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Research Paper

The Diameter and Maximum Link of the Minimum Routing Cost Spanning Tree Problem

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Abstract
The minimum routing cost spanning tree (MRCST) is a spanning tree that minimizes the sum of pairwise distances between its vertices given a weighted graph. In this study, we use Campos Algorithm with slight modifications on the coefficient of spanning potential. Those algorithms were implemented on a random table problem data of complete graphs of order 10 to 100 in increments of 10. The goal is to find the diameter (the largest shortest path distance) and the maximum link (the maximum number of edges connecting two vertices) in the spanning tree solution of MRCST. The result shows that a slight modification of the spanning potential coefficients gives better solutions.

Keywords
Diameter, Cost Routing, Campos Algorithm, Spanning Potential, Maximum Link

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1. INTRODUCTION

One of the mathematical fields with specific birth dates is graph theory (Vasudev, 2006). Graph theory is a mathematical discipline that is used to represent discrete objects and the connections between these objects. A vertex can represent an object, while an edge can represent the relationship between objects. The graph theory concept was first initiated by Leonard Euler in 1736 when he gave a solution to The Konigsberg bridge problem in Kaliningrad, Russia. In the city of Konigsberg, there is the Pregel river which splits the city into four separate landmasses, and there are seven bridges that connect the landmasses as shown below:

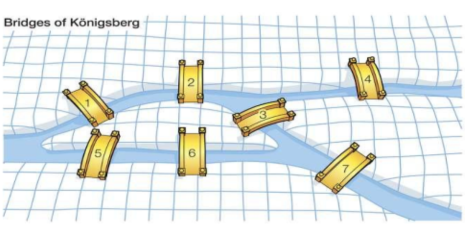

Figure 1. Illustration of Konigsberg Problem

Source: https://www.britannica.com/science/Konigsberg-bridge-problem

The issue is that the people want to start from one of the landmasses, cross each bridge precisely once, and then return to the beginning. By representing the landmasses as nodes (vertices) and bridges as lines (edges), Euler stated that this was impossible because the number of bridges connecting each landmass was odd. This problem is only possible if the number of bridges connecting each land is even. Later, this model using vertices and edges representing lands and bridges became the background for the emergence of the current concept of graph theory. There is a rapid development of graph theory after the solution given by Euler because of its flexibility so that graph theory can be used to represent daily life problems.

There are numerous applications of graph theory. For instance, in network-design problems such as transportation, power supply, water resource management, communication, and many others. In network design problems, some theoretical graph concepts are frequently used, such as Shortest Path (SP), Minimum Spanning Tree (MST), and others.

Among the most popular concepts in graph theory is MST which is commonly used as a backbone in network design problems and has been extensively studied. Kruskal (1956) introduced Kruskal's algorithms, and Prim (1957) introduced Prim's algorithm to solve MST. Those two algorithms are used extensively and are very famous. However, the first algorithm to solve the MST was proposed by Borůvka (1926) when he solved the problem of constructing Moravia's power network in the Czech Republic. Some fast algorithms have been proposed to solve MST due to its unique structure and application in numerous network design issues. The MST problem is commonly encountered in applications for network design where other graph parameters like distance, degree, diameter, flow, connectivity, and others must be satisfied. A distance constraint on commodity flow, for example, could represent the maximum delivery distance in a transportation network. The Degree Constrained Minimum Spanning Tree (DCMST) is the problem of determining an MST whilst having to satisfy the degree restriction on each vertex. If, in addition to the degree constraint, there are additional constraints, namely the period, then the problem becomes a multi-stage network installation problem or Multi-Period Degree-Constrained Minimum Spanning Tree (MPDCMST) problem. In DCMST, all vertices can be connected without period restriction, while in MPDCMST there is a period restriction in connecting the vertices. The period restriction occurs because of some reasons, such as weather conditions, fund limitations, and so on. This problem was investigated by Kawatra (2002) for implementation in a digraph, while Wamiliana et al. (2015b); Wamiliana et al. (2015a); Wamiliana et al. (2020) implemented it in an undirected graph.

Given a connected weighted graph, the routing cost of the spanning tree is defined as the sum of the total path lengths of all pairs of vertices in the spanning tree of that graph. The MRCST aims to find the lowest routing cost of the spanning tree. The MRCST is also known as the shortest average distance-spanning tree and in an unweighted graph is called the minimum Wiener index.

Even though not as famous as traveling salesman problems where many researchers have been involved in that topic, some researchers have already investigated the MRCST problem, including Wu (2002); Wolf and Merz (2010); Chen et al. (2013); Tan and Due (2013); Lin et al. (2006) and Sattari and Didehvar (2013). Since the MRCST is an NP-hard problem, then heuristic algorithms are investigated more, for example, Singh (2008) and Tan (2012b). Singh and Sundar (2011) investigated a bee colony algorithm, and Hieu et al. (2011) proposed an ant colony algorithm. Genetic algorithm for solving the MRCST problem has been investigated by Tan (2012a); Julstrom (2001) and Julstrom (2005). Julstrom (2005) also coded the tree using Blob code and showed that representing the tree using Blob code in genetic algorithms performed better than coding the tree as an edge-set which was proposed by Raidl and Julstrom (2003). Sattari and Didehvar (2015) proposed a GRASP with a metaheuristic path-relinking algorithm to solve MRCST. Fischetti et al. (2002) showed that aside from network design, trees with low routing costs have found useful applications in biological computation, where they are applicable for finding good genomic sequence alignments. A heuristic that is based on the recognition of a network core around which it is possible to construct a solution was proposed by Masone et al. (2019). In this study, we will discuss Campos's algorithm for MRCST because Campos's algorithm usually was used as a comparison for developed algorithms in the literature. In the next Section, we will discuss the MRCST.

2. THE PROBLEM

Given an undirected weighted connected graph $G(V,E)$, V denotes the set of vertices. $V= v_1, v_2, v_3, \cdots, v_n$, $V\neq\varnothing$, and E denote the collection of edges that connect the vertices in V, E={$e_{uv}|(v_u,v_v)\in V$}, and for each edge e_{uv} there is a corresponding weight $c_{uv}\geq 0$, A Minimum Routing Cost Tree (MRCT) T represents a spanning tree in such a way that for all spanning trees T that can be computed from G, C_r(T*)= min $(\sum_{u=1}^{n}\sum_{v=1}^{n} c_T(u,v)$, $u\neq v)$, where $c_T(u,v)$ is the cost of vertex u and vertex v,T* is the spanning tree whose the minimal cost routing among other spanning trees in G Campos and Ricardo (2008). C_r(T) is the total cost of the shortest path for every pair of vertices in T where the shortest path is counted twice for every pair of vertices, once from vertex u to v, and once from v to u (routing). Note that vertex u and v may be connected by a path, not by an edge. By defining $d(u,v)$ as the distance of every pair (u,v) vertices in G, an MRCST can be described as a problem of determining the minimum total distance $d(u,v)$ for each pair of vertices (u,v) in spanning tree T of G so that C_r(T*)= $(\sum_{u=1}^{n}\sum_{v=1}^{n} d_{uv}$, $u\neq v)$, where T* is the spanning tree in G which produce the smallest total distance for each pair of vertices. In MRCST both $d(u,v)$ and $d(v,u)$ are take into account. To illustrate the problem, suppose that we are given a spanning tree of an undirected connected weighted graph as in Figure 2.

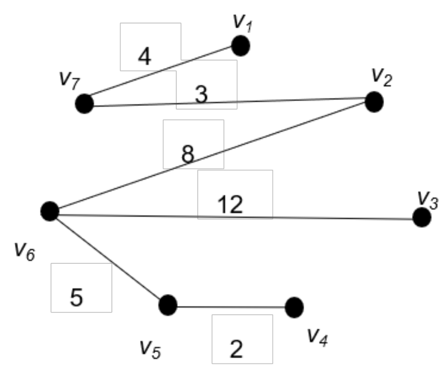

Figure 2. Spanning Tree T

To find the total distance in a graph in Figure 2, we calculate the distance for every pair of vertices in that spanning tree. Let $d(u,v)$ as the distance of vertex u and v, then $d(v_1,v_2)$= 7, $d(v_1,v_3)$= 27, $d(v_1, v_4)$= 22, $d(v_1,v_5)$= 20, $d(v_1,v_6)$= 15, $d(v_1,v_7)$= 4, $d(v_2,v_3)$= 20, $d(v_2,v_4)$= 15, $d(v_2,v_5)$= 13, $d(v_2,v_6)$= 8, $d(v_2,v_7)$= 3, $d(v_3,v_4)$= 19, $d(v_3,v_5)$= 17, $d(v_3,v_6)$= 12, $d(v_3,v_7)$= 23, $d(v_4,v_5)$ = 2, $d(v_4,v_6)$= 7, $d(v_4,v_7)$= 18, $d(v_5,v_6)$= 5, $d(v_5,v_7)$= 16, $d(v_6,v_7)$= 11.

Since in MRCST both $d(u,v)$ and $d(v,u)$ are under consideration, then the value of MRCST of the graph in example 1 is $2\times(7+27+22+20+15+4+20+15+13+8+3+19+17+12+23+2+7+18+5+16+11) = 2\times284 = 568$

A graph's diameter is defined as the largest shortest path distance in the graph. It is, in other words, diameter is the maximum value of overall pairs, which denotes the shortest path distance from a vertex to another vertex. A maximal link is the number of the maximal number of edges that connect a pair of vertices in a graph. For example, in Figure 2, the diameter of that spanning tree T is 27 which is the distance between vertex v_1 and v_3, while the maximal link of that spanning tree is 5 which is the number of edges that connect vertex v_1 and v_4. For other pairs of vertices in that tree, the number of edges connecting them is less than 5. Note that in a weighted spanning tree the diameter and the link of a spanning tree may not be the same, however, in an unweighted graph the diameter and the link are similar which is the maximal number of edges that connect a pair of vertices in the graph.

3. CAMPOS' ALGORITHM

Campos' algorithm begins by choosing an initial vertex with the criteria that the vertex has the largest spanning potential which is defined as: $S_{pv} = C_1\, d_v + C_2\, \frac{d_v}{s_v} + C_3\, \frac{1}{m_v}$, where d_v is the degree of vertex v, s_v is the sum of the weight edges that incidence to v, and m_v is the maximum weight of incidence edges, and C_1, C_2, and C_3 are coefficients. Suppose v is the vertex with the largest spanning potential, then calculate w_v, all the weight of incidence edges to v, pd_v as the degree of the candidate parent vertex in T, ps_v as the sum of the adjacent edge weights of the candidate parent vertex in T, cf_v as cost estimation for the path between vertex v and f in T. Next, calculate parameters: wd_v, jsp_v, sd_v and sw_v as follow:

$wd_v = C_4\, w_v + C_5\, cf_v$ (set $C_4 = C_5 = 1$), jsp_v: $sd_v + \frac{sd_v}{sw_v}$, $sd_v = d_v + pd_v$ and $sw_v = s_v + ps_v$.

Next step is make list L = $\{w_v, d_v, s_v, p_v, pd_v, ps_v, cf_v, wd_v, jsp_v\}$, and put in v to L.

L= $\{v, w_v, d_v, s_v, p_v, pd_v, ps_v, cf_v, wd_v, jsp_v\}$. Choose the highest value of wd_v and jsp_v. If there is no value of wd_v and jsp_v, remove all $w_v, d_v, s_v, p_v, pd_v, ps_v, cf_v, wd_v$ which relate to v and choose another vertex adjacent to v, and calculate again the parameter and continue the process as before. If wdv and jspv have values, then put v as the initial vertex and the smallest adjacent edge to v put into T. Remove all parameters related to the current chosen edge and vertex. Next, choose the smallest edge incidence with vertices in T and check if the addition of that edge in T constitutes a cycle. If yes, remove that edge, and choose the next smallest. If not, check if the spanning tree has already been obtained (|T| = n-1). If yes, stop, spanning tree T is obtained, otherwise, put in the chosen edge in L and repeat the process.

4. RESULT AND DISCUSSSION

We run Campos's algorithm and slight modifications on the coefficient of spanning potential of Campos' Algorithm on the data set problems which are complete weighted graphs of order 10 to 100 in increments of 10. The edge weights are integers generated at random from a uniform distribution (1, 1000), and 30 random problems are generated for each order. The algorithm is implemented using three different coefficients of spanning potential. Based on the simulation, we find that the value of MRCST is smaller if the value of C_2 is between 0.8 and 0.89. If the value of C_2 is smaller than 0.8 or bigger than 0.89, the value of MRCST is bigger than the value of C_2 recommended by Campos and Ricardo (2008) which is 0.6. Based on implementation we get the following result:

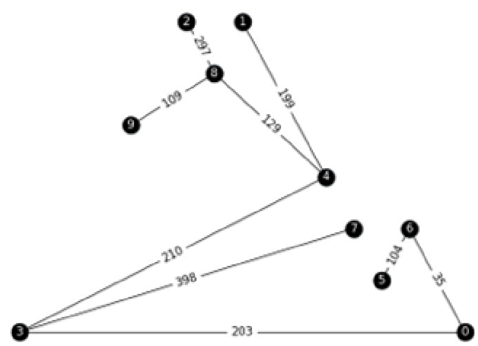

Figure 3. Spanning Tree of n=10 for Problem Dat.10 using $C_1 = 0.2$, $C_2 = 0.6$ and $C_3 = 0.2$, and the Value of MRCST is 22,745, with Maximal Link 6 and Diameter 1034

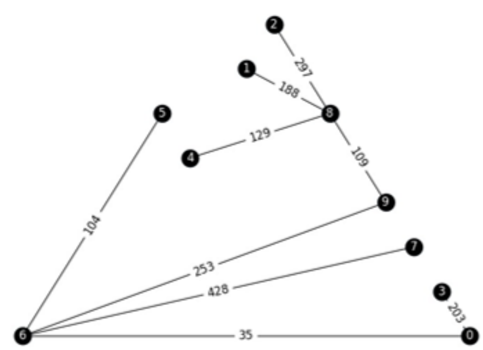

Figure 4. Spanning Tree of n=10 for Problem Dat.10 using $C_1 = 0.1$, $C_2 = 0.8$ and $C_3 = 0.1$, and the Value of MRCST is 21,642, with Maximal Link 5 and Diameter 1087

The coefficient of $C_1 = 0.2$, $C_2 = 0.6$, and $C_3 = 0.2$ are suggested as the best coefficients by Campos and Ricardo (2008), $C_1 = 0.01$, $C_2 = 0.8$, $C_3 = 0.1$ and $C_1 = 0.1$, $C_2 = 0.89$, and $C_3 = 0.2$ are two combinations of coefficients to determine the value of spanning potential. From the simulation, we found that those two combinations are the best. From the result, we found that the highest number of edges connecting a pair of vertices is the same for all three variations. The smallest number occurs

Table 1. The Results of Implementation on Complete Graphs with Order 10 to 100 in Increments of 10

n	C_1=0.2, C_2= 0.6, C_3= 0.2				C_1= 0.02, C_2= 0.89, C_3= 0.1				C_1= 0.1, C_2= 0.8, C_3= 0.1			
	The average value of MRCT	The max link	The largest diameter	The average of the largest diameter	The average value of MRCT	The max link	The largest diameter	The average of the largest diameter	The average value of MRCT	The max link	The largest diameter	The average of the largest diameter
10	15,228.7	8	1429	770.8	15,179.3	8	1429	771.27	15,179.3	8	1429	771.27
20	44,262.4	12	799	551.1	44,119.13	11	799	542.2	44,119.13	11	799	542.2
30	75,626.03	14	573	429.5	75,581.5	14	573	429.5	75,581.5	14	573	429.5
40	113,285.2	17	478	354.97	113,285.2	17	478	354.97	113,285.2	17	478	354.97
50	167,045.8	16	427	318.133	166,982.1	16	427	319.23	166,982.07	16	427	319.23
60	200,377.4	21	364	274.67	200,381.4	21	364	275.2	200,381.4	21	364	275.2
70	241,589.7	17	332	243.3	239,946	17	332	242.5	239,946	17	332	242.5
80	292,620.7	20	299	221.37	293,996.5	20	299	220.8	293,996.53	20	299	220.8
90	336,946.3	20	293	198.8	336,731.2	20	293	198.8	336,731.2	20	293	293
100	395,334.4	20	261	191.9	394,547.6	20	261	191.9	394,547.57	20	261	191.9

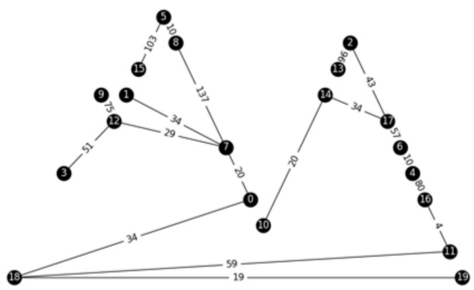

Figure 5. Spanning Tree of n=20 for Problem Dat.3 using C_1= 0.2, C_2= 0.6 and C_3= 0.2, and the Value of MRCST is 43,066, with Maximal Link 12 and Diameter 653

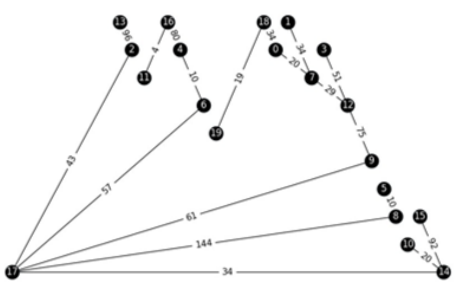

Figure 6. Spanning Tree of n=20 for Problem Dat.3 using C_1= 0.1, C_2= 0.8 and C_3= 0.1, and the Value of MRCST is 38,798, with Maximal Link 10 and Diameter 392

in order 10 with the maximum number of edges connecting two vertices in the solution (spanning tree) being 8, while the highest number of edges occurs in order 21. Note that in the table we present the average value for MRCT (the value of MRCT for 30 problems in every order and taking the average) and the average value of the average of the largest diameter. The largest diameter recorded in Table 1 is the largest diameter of the 30 problems in every graph order, and it is also similar to a maximal link which is taken from the maximal link of the 30 problems in every graph order. Figure 3 to Figure 6 above shows the visualization of examples of the solutions for vertex orders 10 and 20. Due to the space limitation, the visualization for higher-order graphs is not given.

Figure 3 and Figure 4 are the two spanning trees obtained by implementing Campos' Algorithm with different combinations of values of coefficients spanning potential on a graph of order 10. With C_1= 0.2, C_2= 0.6, and C_3= 0.2 in Figure 3, the graph shows that the maximal link is 6 and diameter is 1034 and MRCST value is 22,745, while with C_1= 0.1, C_2= 0.8, C_3= 0.1 in Figure 4 the graph shows that the maximal link is 5 and diameter is 1087 and MRCST value is 21,642. Figures 5 and 6 show the result of implementation on order 20. With C_1= 0.2, C_2= 0.6, and C_3= 0.2 the graph shows that the maximal link is 12 and diameter is 653 and MRCST value is 43,066, while with C_1= 0.1, C_2= 0.8, C_3= 0.1 the graph shows that the maximal link is 10, diameter is 392 and MRCST value is 38,798. Moreover, the diameter (the largest shortest path distance) does not always occur in the maximal link. In Figure 3 for example, the maximal link is 6 which is the path that connects vertices 5–6–0–3–4–8–2 with a distance of 978, while the diameter is 1034 which is the path that connects vertices 2–8–4–3–7.

5. CONCLUSION

We can conclude from the preceding discussion that implemented on a complete weighted connected graph, the edges that form the diameter of the spanning tree may not be the same in the maximal links. Moreover, the combination of the coefficients of the spanning potential (C_2= 0.8 or C_2= 0.89) performed slightly better than the value of the coefficient suggested by Campos and Ricardo (2008). From the results, we found that the highest number of edges connecting a pair of vertices is the same for all three variations. The smallest number occurs in order 10 with the maximum number of edges connecting two vertices in the spanning tree being 8, while the highest number of edges occurs in order 60 which is 21.

6. ACKNOWLEDGMENT

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