $(\psi = 1)$ such conditions occur for the square $(\lambda = 1)$ plate (Fig.5e). For the V_1 support variant and $\psi = 0.75$ this point moves towards the greater values of λ (Fig.5d), while for $\psi = 1.5$ and the V_1, V_2, V_3 support variants it moves towards the lower values of λ parameter (Fig.5a,b,c). The first four frequencies of free vibrations ω_{11}, $\omega_{12}, \omega_{21}, \omega_{22}$ were determined under the V_1, V_2, V_3 boundary conditions and when changing the value of $2h$ from 3.6 mm to 10 mm. Calculations were done assuming the anisotropic core $\psi = 1.5$ and the constant $c = 0.3$ mm. Fig.6 present the influence of the filler thickness under V_1, V_2, V_3 boundary conditions on the adequate frequencies of free vibrations.

The investigation results prove, according to the anticipation, that the

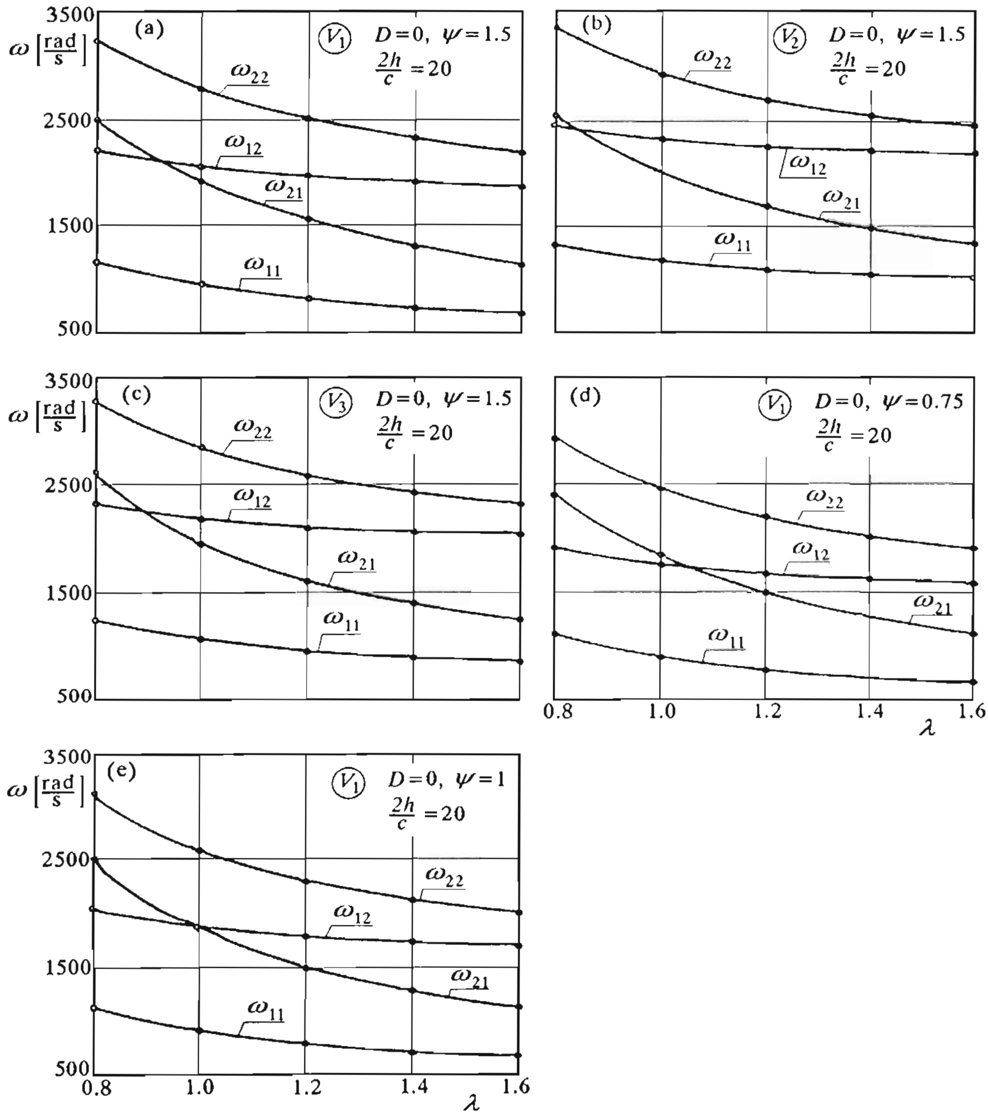

Fig. 5. Free vibration frequency versus λ plots

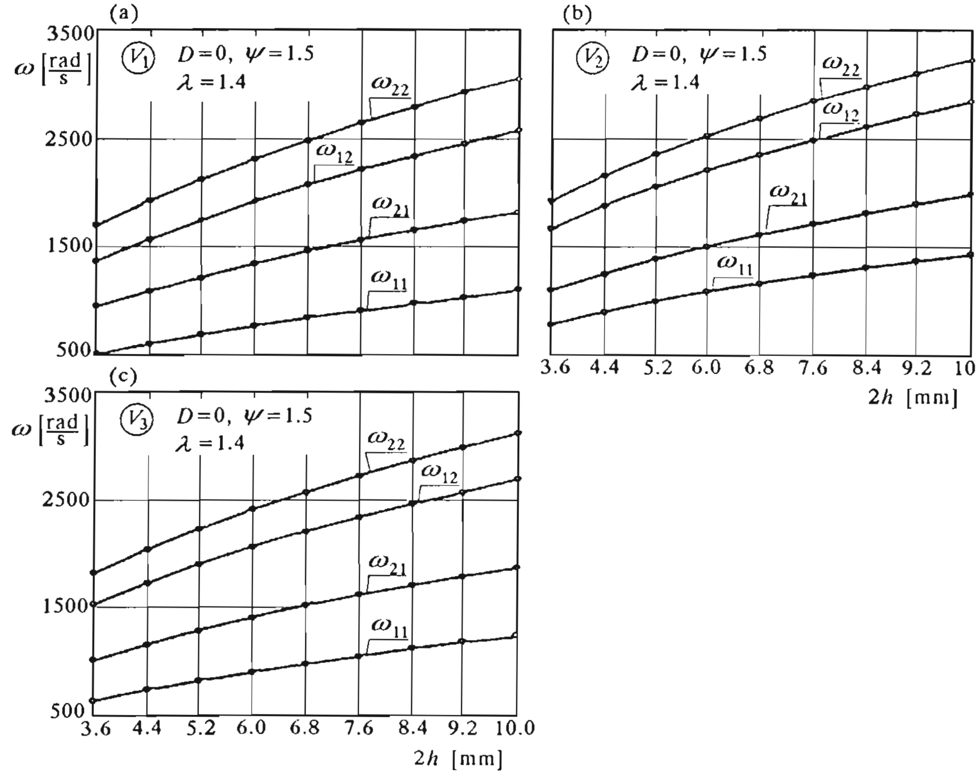

Fig. 6. Frequency of free vibration versus filler thickness $(2h)$ plots

frequency values of free vibrations under varied boundary conditions increase as the thickness of filler grows. The $\omega(\lambda)$ curves courses (Fig.5) point out that the freqencies ω_{21} and ω_{22} undergo more significant changes then ω_{11} and ω_{12} for the same changes of λ parameter. The $\omega(2h)$ curves (Fig.6) testify that all four frequencies of free vibrations present almost the same sensitiveness to changes of the $2h$ parameter. The research results (Fig.5 and Fig.6) make possible the choice of geometrical parameters of plate avoiding resonances.

6. Verifying and comparative investigations

In the paper by Raville and Ueng (1967) the experimental research results of the frequencies of free vibrations for rectangular three-layered plate with joint support along all its edges were presented. The core of plate was anisotropic. Geometrical and material data of that plate were as follows: $a = 1.829$ m, $b = 1.219$ m, $c = 0.00041$ m, $h = 0.00318$ m, $E = 0.6887 \cdot 10^{11}$ N/m^2, $G_{xz} = 0.5167 \cdot 10^8$ N/m^2, $G_{yz} = 0.1344 \cdot 10^9$ N/m^2, $\rho = 2820$ kg/m^3, $\rho_3 = 124$ kg/m^3. In the paper by Cheung and Khatua (1973), despite of the experimental results obtained by Raville and Ueng (1967), the plate analysis by means of the finite elements method was presented. For the same data, using the method presented in this paper, the first frequencies of free vibrations were calculated. The experimetal research results Raville and Ueng (1967), the FEM results (Cheung and Khatua (1973)), and the results obtained by the Authors were set up in Table 3. The columns contain following results: (E) – experimental, (F) – FEM, (A) – our research, respectively.

Table 3

		ω_{mn} [rad/s]			δ_{AE}	δ_{FE}
m	n	(E)	(A)	(F)	[%]	[%]
1	1	–	145.82	144.51	–	–
2	1	282.74	278.02	282.74	−1.7	0
1	2	433.54	445.46	446.11	2.7	2.8
3	1	490.08	493.55	515.22	0.8	5.1
2	2	578.05	574.25	578.05	−0.7	0
3	2	810.53	784.37	804.25	−3.2	−0.7
4	1	835.66	785.82	854.51	−6.0	2.3
1	3	955.04	935.37	942.48	−2.1	−1.3
2	3	1061.85	1058.82	1061.86	−0.3	0
4	2	1112.12	1069.56	1124.69	−3.8	−1.1

The relative proportional scatter of frequencies of free vibrations δ_{AE}, δ_{FE} were calculated using the formulae

$$\delta_{AE} = \frac{\omega_{mn}^{(A)} - \omega_{mn}^{(E)}}{\omega_{mn}^{(E)}} \cdot 100\% \tag{6.1}$$

$$\delta_{FE} = \frac{\omega_{mn}^{(F)} - \omega_{mn}^{(E)}}{\omega_{mn}^{(E)}} \cdot 100\% \tag{6.2}$$

With the exception of ω_{41} frequency, the relative proportional deviation $|\delta_{AE}|$ of results obtained by the analytical method in comparison with the experimental results is inside the $0.3\% \div 3.8\%$ range, however $|\delta_{FE}|$ changes from 0% to 5.1%. Some of the frequency values obtained analytically (A) and by the FEM (F) are greater, and some are lower then the experimental frequencies (E). Such good agreement of compared results testifies the correctness of investigations into frequencies of free vibrations of three-layered plates.

7. Conclusions

The research conducted confirmed the possibility of the omitting, in the case of thin linings, their flexural rigidity in calculations of frequencies of free vibrations. Further calculations with neglected flexural rigidity enable to evaluate the influence of changes in the plate geometrical parameters, the orthotropic properties of core material and support conditions of three-layered plate, respectively, on the frequencies of free vibrations. For the simple support of a plate, the increase of core orthotropic coefficient causes insignificant increase in free vibrations frequency. For the specified geometrical and material parameters such frequencies were detected, at which two different forms of vibrations are possible. Presented research results have a cognitive character, however they can also be used in construction design using the three-layered plates. Correctness of the method was verified by the comparison of our results with the results obtained during experiment (Raville and Ueng (1967)) and calculations obtained by means of the FEM (Cheung and Khatua (1973)).

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Analiza porównawcza wpływu parametrów konstrukcyjnych i podparcia płyty trójwarstwowej na częstości jej drgań własnych

Streszczenie

W pracy przedstawiono badania analityczno-numeryczne częstości drgań własnych symetrycznej płyty trójwarstwowej z lekkim wypełniaczem. Założono, że okładziny mają własności izotropowe, natomiast wypełniacz może mieć własności ortotropowe. Oceniając wpływ sztywności giętnej okładzin badania częstości drgań przeprowadzono dla trzech różnych wariantów podparcia płyty. Zbadano wpływ wartości niektórych parametrów geometrycznych i materiałowych na częstości jej drgań własnych. Wyniki badań zweryfikowano poprzez porównanie wyznaczonych częstości drgań płyty przy podparciu przegubowym, z wynikami eksperymentu fizycznego (Raville and Ueng (1967)) oraz badań częstości drgań metodą elementów skończonych (Cheung and Khatua (1973)).

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