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UNCONVENTIONAL METHOD FOR FLEXIBLE ROTORS BALANCING

CESARE ROSSI

DOMENICO DE FALCO

Department of Mechanical Engineering for Energetics

University "Federico II", Naples, Italy

An analitcal-experimental method for balancing flexible rotors is proposed and some experimental results are presented. By this method it is possible, in principle, to make balanced a flexible rotor, by only a single test run determining the masses that eliminate, in every considered rotor section, the unbalancing. First experimental results, on laboratory test rig, confirm the possibility to reach a good balancing of the rotor within an extended range of rotational speed.

1. Introduction

In the present paper a method for balancing flexible rotors is introduced and subjected to inital experimental analysis. The method in question is analytical-experimental in nature and presents the advantage that its experimental part is considerably simplified, as compared to other methods so far proposed.

Technically, the term "rigid rotor" refers to a rotor whose maximum rotational speed does not exceed 1/3 of the first transverse critical speed so that the rotor's flexional vibration modes are excited to a negligible extent throughout the operating range. Such a rotor can thus be balanced simply by eliminating the resultant of the forces and the moments due to the distributed residual unbalance using only two balancing masses, which can be determined with a low speed balancing machine.

This is not the case if the rotor rotates at a speed that is not sufficiently far from the flexional critical speeds. Such a rotor, called a "flexible rotor" must be balanced using the techniques different from the one mentioned above and which are referred to with the names high-speed, multiplane or modal balancing, respectively.

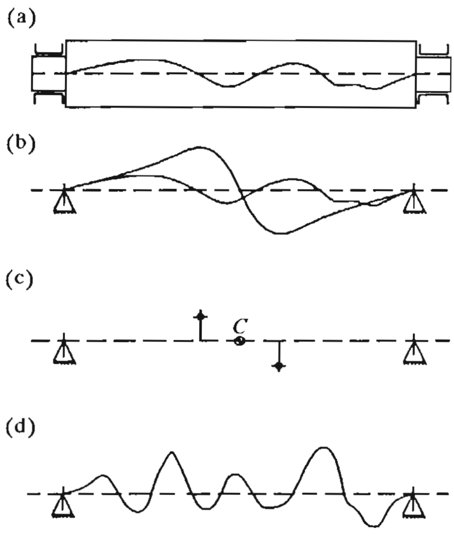

Fig. 1.

For example, consider the solid rotor in Fig.1a for which the center of gravity locus curve of the (infinite) orthogonal sections to the rotation axis is, for instance, of the type shown in the figure. After static balancing (achieved with a single balancing mass such as, ideally, to make the rotation axis through the center of gravity) the center of gravity locus curve of the rotor sections will, in qualitative terms, be like the one shown in Fig.1b.

Obviously, if the rotor were to reach a rotational speed that is not sufficiently far from the 2nd flexional critical speed, the centrifugal forces could considerably excite the 2nd vibration mode, thus resulting in elastic curves of the type shown in Fig.1b.

The amplitudes of these deformations could be significantly reduced by setting two equal balancing masses, as shown in Fig.1c in the same axial plane and at the same distance from the rotation axis and from the center of gravity so as not to introduce static unbalancing but only a moment rotating with the system.

Once this operation has been performed, the first and second rotor vibration modes will both be excited to a negligible degree but the center of gravity lotus curve will now be of the type shown in Fig.1d, and the centrifugal forces will be able to excite vibration modes above the 2nd one; furthermore, the closer the rotational speed is to each of the critical speeds, the more significant this excitation will be.

It is thus clear that a balancing operation such as not to excite any of the vibration modes of a solid rotor could, in principle, be optained simply by ensuring that the center of gravity of each of the infinite sections of the rotor lies on the rotation axis.

However, it is obvious that the vibration modes whose frequency is much greater then the rotor maximum rotational speed would, in any case, be excited to an entirely negligible extent regardless of the rotor's residual unbalance.

In practical terms, it will therefore be sufficient to balance the rotor in such a way as to obtain a center of gravity locus curve such that the centrifugal forces related to it are unable to excite any vibration mode whose frequency is lower then the rotor's maximum rotational speed or the first 2÷4 modes with a higher frequency.

The balancing techniques that allow this result to be obtained can be conceptually referred to two main methods:

1. *Flexibility coefficients method* (cf Diana (1984), Rao (1985), Badgley and Rieger (1972))

   This is a pure experimental method with which to determine a "dynamic" flexibility coefficients matrix, obtained through the experimental detection of the elastic curves of a system rotating at a given speed, both with the addition of test masses and without them. This matrix makes it possible to calculate a system of balancing masses such as to eliminate the deformations at that speed. As the above matrix refers only to the test speed, if the rotor can rotate at more than one steady state speed, then this procedure will have to be repeated at several rotational speeds and the system of balancing masses will have to be determined using a dynamic deformation minimization criterion.

   With this method, therefore, if n correction planes are to be considered, $(n+1)$ test runs will be needed for each speed conidered.

2. *Modal method* (cf Diana (1984), Rao (1985), Bishop and Gladwell (1959), Zhon and Rieger (1982))

   This is analytical-experimental method that requires the prior discretization of the system and the calculation of the modal matrix and the system natural frequencies. It will then be necessary to measure the amplitudes of the system vibrations in n measurement planes while the rotor rotates at a speed near the transverse critical speed both with and without a known test mass.

The modal matrix makes it possible to calculate the work made from the initial unbalance for the considered vibration mode and, from a knowledge of this work, the system of balancing masses that do (for this vibration mode) equal and opposite works, respectively.

Balancing is thus achieved by successively balancing (from the 1st to the nth) the considered vibration modes, introducing for each of them a system of masses that are such as to also satisfy the condition not to excite the already balanced modes.

As can be seen, this method also requires a large number of experimental elastic curve readings.

2. The forces method

The proposed method makes it possible to balance a flexible rotor with a very small number of test runs (possibly one only). This is derived from the consideration of the fact that a rotor elastic curve occurs at a rotational speed Ω between the two critical speeds ω_i and ω_{i+1} (but sufficiently far from each of them). This elastic curve will include the contributions of all the vibrations modes excited by the unbalance (static and dynamic) in the various sections of the rotor itself.

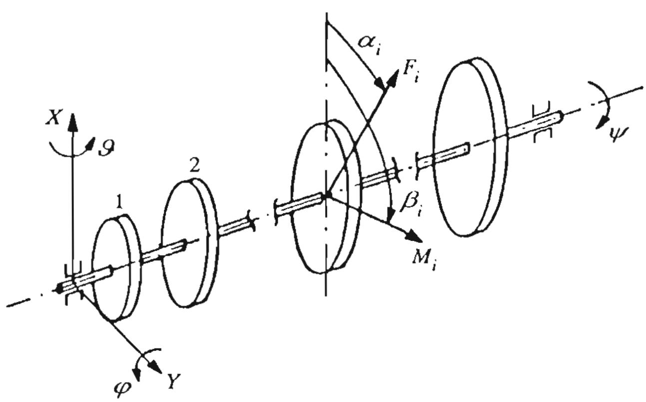

Fig. 2.

Assuming that the rotor-bearing system can be considered linear, the system of forces and moments due to unbalancing (in the various sections) can be straightforwardly calculated by measuring a single deformation of the revolving rotor at a speed Ω satisfying the condition $\omega_i < \Omega < \omega_{i+1}$.

In order to perform this balancing, the following procedure is assumed: the rotor to be balanced must be made discrete as in Fig.2; the following act on each of the n disks in the discrete model:

(a) the centrifugal force, due to static unbalance

$$F_i = -m_i e_i \Omega^2$$

where m_i indicates the mass of ith disk in the discrete model and e_i indicates center of gravity eccentricity. The components on the two axial planes xz and yz have the following values, respectively

$$\begin{aligned} F_{ix} &= F_i \cos(\Omega t + \alpha_i) \\ F_{iy} &= F_i \sin(\Omega t + \alpha_i) \end{aligned} \tag{2.1}$$

(b) the moment, due to the dynamic unbalance M_i, of components on the two axial planes considered

$$\begin{aligned} M_{ix} &= F_i \cos(\Omega t + \beta_i) \\ M_{iy} &= F_i \sin(\Omega t + \beta_i) \end{aligned} \tag{2.2}$$

(c) the gyrostatic moment, due to the precession motion of the disk resulting from the rotations ϑ_i and $varphi_i$ (see Fig.3) of the rotor sections of components

$$\begin{aligned} \tau_{ix} &= I_{pi} \Omega \dot{\varphi} \\ \tau_{iy} &= I_{pi} \Omega \dot{\vartheta} \end{aligned} \tag{2.3}$$

where I_{pi} indicates the polar mass moment of inertia of the ith disk in the discrete model.

For the sake of simplicity it is initially supposed that only the ith disk is mounted on the rotor (see Fig.4). The forced motion equations of the system,

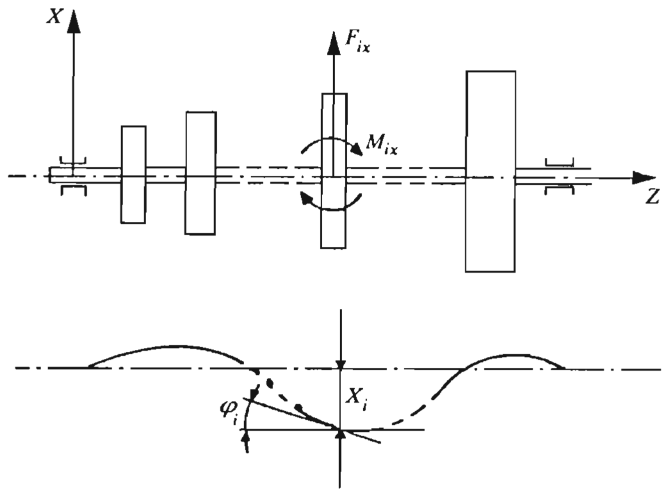

Fig. 3.

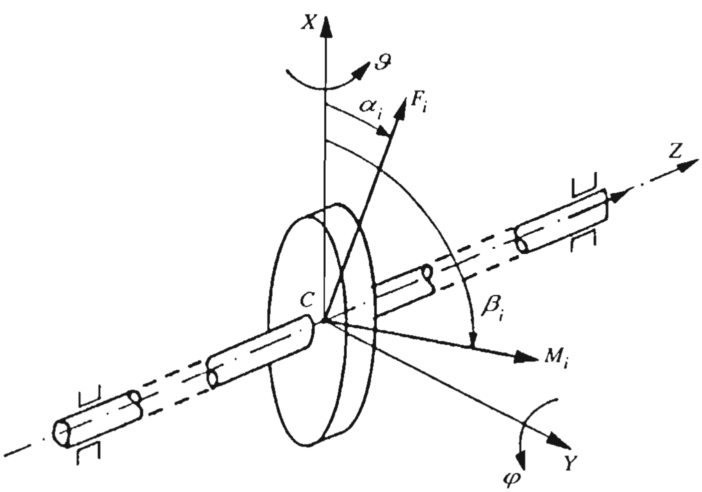

Fig. 4.

revolving at a speed Ω, can thus be written

$$\begin{aligned} m_i\ddot{x}_i + k_{xx}x_i + k_{x\varphi}\varphi_i &= F_{ix} \\ I_{di}\ddot{\varphi} + k_{\varphi x}x_i + k_{\varphi\varphi}\varphi_i &= M_{iy} - I_{pi}\Omega\dot{\vartheta}_i \\ m_i\ddot{y}_i + k_{yy}y_i + k_{y\vartheta}\vartheta_i &= F_{iy} \\ I_{di}\ddot{\vartheta} + k_{\vartheta y}y_i + k_{\vartheta\vartheta}\vartheta_i &= M_{ix} - I_{pi}\Omega\dot{\varphi}_i \end{aligned} \tag{2.4}$$

where I_{di} is the diametral mass moment of inertia of the ith disk in the discrete model.

In Eqs (2.4) the damping coefficients have cleary been omitted since, as already stated above, the rotational speed Ω (and thus the frequency of the forcing actions) is sufficiently far from the frequencies of the vibration modes, between which it is included. Under these particular conditions, the damping coefficients (which in any case give rise to small reactions compared to the other forces involved) do not significantly influence either the amplitudes or forced motion phases of the system.

Eqs (2.4) can be written in the matrix form

$$\begin{bmatrix} m_i & 0 & 0 & 0 \\ 0 & I_{di} & 0 & 0 \\ 0 & 0 & m_i & 0 \\ 0 & 0 & 0 & I_{di} \end{bmatrix} \begin{bmatrix} \ddot{x}_i \\ \ddot{\varphi}_i \\ \ddot{y}_i \\ \ddot{\vartheta}_i \end{bmatrix} + \begin{bmatrix} k_{xx} & k_{x\varphi} & 0 & 0 \\ k_{\varphi x} & k_{\varphi\varphi} & 0 & 0 \\ 0 & 0 & k_{yy} & k_{y\vartheta} \\ 0 & 0 & k_{\vartheta y} & k_{\vartheta\vartheta} \end{bmatrix} \begin{bmatrix} x_i \\ \varphi_i \\ y_i \\ \vartheta_i \end{bmatrix} =$$

$$= \begin{bmatrix} F_{ix} \\ M_{iy} \\ F_{iy} \\ M_{ix} \end{bmatrix} + \begin{bmatrix} 0 \\ I_{pi}\Omega\dot{\vartheta}_i \\ 0 \\ -I_{pi}\Omega\dot{\varphi}_i \end{bmatrix}$$

or in symbolic form

$$\mathbf{M}\ddot{\boldsymbol{x}} + \mathbf{K}\boldsymbol{x} = \boldsymbol{f}_i + \boldsymbol{\tau}_i = (\boldsymbol{f} + \boldsymbol{\tau})_i \tag{2.5}$$

In Eqs (2.4) the gyrostatic moments appear on the right hand side as they can be calculated from the elastic curve reading taken on the rotor revolving at the test speed Ω. The rotor is, in fact, subject to the action of a force F_i and a moment $M = M_i + G_i$ and therefore, if its elastic curves are measured at any two sections, in relation to each of the two axial planes xz and yz during a complete revolution, $x_i(t)$, $\varphi_i(t)$ and $y_i(t)$, $\vartheta_i(t)$ can be calculated. These then make it possible to obtain $\dot{\vartheta}_i(t)$ and $\dot{\varphi}_i(t)$ in a sufficiently approximated way.

The disk motion of each of the two axial planes considered can thus be seen as a forced motion whose forcing actions are due to the unbalances expressed by Eqs (2.1) and (2.2) and to the gyrostatic moments expressed by Eq (2.3). The latter, as already stated above, can be determined with a sufficient degree of accuracy.

The above can be easily extended to a discrete system of n disks, for which the (forced) motion equations can again be expressed, in symbolic matrix form, by Eqs (2.4).

The generalized displacements vector $\boldsymbol{x}$ of the n rotor sections (in which the discrete model's disks (Fig.2) are laid out) and the gyroscopic moments vector $\boldsymbol{\tau}$ can be determined using $2n$ transducers laid out in each of the axial planes xz and yz. The n forcing actions have the same meanings as the ones represented by Eqs (2.1), (2.2) and (2.3).

Making the system discrete enables calculation of the modal matrix $\mathbf{u}$ and the system natural frequencies vector ω_j.

The generalized displacements vector $\boldsymbol{x}$ of the n rotor sections can then be expressed, by means of the expansion theorem, by

$$\boldsymbol{x} = \mathbf{u}\boldsymbol{\eta} \tag{2.6}$$

where $\boldsymbol{\eta}$ represents the normal coordinates vector as functions of time.

If the modal matrix has been normalized with the condition

$$\mathbf{u}^\top \mathbf{m}\mathbf{u} = \mathbf{1} \tag{2.7}$$

we will have

$$[\omega_j - \Omega]\boldsymbol{\eta} = \mathbf{u}^\top(\boldsymbol{f} + \boldsymbol{\tau})_i \tag{2.8}$$

Eq (2.8) makes it possible to derive the forcing actions vector

$$(\boldsymbol{f} + \boldsymbol{\tau})_i = [\mathbf{u}^\top]^{-1}[\omega_j - \Omega]\boldsymbol{\eta} \tag{2.9}$$

Substituting for the vector $\boldsymbol{\eta}$ and the matrix $[\mathbf{u}^\top]^{-1}$ the quantities obtained from Eqs (2.6) and (2.7), respectively, we get

$$(\boldsymbol{f} + \boldsymbol{\tau})_i = \mathbf{m}\mathbf{u}[\omega_j - \Omega]^2\mathbf{u}^{-1}\boldsymbol{x} \tag{2.10}$$

Eqs (2.10) make it possible to see that if rotor deformations measured at a time t^* in the two axial planes mentioned above are available, these can be used to calculate the components on these planes of the forcing actions at this time (or at the angular position $\mu^* = \Omega t^*$). As the reading of the rotor deformations during a complete revolution of the rotor allows the gyrostatic moments vector $\boldsymbol{\tau}$ to be determined, it is finally possible to determine amplitudes and phases compared to μ^* of the unbalance at each of the n sections of the discrete model

$$\boldsymbol{f}_i = (\boldsymbol{f} + \boldsymbol{\tau})_i - \boldsymbol{\tau}_i \tag{2.11}$$

which can thus be corrected with $2n$ masses arranged two by two on the disks.

In conclusion, therefore, the proposed method not only allows (in principle) a flexible rotor to be balanced *with a single test run*, but also presents a particular feature that is conceptually worth examining. The balancing masses that are determined using this method can eliminate the forces and the torques due to the unbalance in each of the rotor sections considered so that, for each of these sections, the rotation axis is the center of inertia. And this is different from other methods by means of which the system of balancing masses that is determined has the effect of simply eliminating (or minimizing) the rotor dynamic deformations.

Finally, it should be pointed out that in the case of an axial symmetrical system (for which the orbits described by the disk centers are approximately circumferences) rather than $2n$ transducers in each of the two axial planes it is sufficient to have $2n$ transducers in a single axial plane and a phase indicator.

3. Experimental results

In light of the above, it should be worth examining the results of a first experimental investigation, which was mentioned by della Valle and Rossi (1990).

The aim of this investigation was to check the conceptual validity of the method and so the information collected was predominantly qualitative.

For this purpose a relatively simple test apparatus was designed and built so as to make it possible to test the proposed balancing method in its simplest form.

3.1. Test apparatus and procedure

Fig.5 shows the experimental apparatus used.

The three identical steel disks have a mass of 1.6 kg and their mass moment of inertia with respect to the rotation axis I_p is approximately the same as the diametral one I_d. Eight holes were made in each of the disks (having an axis parallel to the rotation axis and equally distributed over a circumference of about 50 mm in diameter) in order to make it possible to fasten the balancing masses.

The elastic curves were detected using parasite current transducers (KAMAN KO 2300). In order to avoid measurement errors that are typically

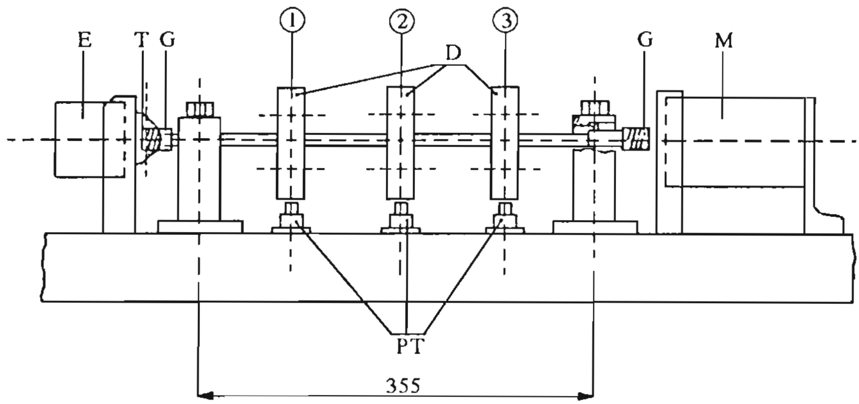

Fig. 5. Experimental test rig scheme; D – disks, E – encoder, G – elastic joints, M – D.C. motor, T – trigger, PT – proximity transducers, 1,2,3 – measurement planes

made when this kind of transducer is placed on a ferromagnetic surface (electrical runout), on the external surface of each one of the disks was clamped an aluminium ring.

The transducer signals were collected by a wave form analyzes (DATA 6000 mod. 611) with signal sampling controlled with signal from an encoder. The trigger and encoder acted as a phase indicator, thus making it possible to measure the amplitudes and the phases of the elastic curve, even although there was only one transducer in each measurement plane.

Since for each disk $I_p = I_d$ the disk effect was not taken into account in this phase in determining the rotor modal parameters and the dynamic unbalances of each disk were also ignored.

An experimentally determined flexibility coefficients matrix $\boldsymbol{\alpha}$ was used to calculate the modal matrix $\mathbf{u}$ and the natural frequencies ω_j of the system's flexional motion.

The next step was to analyze the frequency of system response to an impulse: Fig.6 shows the result of this analysis. Table 1 shows the values of the natural frequencies ω_j, calculated and measured experimentally for the three modes considered.

Table 1

	Calculated*	Measured
ω_1 [rad/s]	183.6	180.6
ω_2 [rad/s]	573.5	559.3
ω_3 [rad/s]	1160.3	1162.4

* before discretization

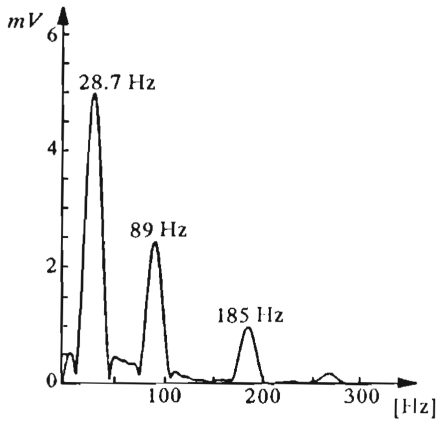

Fig. 6. Frequency analysis of free motion signal

Finally, it should be noted that in the experimental system, imperfections due to manufacture and assembly led to the external surface of the disks being eccentric to the rotation axis. Therefore, in order to measure the system's "effective" elastic curve at a certain speed, the elastic curve recorded when the rotor rotated at a speed of a few rad/s was subtracted, at each point, from the elastic curve recorded at the speed. The latter reading thus represents, with a fair degree of approximation, the geometric deformation of the system.

The elastic curve obtained (with a good degree of repeatability) as described above, was then used to calculate the balancing masses to be applied on the three disks in implementing the proposed method in its simplest form.

3.2. Test results

Fig.7a shows the elastic curves recorded in the three measurement planes at a speed of 3900 rpm. This single recording made it possible calculate the

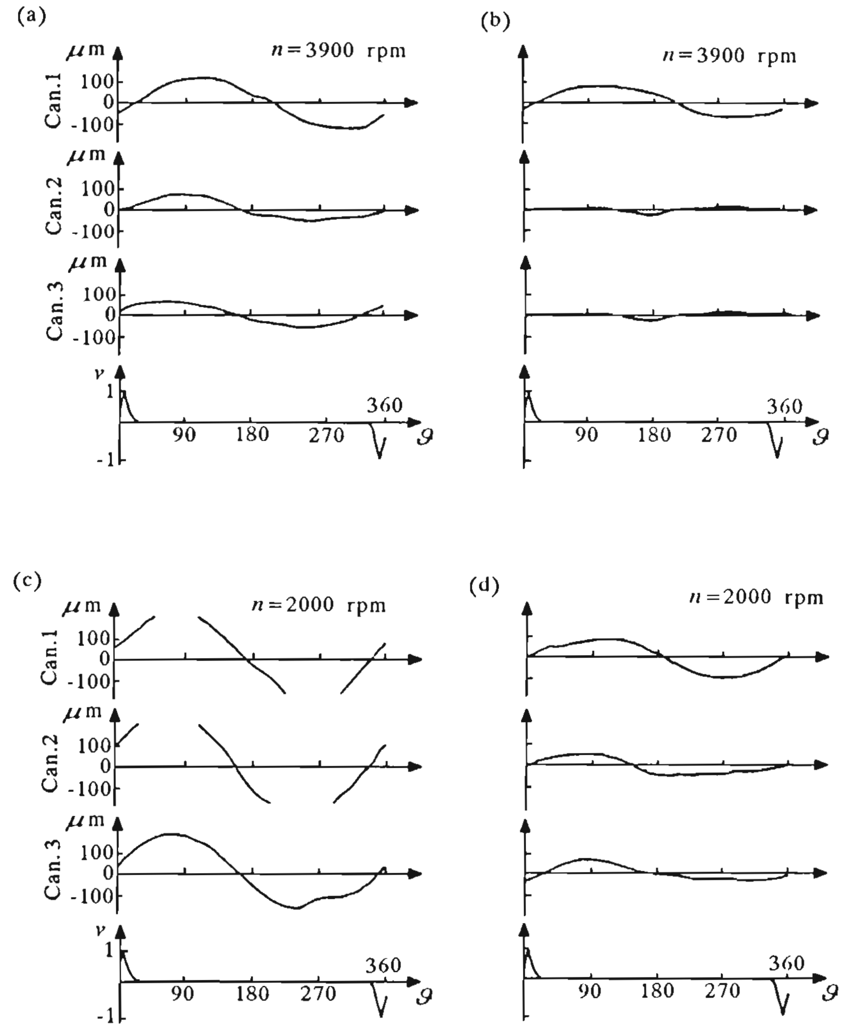

Fig. 7. (a), (c) – before balancing; (b), (d) – after balancing

balancing masses

$$m_1 = 11.95\text{g} \qquad \varphi_1 = 234.9^\circ$$
$$m_2 = 18.84\text{g} \qquad \varphi_2 = 257.1^\circ$$
$$m_3 = 5.52\text{g} \qquad \varphi_3 = 246.9^\circ$$

Fig.7b shows the elastic curves recorded at the same speed after the above masses were applied to the rotor.

Fig.7c and Fig.7d show the elastic curves at 2000 rpm before and after balancing was performed with the above masses, respectively.

Finally, Table 2 shows, for each measurement plane, the peak-to-peak amplitudes of the transducer signals referring to the elastic curves shown in Fig.7.

Table 2

Plane	$n = 3900$ rpm		$n = 2000$ rpm	
	A	B	A	B
1	257.7	162.7	517.3	195.3
2	140.7	49.3	149.3	103.7
3	140	31	373	114

Peak-to-peak oscilation amplitudes in the measurement planes [μm]
A – before balancing
B – after balancing.

Table 2 and Fig.7 cleary show that the balancing masses, calculated with a single measurement of the elastic curve at a single test speed, make it possible significantly contain the amplitudes of the elastic curves in a wide range of speeds.

4. Conclusions

A flexible rotor balancing method has been proposed which enables gleat advantages to be gained both on th e conceptual and the operative levels, compared to other more commonly used methods.

The method is analytical-experimental in nature and entails first making the rotor to be balanced discrete using a model with n masses and determining its eigenvalues and eigenvectors. The rotor system elastic curve is then recorded at a speed between two flexional critical speeds, using $4n$ transducers (or $2n$ if the rotor is axial-symmetrical).

The recording, which is performed for the time it takes the rotor to complete one revolution, makes it possible to determine generalized displacements and speeds of the rotor sections corresponding to the discrete model nodes. These can then be used, applying the proposed relations, to determine the forces and torques due to unbalance that act the above rotor sections.

The particular feature of the proposed method thus lies in its being able to make the balancing, in principle, with a single test run and in its enabling the determination, and thus the elimination, of the forces and torques due to unbalance in each of the sections considered.

Results are the given for the first experimental investigation performed on a relatively simple test apparatus. This was comprised of an axial-symmetrical rotoe on rigid bearings made discrete with three degrees of freedom and enabled the detection of the elastic curve by measuring displacements in only three planes.

In this way the method was applied in its simplest form, i.e. without taking into account the torques due to unbalancing or gyrostatic moments. The latter were certainly negligible as, for each of the rotor disks, I_p, I_d.

Despite the simplicity of the test apparatus, the experimental results were extremaly encouraging. They seem to show that, in qualitative terms, the elastic curve recorded at a single test speed makes it possible to determine a group of balancing masses that is such as to bring about a significant reduction in the amplitude of the rotor vibrations in a wide range of rotational speeds.

Note that the method "structure" is such that its accuracy is dependent both upon the correct discretization of the system and upon the accuracy and the repeatability of the elastic curve measurement. Because of this, the method would not seen to be easily applicable for "on-site" balancing of rotors moment on oil-film bearings but could profitably be used for rapid "in factory" balancing of rotors mounted on "rigid" bearings.

Finally, it should be noted that, as a result of the inevitable errors regarding discretization and experimental readings, in many cases it may be worth performing a number of test runs, at speed that are in any case far from the system critical speeds, in order to determine the balancing masses with aid of optimization criteria.

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Niekonwencjonalna metoda wyważania podatnych wirników

Streszczenie

Zaproponowano pewną analityczno-doświadczalną metodę wyważania podatnych *wirników oraz przedstawiono wyniki doświadczeń. Wykorzystując metodę* możliwe jest, zasadniczo, wyważenie podatnego wirnika podczas zaledwie jednego testu określającego masę która eliminuje niewyważenie w każdej badanej części wirnika. Pierwsze wyniki doświadczeń przeprowadzonych na stanowisku laboratoryjnym potwierdzają możliwość osiągnięcia właściwego wyważenia wirnika w szerokim zakresie prędkości obrotowych.

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