

ROBUSTNESS PROBLEM IN CONTROL OF WHEELED MOBILE ROBOT

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This paper considers the robustness of trajectory – control problem of the nonlinear Wheeled Mobile Robot. The equation of motion for the front steered wheels is used via the Lagrange's equation. In this paper we design a control algorithm in which we detect trajectory errors by finding the difference between desired and actual quantities determined in joint space. A trajectory following control algorithm contains PD control law and dynamic model of the WMR. This paper demonstrates the computed torque method proposing a new approach to the tracking control of rigid WMR. The effect of the nonlinear model-based control is illustrated by simulation results which are presented.

1. Introduction

Wheeled Mobile Robots (WMRs) and Automated Guide Vehicles (AGVs) have been used in automated factories for material handling and especially the nuclear and explosive ones. These vehicles require some types of guide path to follow, such as reflective tape or electromagnetic cables. Alternatively, they can use a route map stored in the computer memory. A comprehensive survey of the latest control results for rigid WMR is presented by Hemami et al. (1992). Authors studied a synthesis of the optimal control law for three-wheeled vehicles. This synthesis was based on the linearized dynamic model of a vehicle and minimization of a quadratic criterion, assuming that the process model is known in details. However, this is never the case in practice. By nature, most WMR remove various parts and tools so they have varying loads. The robust and adaptive tracking control problem for improving dynamic performance of the WMR in the presence of parameter uncertainties

and unknown disturbances are being intensively studied by the author (cf Hendzel (1993a,b) and (1994)).

It is known that nonlinear, uncertain systems are difficult to control by a conventional regulator, which is the main reason for using relatively modern methods in synthesis of the control algorithm. In this paper based on a dynamic model of a the four-wheeled cart with front steering wheels, robustness of the nonlinear model-based control is presented. We assume that the desired trajectory is available in terms of the time histories of joint position, velocity and acceleration, respectively. We design a joint-based control scheme, that is the scheme, in which we detect the trajectory errors by finding the difference between the desired and actual quantities, respectively determined in a joint space. A trajectory – following control algorithm contains PD control law. The effect of nonlinear model-based control is illustrated by the presented simulation results.

2. Dynamic model of the system

The class of system presented in this paper consists of multibody mechanisms. Fig.1 illustrates the scheme of WMR C80¹.

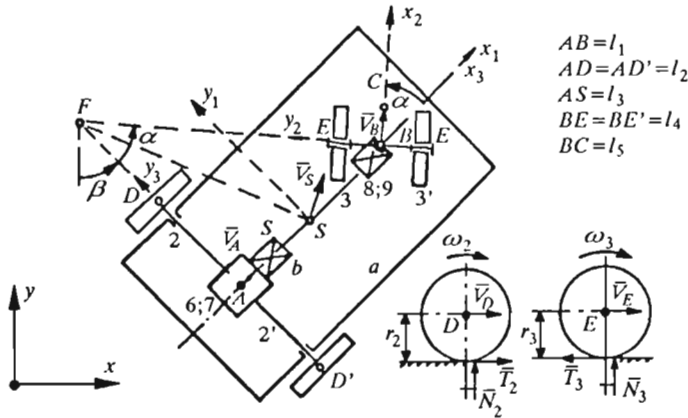


Fig. 1. Scheme of the vehicle

It consists of three units: track of the vehicle (a), drive mechanism (b), and steering unit (c). The motion of vehicle is fully described when we know the

¹The WMR C80 was designed and built in the Rzeszów University of Technology

velocity of point A and rotation angle α . The motion of vehicle is described by the system two generalized coordinates $q_1 = S_A$ and $q_2 = \alpha$. Fig.1 shows dimensions and presents definitions of some other variables. We use a standard method for deriving the dynamic equations of mechanical system via the Lagrange's equations

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} = \mathbf{Q} \tag{2.1}$$

where

- \mathbf{q} - set of generalized coordinates, $\mathbf{q} \in R^2$
- L - Lagrangian
- \mathbf{Q} - vector of generalized forces acting upon the system, $\mathbf{Q} \in R^2$

The potential energy P of WMR is constant. The kinetic energy being a quadratic function of the vector $\dot{\mathbf{q}}$ can be written by Hendzel et al. (1992) as

$$E = AV_A + BV_A^2 + \tan^2 \alpha + CV_A \dot{\alpha} \tan \alpha + D\dot{\alpha}^2 \tag{2.2}$$

where

$$\begin{aligned} A &= \frac{1}{2}m_1 + m_2 + m_3 + I_{y2} \left(\frac{1}{r_2}\right)^2 + I_{y3} \left(\frac{l_4}{l_3 r_3}\right)^2 \\ B &= \frac{1}{2}m_1 \left(\frac{l_3}{l_1}\right)^2 + m_3 + \left(\frac{1}{2}I_{z1} + I_{z2}\right) \left(\frac{1}{l_1}\right)^2 + I_{y2} \left(\frac{l_2}{l_1 r_2}\right)^2 + \\ &+ I_{y3} \left(\frac{l_4}{l_3}\right)^2 \left(\frac{1}{r_3^2} + \frac{1}{l_1^2}\right) + I_{z3} \left(\frac{1}{l_1}\right)^2 \\ C &= 2I_{z3} \frac{1}{l_1} + 2I_{y3} \frac{l_4}{l_3} \frac{1}{l_1} \\ D &= I_{z3} + I_{y3} \left(\frac{l_4}{l_3}\right)^2 \end{aligned} \tag{2.3}$$

In Eq (2.3) m_1, m_2, m_3 stand for substitutional masses of units a, b, c , respectively. I_{yi}, I_{zi} represent elements of the inertia matrix. Taking partial derivatives we can rewrite Eq (2.1) as follows

$$\begin{aligned} &\begin{bmatrix} 2A + 2B \tan^2 q_2 & C \tan q_2 \\ C \tan q_2 & 2D \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \\ &+ \begin{bmatrix} 4Bq_2 \tan q_2 & C\dot{q} \frac{1}{\cos q_2} \\ -2B\dot{q}_1 \sin q_2 \frac{1}{\cos^3 q_2} & 0 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} \end{aligned} \tag{2.4}$$

where

$$q_1 = S_A \quad \dot{q}_1 = \dot{S}_A = V_A \quad q_2 = \alpha \quad \dot{q}_2 = \dot{\alpha}$$

The generalized forces in Eq (2.4) we can write as follows

$$\begin{aligned} \delta q_1 &= \delta S_A \neq 0 & \delta q_2 &= \delta \alpha = 0 \\ Q_1 \delta S_A &= \left[M_1 \frac{i_1}{r_2} - \left(\frac{l_1 - l_3}{r_2} f_2 + \frac{l_3}{r_3} f_3 \sqrt{1 + \tan^2 \alpha} \right) \frac{Q}{l_1} \right] \delta S_A \\ \delta q_2 &= \delta \alpha \neq 0 & \delta S_A &= 0 \\ Q_2 \delta \alpha &= \left[M_2 i_2 - \frac{l_3 l_4}{l_1} \mu Q \operatorname{sgn} \dot{\alpha} \right] \delta \alpha \end{aligned} \quad (2.5)$$

where

- M_1 – motor-torque of the drive unit
- M_2 – actuator-torque
- Q – gravity force of the WMR
- μ – Coulomb friction coefficient
- f_2 – flowing friction coefficient of wheels of the drive unit
- f_3 – flowing friction coefficient of the steering wheels
- i_1, i_2 – gear ratio of the drive and steering unit, respectively.

In Eq (2.5) viscous friction appearing in kinematics pairs is neglected. Using Eqs (2.5), (2.4) and assuming the forward velocity V_A , to be constant we obtain

$$\frac{2D}{i_2} \ddot{\alpha} - \frac{2BV_A^2}{i_2} \frac{\sin \alpha}{\cos^3 \alpha} + \frac{l_3 l_4}{l_1 i_2} \mu Q \operatorname{sgn} \dot{\alpha} = M_2 \quad (2.6)$$

From Eq (2.6) it can be seen that

$$\begin{aligned} H &= \frac{2D}{i_2} \\ V(\alpha) &= \frac{2BV_A^2}{i_2} \frac{\sin \alpha}{\cos^3 \alpha} \\ F(\dot{\alpha}) &= \frac{l_3 l_4}{l_1 i_2} \mu Q \operatorname{sgn} \dot{\alpha} \\ M &= M_2 \end{aligned} \quad (2.7)$$

Finally, substituting Eq (2.7) into Eq (2.6) we have the scalar equation

$$H \ddot{\alpha} + F(\dot{\alpha}) + V(\alpha) = M \quad (2.8)$$

Eq (2.8) describes the WMR motion.

The actuator used in this vehicle is the direct current (DC) motor with permanent magnet. The dynamics of DC motor is expressed as Luh (1983) and Parkin (1991)

$$\begin{aligned} u_a &= R_a I_a + k_b \dot{\alpha}_m \\ M &= k_i I_a \end{aligned} \quad (2.9)$$

where

- u_a, I_a – armature voltage and current respectively
- R_a – armature resistance
- k_b, k_i – proportionality constants
- $\dot{\alpha}_m$ – angular velocity of the actuator shaft.

The DC motor shaft is mechanically connected with the actuator-gear-load assembly so that

$$\alpha_m = i_2 \alpha \quad (2.10)$$

The assembly is described by the equations

$$\begin{aligned} H \ddot{\alpha} + F(\dot{\alpha}) + V(\alpha) &= M \\ M &= k_i I_a \\ u_a &= R_a I_a + k_b i_2 \dot{\alpha}_m \end{aligned} \quad (2.11)$$

We can rewrite Eq (2.11) as

$$P \ddot{\alpha} + G(\dot{\alpha}) + K(\alpha) = u \quad (2.12)$$

where

$$\begin{aligned} P &= \frac{R_a}{k_i} H & K(\alpha) &= \frac{R_a}{k_i} V(\alpha) \\ G(\dot{\alpha}) &= \frac{R_a}{k_i} F(\dot{\alpha}) + k_b i_2 \dot{\alpha} \end{aligned} \quad (2.13)$$

3. Trajectory-following control

The WMR control problem is considered. To solve this problem we will divide the controller into a model-based part and a servo part (cf Craig (1986)

and (1988)). Such a formulation of control yields a controller that suppresses disturbances and tracks desired trajectories. However, this desirable performance is only achieved when all the vehicle parameters are known. Suppose that our model of the vehicle is imperfect. We define the following notation

$$\tilde{P} = P - \hat{P} \quad \tilde{K} = K - \hat{K} \quad \tilde{G} = G - \hat{G} \quad (3.1)$$

where \hat{P} , \hat{K} , \hat{G} stand for our model parameters. We partition the controller into a model-based part and servo part. The model-based part of the controller appears in a control law of the form

$$u = A\dot{u} + B \quad (3.2)$$

where

$$A = \hat{P} \quad B = \hat{K}(\alpha) + \hat{G}(\dot{\alpha}) \quad (3.3)$$

and the servo part is

$$\dot{u} = \ddot{\alpha}_0 + K_D(\dot{\alpha}_0 - \dot{\alpha}) + K_P(\alpha_0 - \alpha)$$

In the control law partition the system parameters appear only in the model-based part while and the servo part is independent of these parameters. We assume that the trajectory is smooth and the trajectory generator provides α_0 , $\dot{\alpha}_0$, $\ddot{\alpha}_0$ at all moments of time t . We define the servo error between the desired and the actual trajectories as $E = \alpha_0 - \alpha$, $\dot{E} = \dot{\alpha}_0 - \dot{\alpha}$. Combining Eq (3.2) with Eqs (2.12) yields

$$\ddot{E} + K_D\dot{E} + K_P E = \hat{P}^{-1}[\tilde{P}\ddot{\alpha} + \tilde{G}(\dot{\alpha}) + \tilde{K}(\alpha)] \quad (3.4)$$

For the exact model the right-hand side of Eq (3.4) is zero. The closed loop system is then characterized by the error equation

$$\ddot{E} + K_D\dot{E} + K_P E = 0 \quad (3.5)$$

Since this is a second-order differential equation, we can design any response we wish. In the trajectory – following problem critical damping is often the choice made. In order to do this, gains in Eq (3.5) of a PD control law should be

$$K_P = \frac{1}{4}\omega_r^2 \quad K_D = 2\sqrt{K_P} = \omega_r \quad (3.6)$$

where ω_r is the lowest structural resonance frequency. Fig.2 shows a block diagram of our trajectory controller.

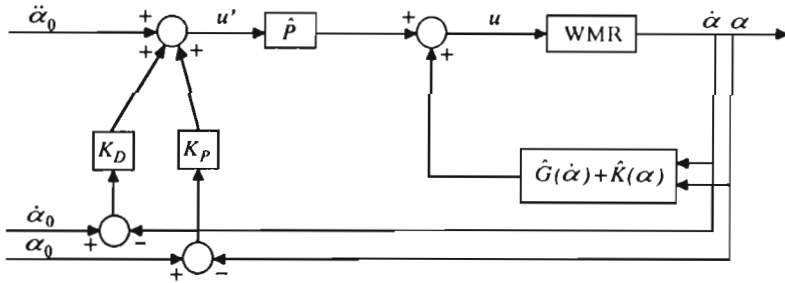


Fig. 2. Trajectory following controller

4. Approach to the robustness analysis

When the parameters of dynamic functions are not known exactly, the mismatch between actual and modeled parameters will cause servo errors. Our model-based controller employs estimates of the parameters \hat{V} with errors denoted as

$$\tilde{V} = V - \hat{V} \tag{4.1}$$

Hence, we have a limit imposed on the magnitude of any parameter error

$$|\tilde{V}_i| \leq V_{im} - V_{iM} \tag{4.2}$$

Now, after Craig (1986), we will check the influence of these uncertainty on the quality of control and stability. This method is similar to that used by Craig (1988). Using

$$\hat{P}^{-1} \tilde{P} \ddot{\alpha} = \hat{P}^{-1} \tilde{P} \ddot{\alpha}_0 - \hat{P}^{-1} \tilde{P} \ddot{E} \tag{4.3}$$

we have

$$\hat{P}^{-1} P \ddot{E} + K_D \dot{E} + K_P E = \hat{P} [\tilde{P} \ddot{\alpha}_0 + \tilde{G}(\dot{\alpha}) + \tilde{K}(\alpha)] \tag{4.4}$$

Now, we multiply both sides by $P^{-1} \hat{P}$ to obtain

$$\ddot{E} + P^{-1} \hat{P} K_D \dot{E} + P^{-1} \hat{P} K_P E = P^{-1} [\tilde{P} \ddot{\alpha}_0 + \tilde{G}(\dot{\alpha}) + \tilde{K}(\alpha)] \tag{4.5}$$

Writing

$$\tilde{G}(\dot{\alpha}) = \tilde{F}(\dot{\alpha}) + \tilde{C}_V \dot{\alpha} \tag{4.6}$$

$$\tilde{C}_V \dot{\alpha} = \tilde{C}_V \dot{\alpha}_0 - \hat{C}_V \dot{E}$$

we can rewrite Eq (4.5) as

$$\ddot{E} + P^{-1} \hat{P} K_D \dot{E} + P^{-1} \hat{P} K_P E = P^{-1} [\tilde{P} \ddot{\alpha}_0 + \tilde{F}(\dot{\alpha}) + \tilde{K}(\alpha) + \tilde{C}_V \dot{\alpha}_0 - \tilde{C}_V \dot{E}] \quad (4.7)$$

We define N to be the right-hand side of Eq (4.7). We have

$$N = P^{-1} [\tilde{P} \ddot{\alpha}_0 + \tilde{F}(\dot{\alpha}) + \tilde{K}(\alpha) + \tilde{C}_V (\dot{\alpha}_0 - \dot{E})] \quad (4.8)$$

The form of Eq (4.8) is independent of $\ddot{\alpha}$ and depends on $\dot{\alpha}$, α only occurs through $\operatorname{sgn} \dot{\alpha}$, $\sin \alpha$, $\cos \alpha$, $|\alpha| \leq \frac{\pi}{6}$.

Hence, N has a limit that is independent of the trajectory $(\ddot{\alpha}, \dot{\alpha}, \alpha)$. We now rewrite Eq (4.7) in such a form that the left-hand side is a linear differential equation. Writing

$$P^{-1} \hat{P} K_D = K_D + P^{-1} \hat{P} K_D - K_D \quad (4.9)$$

$$P^{-1} \hat{P} K_P = K_P + P^{-1} \hat{P} K_P - K_P$$

we have

$$\ddot{E} + K_D \dot{E} + K_P E = \eta \quad (4.10)$$

$$\eta = N + (1 - P^{-1} \hat{P}) K_D \dot{E} + (1 - P^{-1} \hat{P}) K_P E$$

Consider the convolution operators $H_1 : \eta \rightarrow E$, $H_2 : \eta \rightarrow \dot{E}$. Now, we write the transfer function from input η to output E as

$$\frac{E(s)}{\eta(s)} = \frac{1}{s^2 + K_D s + K_P} \quad (4.11)$$

and the transfer function from input η to output \dot{E} as

$$\frac{\dot{E}(s)}{\eta(s)} = \frac{s}{s^2 + K_D s + K_P} \quad (4.12)$$

From Eqs (4.11) and (4.12) we have

$$e(t) = \int_0^t h(t - \tau) \eta(\tau) d\tau \quad (4.13)$$

$$\dot{e}(t) = \int_0^t g(t - \tau) \eta(\tau) d\tau$$

where g and h are the impulse responses of Eqs (4.11) and (4.12), respectively. K_D, K_P are given by Eq (3.6). We can bound Eqs (4.13)

$$\|H_1\|_\infty = \int_0^\infty |h(t)| dt \tag{4.14}$$

$$\|H_2\|_\infty = \int_0^\infty |g(t)| dt$$

where

$$h(t) = te^{-\sqrt{K_P}t} \qquad g(t) = (1 - \sqrt{K_P}t)e^{-\sqrt{K_P}t} \tag{4.15}$$

$$\|H_1\|_\infty = K_P^{-1} = \gamma_1 \qquad \|H_2\|_\infty = \frac{4e^{-1}}{K_D} = \gamma_2 \tag{4.16}$$

Since the norms of the operators are $\|H_1\|, \|H_2\| < \infty$, we say that $H_1, H_2 \in L_\infty$ (cf Craig (1988)). Hence, we have

$$\|E\|_{T_\infty} \leq \gamma_1 \|\eta\|_{T_\infty} \qquad \|\dot{E}\|_{T_\infty} \leq \gamma_2 \|\eta\|_{T_\infty} \tag{4.17}$$

where $\|\eta\|_{T_\infty}$ denotes the L_∞ norm of $\eta(t)$ truncated at the time T . For brevity, the sign T_∞ will be dropped below.

We develop a limit on $\|\eta\|$ as a function of $\|E\|, \|\dot{E}\|$. The term on the right-hand side that is independent of E, \dot{E} has a limit

$$\delta_1 = \|P^{-1}[\tilde{P}\ddot{\alpha}_0 + \tilde{F}(\dot{\alpha}) + \tilde{C}_V\dot{\alpha}_0 + \tilde{K}(\alpha)]\| \tag{4.18}$$

hence we can bound η as follows

$$\|\eta\| \leq \delta_1 + \delta_2\|E\| + \delta_3\|\dot{E}\| \tag{4.19}$$

where

$$\delta_2 = \|(1 - P^{-1}\hat{P})K_D - \tilde{C}_V\| \tag{4.20}$$

$$\delta_3 = \|(1 - P^{-1}\hat{P})K_P\|$$

Combining Eqs (4.17) and (4.19) results in two linear inequalities within which the error magnitudes must lie

$$\|E\| \leq \frac{\gamma_1\delta_1}{1 - \gamma_1\delta_2} + \frac{\gamma_1\delta_3}{1 - \gamma_1\delta_2}\|\dot{E}\| \tag{4.21}$$

$$\|E\| \geq -\frac{\delta_1}{\delta_2} + \frac{1 - \gamma_2\delta_3}{\gamma_2\delta_2}\|\dot{E}\|$$

where we have assumed that

$$\gamma_1 \delta_2 < 1 \qquad \gamma_2 \delta_3 < 1 \qquad (4.22)$$

To determine condition on which a closed region in the magnitude plane exists (see Fig.3) we solve the system of inequalities (4.21). This condition can be written as

$$\gamma_1 \delta_2 + \gamma_2 \delta_3 < 1 \qquad (4.23)$$

Hence, when Eq (4.23) is satisfied the system represented by Eq (4.10) is L_∞ stable.

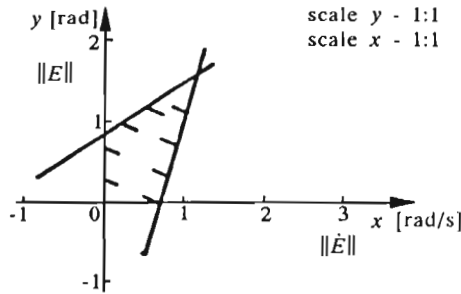


Fig. 3. Stability region for $\|E\|$, $\|\dot{E}\|$

5. Simulation results

In this section, a second-order nonlinear system (2.12) will be used to illustrate robustness of computed torque controller. We will regard the motion of WMR, point C of which. Fig.4 follows the desired circle trajectory with radius $R = 1$ [m]. The WMR moves at speed $V_A = 0.2$ [m/s]. Fig.5 and Fig.6 show the reference generalized coordinate α_0 and generalized velocity $\dot{\alpha}$, respectively. The input data are the range of uncertainty in parameter values, the servo gains, and the bounds on the velocity and acceleration of the desired trajectory. The following values were used

$$\begin{aligned} \|\ddot{\alpha}_0\| &= 0.4 \text{ [rad/s}^2\text{]} & K &\in (5.29, 5.38) \text{ [V]} \\ \|\dot{\alpha}_0\| &= 0.23 \text{ [rad/s]} & P &\in (0.37, 0.4) \text{ [Vs}^2\text{/rad]} \\ \left\| \frac{\sin \alpha}{\cos^3 \alpha} \right\|_{t < T} &= 1.24 & C_V &\in (4.73, 4.81) \text{ [Vs/rad]} \\ K_D &= 9 & F &\in (1.5, 1.53) \text{ [Vs/rad]} \\ K_P &= 6 & m_1 &\in (550, 750) \text{ [kg]} \end{aligned} \qquad (5.1)$$

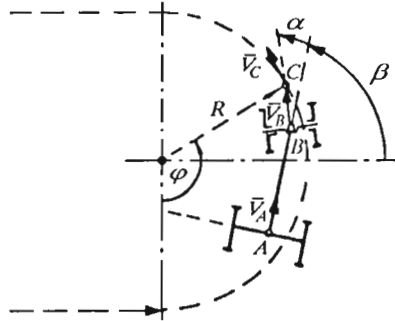


Fig. 4. Desired trajectory

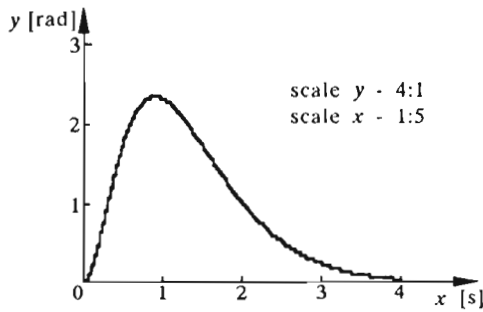


Fig. 5. Generalized coordinate α_0

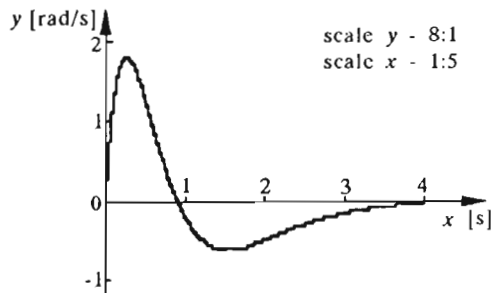


Fig. 6. Generalized velocity $\dot{\alpha}_0$

With these values the robustness condition (4.23) was met. The left-hand side evaluated to 0.2613. Fig.7 shows the regions of interest and the trajectory error of the control system in the error magnitude plane.

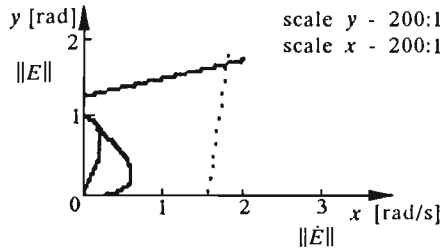


Fig. 7. Regions for a numerical example

The numerical analysis performed shows that presented method is robust to parameter errors. That is, for moderately mistuned parameters the designed algorithm keeps satisfactory performance.

6. Conclusions

In this study, we present a trajectory-following control for four wheeled, front steering vehicle. The analyzed class of system consists of multibody mechanisms. Uncertainties result from unknown system parameters. In designing a trajectory following algorithm we divided the controller into the model based part and the servo part. In the result the system parameters appear only in the model – based part and the servo part is independent of these parameters. Using this methodology we have chosen the PD servo law with gains, so the system is critically damped. We have shown that the designed controller is robust in the presence of unknown system parameters. In order to test the designed controller we made a series of simulation experiments. It was observed from numerical studies that the control algorithm we presented worked well when the bounds of parameters were small. If the bounds are large then the system is still stable but the quality of control will degenerates.

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Sterowanie nadające kołowym mobilnym robotem z uwzględnieniem niedokładności modelu

Streszczenie

W pracy omówiono zagadnienie śledzenia zadanej trajektorii ruchu przez wybrany punkt mobilnego robota. Do opisu własności ruchu precesyjnego zespołu skręcającego mobilnego robota wykorzystano równanie Lagrang'a drugiego rodzaju. Zakłada się, że zadana trajektoria ruchu wybranego punktu mobilnego robota jest przeliczona z przestrzeni kartezjańskiej do przestrzeni kątowej i jest zapisana w pamięci komputera realizującego sterowanie. Analizowany algorytm sterowania oparty jest na metodzie wyliczania momentu. Własności stabilności oraz jego jakość oceniono na podstawie teorii układów krzepkich.

Przedstawiona metoda stanowi szersze ujęcie problemu sterowania mobilnymi robotami. Rezultaty zastosowanej metody sterowania otrzymano na podstawie symulacji numerycznej.