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Determining The Number of Connected Vertex Labeled Graphs of Order Seven without Loops by Observing The Patterns of Formula for Lower Order Graphs with Similar Property

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Abstract
Given n vertices and m edges, $m \geq 1$, and for every vertex is given a label, there are lots of graphs that can be obtained. The graphs obtained may be simple or not simple, connected or disconnected. A graph G(V,E) is called simple if G(V,E) not containing loops nor paralel edges. An edge which has the same end vertex is called a loop, and paralel edges are two or more edges which connect the same set of vertices. Let $N(G_{7,m,t})$ as the number of connected vertex labeled graphs of order seven with m vertices and t (t is the number edges that connect different pair of vertices). The result shows that $N(G_{7,m,t})$ = $c_t\ C_{t-1}^{(m-1)}$, with c_6=6727, c_7= 30160 , c_8=30765, c_9=21000, c_{10}=28364, c_{11}=26880, c_{12}=26460, c_{13}=20790, c_{14}=10290, c_{15}= 8022, c_{16}=2940, c_{17}=4417, c_{18}=2835, c_{19}=210, c_{20}= 21, c_{21}=1.

Keywords
Graph, Connected, Vertex, Labeled, Order, Loops

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1. INTRODUCTION

Graph theory emerged as a new field in mathematics in 1736 after Leonhard Euler gave solution to the Konigsberg problem, graph theory was used widely in many real-life applications, especially as problems representation. A Graph G (V,E) is a structure which consists of a set V={$v_1, v_2, \ldots, v_n$} of vertices, where V ≠ ∅, and a set of edges E=e_{ij} | i, j ∈ V which connect the vertices of V. Usually the vertices are used to represent cities, depots, train stations, airports, etc., while edges are usually used to represent roads, train tracks, flight paths, etc. A number $c_{ij} \geq 0$ can be assigned to the edge e_{ij} as a nonformal information which can represent the distance, time, cost, flow, etc. Because the flexibility of how to draw a graph, where there is no restriction in drawing an edge (can be a straight line, a curve, or other line), graph becomes an interesting structure to cope with, especially to represent the problem for easily visualization. Some of graph terminologies that commonly used in application is the concept of tree, where tree is a connected graph without cycle.

Some applications that use graph theoretical concept as problems representation include applications in biology, chemistry, engineering, computer science, economics, agriculture, and others. For example, in biology, a leaf labeled tree was used to represent the evolutionary history of a set of taxa which is called as phylogenetic tree (Huson and Bryant, 2006; Brandes and Cornelsen, 2009), and Mathur and Adlakha (2016) used combined tree to represent DNA, in chemistry/pharmaceutical, Gramatica et al. (2014) used graph concept to describe or represent the possible modes of action for any given pharmacological compound; in engineering and computer science, Hsu and Lin (2009) exposed a lot of graph theoretical concepts including Hamiltonian circuits with relation in network design, Al Etaiwi (2014) in order to generate a complex cipher text used the concepts minimum spanning tree, complete graph and cycle graph, Priyadarsini (2015) investigate the use of graph theory concept, extremal and expander graphs in designing some ciphers, while Ni et al. (2021) use bipartite and corona graphs to create ciphers; in economics, Álvarez and Ehnts (2015) used directed graph to represent the dynamic closures of the accounting structure; in agriculture, Kawakura and Shibasaki (2018) used graph theory concepts to group agricultural workers engaging in manual tasks, Kannimuthu et al. (2020) use graph coloring to optimize farmer's objective, and many more.

In 1857 Cayley enumerated the isomer of C_nH_{2n+2} using the concept of tree (Cayley, 1874), and followed by Slomenski (1964) who used graph theory to calculate additive structural properties of hydrocarbon. Bona (2007) discussed how to enumerate trees and forest. If we are given n vertices and m edges, then lots of graphs can be obtained using that information. The graph obtained may be simple graph which does not contain loop nor parallel edges, or maybe not simple. Moreover, the graph obtained also may be connected or disconnected. For connected vertex labeled graph, the number of graph of order five with maximum number of paralel edges is five without loops was investigated by Wamiliana et al. (2019), and the number of graph of order six without paralel edges with ten loops maximum also investigated by Wamiliana et al. (2020). Puri et al. (2021) investigated the number of graphs of order six with maximum thirty edges without loops. For disconnected vertex labeled graph, Wamiliana et al. (2016) investigated the number of graph of order five without paralel edges, Amanto et al. (2017) gave the formula for graph of order maximal four, Putri et al. (2021) observed and gave formula for the number graphs of order six without loops and may contain maximum twenty parallel edges, while Pertiwi et al. (2021) proposed the formula for counting the number graph of order six without loops, especially when the graph obtained only contains maximum seven loops and the number of non loop edges is even. In this study, by observing the patterns of the formula of the number of connected vertex labeled graphs of order five and order six containing no loops, the formula of graphs of order seven with similar property will be discussed.

We organized this paper as follows: Section I is Introduction that describes about what is graph, some applications of graphs, and some researches related with this study. In Section II Observation and Investigation will be discussed while Result and Discussion is provided in Section III, and Conclusion in given in Section IV.

2. OBSERVATION AND INVESTIGATION

Given a graph G(V,E) where n =|V| = 7 and G is connected. Because G is connected, then the number of edges m = |E| ≥ 6. Every vertex in G is labeled, therefore graphs G_1 and G_2 in Figure 1 are two different graphs even though both graphs look similar.

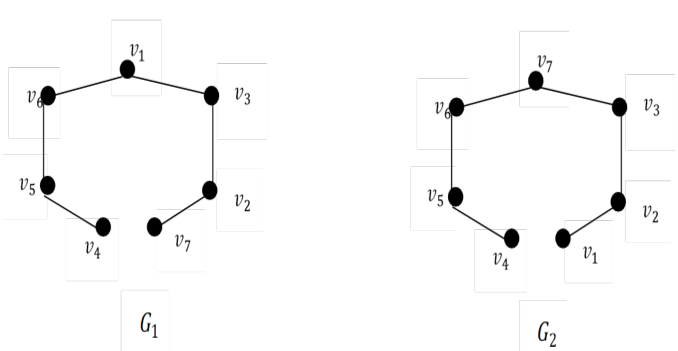


Figure 1. Two Different Graphs that Look The Same but Different because of The Vertex Labeling

Denote N($G_{n,m,t}$) as the number of connected vertex labeled graphs containing no loops of order n, m edges and t, where t is the number of edges that connect different pairs of vertices in G. Edges that connect the same pair of vertices is counted as one. Moreover, isomorphics graphs are counted as one graph.

The result on Table 1 for n=5 and are obtained from Wamiliana et al. (2019), and for n=6 from and Puri et al. (2021). From Table 1 we know that for n = 5, the maximum number of t is 10, and for n= 6 is 15, and since the graph is connected, then m ≥ 4 for n = 5, and m ≥ 5 for n =6. By observing Table 1 we found that there are patterns between those two order graphs. Notice that, for every t, the formula only differ on the coefficients. Let ct is contant with t= 6,7,. . . ,21. By using the patterns on Table 1, we predict that the formula for order seven as $c_t\ C_{t-1}^{(m-1)}$. Note that for order 7, maximum t is 21.

3. RESULTS AND DISCUSSION

Given n=7, t and m, the number of graphs of order seven, connected, and vertex labelled are obtained by: pattern construction, grouping the patterns in term of m and t, and then calculate the graphs. Starting with t=6, we construct for m ≥ 6. The process continue with t=7 until t=21 (maximum possible t for n=7). Table 2 shows some possible patterns for t=n-1.

Note that we do not put all possible patterns here due to space limitation, for example, for t=n-1 and m= n, the paralel edges maybe connect vertex v_1 and v_2 or v_4 and v_5, and so on, and for t=n-1 and m=n+1, that is possible the paralel edges only on one pair of vertices, for example, there are three edges that connect vertex v_1 and v_2, etc. The number of graphs obtained is given in Table 3. By observing the number in every column, Table 3 can be rewrite as in Table 4.

By grouping the graphs by m and t, we notice that every column of Table 4 constitute patterns. Note that in the Table 3 and 4 we are not inputting all the numbers of graph obtained because t is fixed in every column and adding more edges only adding more paralel edges on t, and the pattern continues for the next m, for example: for t= 6, m ≥ 6 we only input the number until m= 12 and the pattern is 1, 6, 21, 56, 126, 252, 462 (if adding more m, the pattern becomes 1, 6, 21, 56, 126, 252, 462, 792, 1287, 2002,. . .).

The sequence of numbers that appear in the first column (t= 6) is 1, 6, 21, 56, 126, 252, 462 and that number is multiplied by 6727. Therefore we can claim that the value of c_6 in Table 3 is 6727.

1		6		21		56		126		252		462
	5		15		35		70		126		210	
		10		20		35		56		84		
			10		15		21		28			
				5		6		7				
					1		1					

Table 1. The Formula of The Number of Connected Vertex Labeled Graph of Order N , N = 5, 6 , M Edges And T, where T is The Number of Edges that Connect Different Pairs of Vertices in Graph, and Containing no Loops

t	n	
	5	6
4	$N(G_{5,m,4}) = 125 \times C_3^{(m-1)}$	
5	$N(G_{5,m,5}) = 222 \times C_4^{(m-1)}$	$N(G_{6,m,5}) = 1296 \times C_4^{(m-1)}$
6	$N(G_{5,m,6}) = 205 \times C_5^{(m-1)}$	$N(G_{6,m,6}) = 1980 \times C_5^{(m-1)}$
7	$N(G_{5,m,7}) = 110 \times C_6^{(m-1)}$	$N(G_{6,m,7}) = 3330 \times C_6^{(m-1)}$
8	$N(G_{5,m,8}) = 45 \times C_7^{(m-1)}$	$N(G_{6,m,8}) = 4620 \times C_7^{(m-1)}$
9	$N(G_{5,m,9}) = 10 \times C_8^{(m-1)}$	$N(G_{6,m,9}) = 6660 \times C_8^{(m-1)}$
10	$N(G_{5,m,10}) = 1 \times C_9^{(m-1)}$	$N(G_{6,m,10}) = 2640 \times C_9^{(m-1)}$
11		$N(G_{6,m,11}) = 1155 \times C_{10}^{(m-1)}$
12		$N(G_{6,m,12}) = 420 \times C_{11}^{(m-1)}$
13		$N(G_{6,m,13}) = 150 \times C_{12}^{(m-1)}$
14		$N(G_{6,m,14}) = 15 \times C_{13}^{(m-1)}$
15		$N(G_{6,m,13}) = 1 \times C_{14}^{(m-1)}$

Table 2. Some Possible Patterns for t=n-1 (n=7)

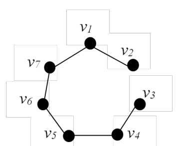
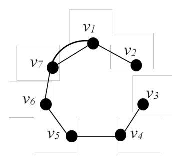
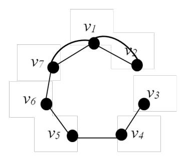
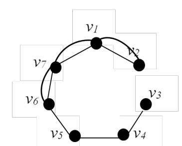
t=n-1, m = n-1	*t=n-1, m = n*	*t=n-1, m=n+1*	*t=n-1, m=n+2*
			

Table 3. Grouping The Number of Connected Vertex Labeled Graph of Order Seven without Loops by m and t

m	The Number of Connected Vertex Labeled Graphs of Order Seven without Loops					
	t					
	6	7	8	9	10	11
6	6727					
7	40362	30160				
8	141267	211120	30765			
9	376712	844480	246120	21000		
10	847602	2533440	1107540	189000	28364	
11	1695204	6333600	3691800	945000	283640	26880
12	3107874	13933920	10152450	3465000	1560020	295680
13		27867840	24365880	10395000	6240080	1774080
14		51754560	52792740	27027000	20280260	7687680
15			105585480	63063000	56784728	26906880
16			197972775	135135000	141961820	80720640
17				270270000	324484160	215255040
18				510510000	689528840	522762240
19					1379057680	1176215040
20					2620209592	2483120640
21						4966241280
22						9481006080

	The Number of Connected Vertex Labeled Graphs of Order Seven without Loops				
			t		
m	12	13	14	15	16
12	26460				
13	317520	20790			
14	2063880	270270	10290		
15	9631440	1891890	144060	8022	
16	36117900	9459450	1080450	120330	5460
17	115577280	37837800	5762400	962640	87360
18	327468960	128648520	24490200	5454960	742560
19	842063040	385945560	88164720	24547320	4455360
20	1999899720	1047566520	279188280	93279816	21162960
21	4444221600	2618916300	797680800	310932720	84651840
22	9332865360	6110804700	2093912100	932798160	296281440
23	18665730720	13443770340	5118451800	2565194940	931170240
24	35775983880	28109701620	11772439140	6555498180	2677114440
25		56219403240	25685321760	15733195632	7138971840
26		108114237000	53511087000	35757262800	17847429600
27			107022174000	77474069400	42184833600
28			206399907000	160907682600	94915875600
29				321815365200	204434193600
30				622176372720	423470829600
31					846941659200
32					1640949464700

	The Number of Connected Vertex Labeled Graphs of Order Seven without Loops				
			t		
m	17	18	19	20	21
17	4417				
18	75089	2835			
19	675801	51030	210		
20	4280073	484785	3990	21	
21	21400365	3231900	39900	420	1
22	89881533	16967475	279300	4410	21
23	329565621	74656890	1536150	32340	231
24	1082858469	286184745	7066290	185955	1771
25	3248575407	981204840	28265160	892584	10626
26	9023820575	3066265125	100947000	3719100	53130
27	23461933495	8858099250	328077750	13813800	230230
28	57588382215	23916867975	984233250	46621575	888030
29	134372891835	60879300300	2755853100	145044900	3108105
30	299754912555	147124975725	7265430900	420630210	10015005
31	642331955475	339519174750	18163577250	1147173300	30045015
32	1327486041315	751792458375	43313145750	2963531025	84672315
33	2654972082630	1603823911200	99001476000	7294845600	225792840
34	5153769336870	3307886816850	217803247200	17194993200	573166440
35		6615773633700	462831900300	38975317920	1391975640
36		12864004287750	952889206500	85258507950	3247943160
37			1905778413000	180547428600	7307872110
38			3711252699000	371125269900	15905368710
39				742250539800	33578000610
40				1447388552610	68923264410
41					137846528820
42					269128937220

Table 4. Another form of Table 3

The Number of Connected Vertex Labeled Graphs of Order Seven without Loops						
	t					
m	6	7	8	9	10	11
6	1 ×6727					
7	6 ×6727	1×30160				
8	21×6727	7×30160	1 ×30765			
9	56×6727	28×30160	8 ×30765	1×21000		
10	126×6727	84×30160	36 ×30765	9 ×21000	1 ×28364	
11	252×6727	210×30160	120 ×30765	45 ×21000	10 ×28364	1× 26880
12	462×6727	462×30160	330 ×30765	165 ×21000	55 ×28364	11 × 26880
13		924×30160	792 ×30765	495 ×21000	220 ×28364	66 × 26880
14		1716×30160	1716 ×30765	1287 ×21000	715 ×28364	286 × 26880
15			3432 ×30765	3003×21000	2002 ×28364	1001 × 26880
16			6435×30765	6435×21000	5005×28364	3003× 26880
17				12870×21000	11440×28364	8008× 26880
18				24310×21000	24310×28364	19448× 26880
19					48620×28364	43758× 26880
20					92378×28364	92378× 26880
21						184756× 26880
22						352716× 26880

The Number of Connected Vertex Labeled Graphs of Order Seven without Loops					
	t				
m	12	13	14	15	16
12	1×26460				
13	12×26460	1×20790			
14	78×26460	13×20790	1×10290		
15	364×26460	91×20790	14×10290	1×8022	
16	1365×26460	455×20790	105×10290	15×8022	1×2940
17	4368×26460	1820×20790	560×10290	120×8022	16×2940
18	12376×26460	6188×20790	2380×10290	680×8022	136×2940
19	31824×26460	18564×20790	8568×10290	3060×8022	816×2940
20	75582×26460	50388×20790	27132×10290	11628×8022	3876×2940
21	167960×26460	125970×20790	77520×10290	38760×8022	15504×2940
22	352716×26460	293930×20790	203490×10290	116280×8022	54264×2940
23	705432×26460	646646×20790	497420×10290	319770×8022	170544×2940
24	1352078×26460	1352078×20790	1144066×10290	817190×8022	490314×2940
25		2704156×20790	2496144×10290	1961256×8022	1307504×2940
26		5200300×20790	5200300×10290	4457400×8022	3268760×2940
27			10400600×10290	9657700×8022	7726160×2940
28			20058300×10290	20058300×8022	17383860×2940
29				40116600×8022	37442160×2940
30				77558760×8022	77558760×2940
31					155117520×2940
32					300540195×2940

Result 1: Given n = 7, $m \geq 6$, t= 6, the number of connected graphs of order seven containing no loops is $N(G_{7,m,6}) = 6727 \times C_5^{(m-1)}$.
Proof:

Look at the sequence of numbers above.
It can be seen that from the sequence above that the fixed difference occur on the fifth level. Therefore the polynomial that can represent this sequence is polynomial of order five:

	The Number of Connected Vertex Labeled Graphs of Order Seven				
	t				
m	17	18	19	20	21
17	1×4417				
18	17×4417	1×2835			
19	153×4417	18×2835	1×210		
20	969×4417	171×2835	19×210	1×21	
21	4845×4417	1140×2835	190×210	20×21	1×1
22	20349×4417	5985×2835	1330×210	210×21	21×1
23	74613×4417	26334×2835	7315×210	1540×21	231×1
24	245157×4417	100947×2835	33649×210	8855×21	1771×1
25	735471×4417	346104×2835	134596×210	42504×21	10626×1
26	2042975×4417	1081575×2835	480700×210	177100×21	53130×1
27	5311735×4417	3124550×2835	1562275×210	657800×21	230230×1
28	13037895×4417	8436285×2835	4686825×210	2220075×21	888030×1
29	30421755×4417	21474180×2835	13123110×210	6906900×21	3108105×1
30	67863915×4417	51895935×2835	34597290×210	20030010×21	10015005×1
31	145422675×4417	119759850×2835	86493225×210	54627300×21	30045015×1
32	300540195×4417	265182525×2835	206253075×210	141120525×21	84672315×1
33	601080390×4417	565722720×2835	471435600×210	347373600×21	225792840×1
34	1166803110×4417	1166803110×2835	1037158320×210	818809200×21	573166440×1
35		2333606220×2835	2203961430×210	1855967520×21	1391975640×1
36		4537567650×2835	4537567650×210	4059928950×21	3247943160×1
37			9075135300×210	8597496600×21	7307872110×1
38			17672631900×210	17672631900×21	15905368710×1
39				35345263800×21	33578000610×1
40				68923264410×21	68923264410×1
41					137846528820×1
42					269128937220×1

$P_5(m) = a_5m^5 + a_4m^4 + a_3m^3 + a_2m^2 + a_1m + a_0$
Substitute m = 6, 7, 8, 9, 10, 11 to the equation we get the following:

$$6727 = 7776a_5 + 1296a_4 + 216a_3 + 36a_2 + 6a_1 + a_0 \quad (1)$$

$$40362 = 16807a_5 + 2401a_4 + 343a_3 + 49a_2 + 7a_1 + a_0 \quad (2)$$

$$141267 = 32768a_5 + 4096a_4 + 512a_3 + 64a_2 + 8a_1 + a_0 \quad (3)$$

$$376712 = 59049a_5 + 6561a_4 + 729a_3 + 81a_2 + 9a_1 + a_0 \quad (4)$$

$$847602 = 100000a_5 + 10000a_4 + 1000a_3 + 100a_2 + 10a_1 + a_0 \quad (5)$$

$$1695204 = 161051a_5 + 14641a_4 + 1331a_3 + 121a_2 + 11a_1 + a_0 \quad (6)$$

Solving this system of linear equations we get $a_5 = \frac{6727}{120}$, $a_4 = -\frac{100905}{120}$, $a_3 = \frac{571795}{120}$, $a_2 = -\frac{1513575}{120}$, $a_1 = \frac{1843198}{120}$ and $a_0 = -\frac{807239}{120}$

$$\begin{aligned} P_5(m) &= a_5m^5 + a_4m^4 + a_3m^3 + a_2m^2 + a_1m + a_0 \\ &= \frac{6727}{120}m^5 - \frac{100905}{120}m^4 + \frac{571795}{120}m^3 - \frac{1513575}{120}m^2 \\ &+ \frac{1843198}{120}m - \frac{807239}{120} \\ &= \frac{6727}{120}(m^5 - 15m^4 + 85m^3 - 225m^2 + 274m - 120) \\ &= \frac{6727}{120}(m-1)(m-2)(m-3)(m-4)(m-5) \\ &= 6727 \times \frac{(m-1)(m-2)(m-3)(m-4)(m-5)}{(5 \times 4 \times 3 \times 2 \times 1)} \\ &= 6727 \times C_5^{(m-1)} \end{aligned} \quad (7)$$

For t=7, we can see from Table 4 that the sequence of numbers is 1, 7, 28, 84, 210, 462, 924, 1716.

1 7 28 84 210 462 924 1716

6 21 56 126 252 462 792

15 35 70 126 210 330

20 35 56 84 120

15 21 28 36

6 7 8

1 1

Result 2: Given n = 7, $m \geq 6$, $t = 7$, the number of connected graphs of order seven containing no loops is N $(G_{7,m,7})$ = $30160 \times C_6^{(m-1)}$.
Proof:
Look at the sequence of numbers above.
It can be seen that from the sequence above that the fixed difference occur on the sixth level. Therefore the polynomial that can represent this sequence is polynomial of order six:
$P_6(m) = a_6m^6 + a_5m^5 + a_4m^4 + a_3m^3 + a_2m^2 + a_1m + a_0$
Substitute m = 7, 8, 9, 10, 11, 12, 13 to the equation we get the following:

$$30160 = 117649a_6+16807a_5+2401a_4+343a_3+49a_2+7a_1+a_0 \tag{8}$$

$$211120 = 262144a_6+32768a_5+4096a_4+512a_3+64a_2+8a_1+a_0 \tag{9}$$

$$844480 = 531441a_6+59049a_5+6561a_4+729a_3+81a_2+9a_1+a_0 \tag{10}$$

$$2533440 = 1000000a_6 + 100000a_5 + 10000a_4 + 1000a_3+ 100a_2 + 10a_1 + a_0 \tag{11}$$

$$6333600 = 1771561a_6 + 161051a_5 + 14641a_4 + 1331a_3+ 121a_2 + 11a_1 + a_0 \tag{12}$$

$$13933920 = 2985984a_6 + 248832a_5 + 20736a_4 + 1728a_3 + 144a_2 + 12a_1 + a_0 \tag{13}$$

$$27867840 = 4826809a_6 + 371293a_5 + 28561a_4 + 2197a_3 + 169a_2 + 13a_1 + a_0 \tag{14}$$

Solving this system of linear equations we get $a_6 = \frac{30160}{720}$, $a_5 = -\frac{633360}{720}$, $a_4 = \frac{5278000}{720}$, $a_3 = -\frac{22167600}{720}$, $a_2 = \frac{48979840}{720}$, $a_1 = -\frac{53202240}{720}$ and $a_0 = \frac{21715200}{720}$

Therefore

$$\begin{aligned} P_6(m) &= a_6m^6 + a_5m^5 + a_4m^4 + a_3m^3 + a_2m^2 + a_1m + a_0 \\ &= \frac{30160}{720}m^6 - \frac{633360/720}{720}m^5 + \frac{5278000}{720}m^4 \\ &- \frac{22167600}{720}m^3 + \frac{48979840}{720}m^2 - \frac{53202240}{720}m \\ &+ \frac{21715200}{720} \\ &= \frac{30160}{720}(m^6 - 21m^5 + 175m^4 - 735m^3 + 1624m^2 \\ &- 1764m + 720) \\ &= \frac{30160}{720}(m-1)(m-2)(m-3)(m-4)(m-5)(m-6) \\ &= 30160 \times \frac{(m-1)(m-2)(m-3)(m-4)(m-5)(m-6)}{(6 \times 5 \times 4 \times 3 \times 2 \times 1)} \\ &= 30160 \times C_6^{(m-1)} \end{aligned} \tag{15}$$

Doing with similar manner we get the following results:

For n=7, $m \geq 8$, t =8, $N(G_{7,m,8}) = 30765 \times C_7^{(m-1)}$
For n=7, $m \geq 9$, t =9, $N(G_{7,m,9}) = 21000 \times C_8^{(m-1)}$
For n=7, $m \geq 10$, t =10, $N(G_{7,m,10}) = 28364 \times C_9^{(m-1)}$
For n=7, $m \geq 11$, t =11, $N(G_{7,m,11}) = 26880 \times C_{10}^{(m-1)}$
For n=7, $m \geq 12$, t =12, $N(G_{7,m,12}) = 26460 \times C_{11}^{(m-1)}$
For n=7, $m \geq 13$, t =13, $N(G_{7,m,13}) = 20790 \times C_{12}^{(m-1)}$
For n=7, $m \geq 14$, t =14, $N(G_{7,m,14}) = 10290 \times C_{13}^{(m-1)}$
For n=7, $m \geq 15$, t =15, $N(G_{7,m,15}) = 8022 \times C_{14}^{(m-1)}$
For n=7, $m \geq 16$, t =16, $N(G_{7,m,16}) = 2940 \times C_{15}^{(m-1)}$
For n=7, $m \geq 17$, t =17, $N(G_{7,m,17}) = 4417 \times C_{16}^{(m-1)}$
For n=7, $m \geq 18$, t =18, $N(G_{7,m,18}) = 2835 \times C_{17}^{(m-1)}$
For n=7, $m \geq 19$, t =19, $N(G_{7,m,19}) = 210 \times C_{18}^{(m-1)}$
For n=7, $m \geq 20$, t =20, $N(G_{7,m,20}) = 21 \times C_{19}^{(m-1)}$
For n=7, $m \geq 21$, t =21, $N(G_{7,m,21}) = 1 \times C_{20}^{(m-1)}$

Base on these result, we get Table 5. From Table 5 it can be seen that for every t, the formula consist of $C_{t-1}^{(m-1)}$, and the difference is on c_t.

Table 5. Comparison for The Number of Connected Vertex Labeled Graphs of Order Five, Six, and Seven Containing no Loops

t	n: 5	n: 6	n: 7
4	N$(G_{5,m,4})$ = 125 × $C_3^{(m-1)}$		
5	N$(G_{5,m,5})$ = 222 × $C_4^{(m-1)}$	N$(G_{6,m,5})$ = 1296 × $C_4^{(m-1)}$	
6	N$(G_{5,m,6})$ = 205 × $C_5^{(m-1)}$	N$(G_{6,m,6})$ = 1980 × $C_5^{(m-1)}$	N$(G_{7,m,6})$ = 6727 × $C_5^{(m-1)}$
7	N$(G_{5,m,7})$ = 110 × $C_6^{(m-1)}$	N$(G_{6,m,7})$ = 3330 × $C_6^{(m-1)}$	N$(G_{7,m,7})$ = 30160 × $C_6^{(m-1)}$
8	N$(G_{5,m,8})$ = 45 × $C_7^{(m-1)}$	N$(G_{6,m,8})$ = 4620 × $C_7^{(m-1)}$	N$(G_{7,m,8})$ = 30765 × $C_7^{(m-1)}$
9	N$(G_{5,m,9})$ = 10 × $C_8^{(m-1)}$	N$(G_{6,m,9})$ = 6660 × $C_8^{(m-1)}$	N$(G_{7,m,9})$ = 21000 × $C_8^{(m-1)}$
10	N$(G_{5,m,10})$ = 1 × $C_9^{(m-1)}$	N$(G_{6,m,10})$ = 2640 × $C_9^{(m-1)}$	N$(G_{7,m,10})$ = 28634 × $C_9^{(m-1)}$
11		N$(G_{6,m,11})$ = 1155 × $C_{10}^{(m-1)}$	N$(G_{7,m,11})$ = 26880 × $C_{10}^{(m-1)}$
12		N$(G_{6,m,12})$ = 420 × $C_{11}^{(m-1)}$	N$(G_{7,m,12})$ = 26460 × $C_{11}^{(m-1)}$
13		N$(G_{6,m,13})$ = 150 × $C_{12}^{(m-1)}$	N$(G_{7,m,13})$ = 20790 × $C_{12}^{(m-1)}$
14		N$(G_{6,m,14})$ = 15 × $C_{13}^{(m-1)}$	N$(G_{7,m,14})$ = 10290 × $C_{13}^{(m-1)}$
15		N$(G_{6,m,13})$ = 1 × $C_{14}^{(m-1)}$	N$(G_{7,m,15})$ = 8022 × $C_{14}^{(m-1)}$
16			N$(G_{7,m,16})$ = 2940 × $C_{15}^{(m-1)}$
17			N$(G_{7,m,17})$ = 4417 × $C_{16}^{(m-1)}$
18			N$(G_{7,m,18})$ = 2835 × $C_{17}^{(m-1)}$
19			N$(G_{7,m,19})$ = 210 × $C_{18}^{(m-1)}$
20			N$(G_{7,m,20})$ = 21 × $C_{19}^{(m-1)}$
21			N$(G_{7,m,21})$ = 1 × $C_{20}^{(m-1)}$

4. CONCLUSIONS

From the discussion above we can conclude that the formula to count the number of connected vertex labeled graph of order seven has a similar pattern with the lower order graph with the similar property. The difference of the formulas is on the coeficient for every t. The result shows that the number of connected vertex labeled graphs of order seven containing no loops is $N(G_{7,m,t})= c_t\ C_{t-1}^{(m-1)}$, with c_6=6727, c_7= 30160 , c_8=30765, c_9=21000, c_{10}=28364, c_{11}=26880, c_{12}=26460, c_{13}=20790, c_{14}=10290, c_{15}=8022, c_{16}=2940, c_{17}=4417, c_{18}=2835, c_{19}=210, c_{20}=21, c_{21}=1.

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