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Research Paper

Branch and Cut Method for Solving Capacitated Vehicle Routing Problem (CVRP) Model of LPG Gas Distribution Routes

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Abstract

Capacitated Vehicle Routing Problem (CVRP) is a problem that discusses how to choose several routes that must be passed by a number of transport vehicles in the process of distributing goods that combine customer demand with regard to transport capacity. CVRP designs an optimal delivery route where each vehicle only takes one route, each vehicle has the same characteristics, each customer has a request and there is only one depot. In this paper, two CVRP models were formulated. Formulation of the first CVRP model without regard to vehicle loads and vehicles returned to the depot. The second CVRP model formulation takes into account the vehicle load and the vehicle does not return to the depot. Determination of LPG gas distribution routes is completed using the Branch and Cut method.

Keywords

Capacitated Vehicle Routing Problem, Branch and Cut

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1. INTRODUCTION

The Vehicle Routing Problem (VRP) and its variants hae grown popular. The capacitated vehicle routing problem (CVRP) deals with the distribution of a single commodity from a centralized depot to a number of specified customer locations with known demands (Achuthan et al., 2003; Baldacci et al., 2004; Ralphs et al., 2003). Vehicle Routing Problem (CVRP) is a VRP subcase, where vehicles have limited capacity (Beresneva and Avdoshin, 2018). Optimal route search in daily life is needed to minimize the time and costs incurred, especially for large companies that distribute their products every day by using a vehicle. The problem of finding the optimal vehicle route or the Vehicle Routing Problem (VRP) is a problem that discusses how to choose several routes that must be passed by a number of transport vehicles in the process of distributing goods that combine customer demand with regard to transport capacity. VRP was introduced by Dantzig and Ramser in 1959 on the problem of modeling truck deliveries. The problem under investigation is, how a homogeneous fleet of trucks can serve oil demand from a number of gas stations from the depot and minimize mileage. The purpose of this optimization is to find a set of routes that include n customers with a minimum overall distance (Jepsen, 2011). The problem under study is how to serve a set of customers scattered around a central depot, using a fleet of trucks with various capacities (Braekers et al., 2016). VRP has experienced developments that are tailored to real- world problems. A company faces a vehicle route problem to determine the optimal route. Classic VRP aims to find a set of vehicle routes that start and end at a depot for vehicles with the same capacity so that each customer is visited exactly once (Atefi et al., 2018). Classic VRP is also known as Capacitated Vehicle Routing Problem (CVRP). CVRP designs an optimal delivery route where each vehicle only takes one route, each vehicle has the same characteristics, each customer has a demand and there is only one central depot to meet customer demand with the number of vehicle loads that does not exceed capacity. There are various Integer Linear Programming (ILP) models from CVRP, as already examined (Borčinova, 2017; Alipour, 2012). One of the main differences lies in eliminating sub-tours, namely cycles that do not go through depots. Cacceta et al. investigated that reducing the Clarke and Wright algorithm with a hybrid approach to CVRP completion (Caccetta et al., 2012). Achuthan et al. (2003) has developed an exact branch and cut algorithm. Letchford et al. (2007) presents the branch and cut algorithm for the open routing problem, the vehicles are not required to return to the depot after completing service. In this study, the ILP model formulated from CVRP was solved by the Branch and Cut method. Distribution of LPG gas products to several customers in the city of Palembang. Basically, this method attempts to strengthen the lower bounds by the addition of constraints (cuts) at each node within a Branch and Bound procedure (Yang et al., 2000). PT. Lebong Terang is expected to be able to create reliable shipping performance in the distribution of gas products. The conversion of kerosene to gas makes LPG gas demand continues to increase. During this time the distribution process that has been good, but not yet maximum which resulted in the long enough shipping distance and resulting in greater distribution costs, for this reason the company is expected to have a plan in determining the distribution channel so that the product distribution process can run optimally at a low cost . This research will formulate a CVRP model to determine the LPG gas distribution route using the branch and cut method.

2. EXPERIMENTAL SECTION

The research is a case study, LPG gas distribution routes at PT Terang Lebong in Palembang. The data is the result of a survey of 3 LPG gas base data collected by the surveyor, in the form of a list of verified gas bases.

Table 1. Parameters

Parameter	Description
d1	demand for gas station 1
d2	demand for gas station 2
d3	demand for gas station 3
d4	demand for gas station 4
d5	demand for gas station 5
d6	demand for gas station 6
d7	demand for gas station 7
d8	demand for gas station 8
d9	demand for gas station 9
d10	demand for gas station 10
d11	demand for gas station 11
d12	demand for gas station 12
d13	demand for gas station 13
d14	demand for gas station 14
d15	demand for gas station 15
d16	demand for gas station 16
d17	demand for gas station 17
d18	demand for gas station 18
d19	demand for gas station 19
d20	demand for gas station 20
d21	demand for gas station 21
d22	demand for gas station 22
d23	demand for gas station 23
d24	demand for gas station 24
Q	Vehicle capacity

2.1 Materials

The data used in this study is the distribution of 3 kg LPG gas. The gas is distributed to 24 gas bases (Yuliza et al., 2019).

2.2 Methods

Following are the research steps:

1. LPG gas demand data and distance from the agent to each gas base.
2. Determine the distance matrix.
3. Formulate the CVRP model.
4. Solve the CVRP model using the branch and cut method.

The calculation process uses the help of LINGO 13.0 software.

3. RESULTS AND DISCUSSION

3.1 Mixed Linear Programming Formulation of CVRP

Given the definition of variables and parameters. For each (i, j) ϵ V, i ≠ j and for each vehicle r defined variable :

$$x_{ij} = \left\{ \begin{array}{c} 1; if there is a trip from the gas base to i to the gas base to j \\ 0; otherwisw \end{array} \right\}$$

From Table 1 defined parametres.

VRP problems in LPG gas distribution can be determined as a graph G=(V,E). The set V consists of a combined set of P gas bases and warehouses, V={0,1,2,3,. . . ,24}. The set P is in the first gas base, the second gas base, ..., the 24th gas base, P={1,2,3,. . . ,24}. The set of vehicles is a collection of vehicles that are homogeneous with capacity. Every base i for every i ϵ V has a demand in so that the length of the route is limited by the capacity of the vehicle (Q). For all pairs i,jϵV,i≠j, we calculate the savings s_ij for joining the cycles 0⟶i⟶0 and 0⟶j⟶0 using arc (i,j); $s_{ij}=c_{i0}+c_{0j}-c_{ij}$ (Borčinova, 2017).

Table 2. Solution of CVRP model with branch and cut method

The MILP Model Distribution Route from CVRP	Distance Traveled (km)
0-5-11	0.4
0-3-13	7.9
0-17-15	5,5
0-2-20	0.7
0-8-10	2.4
0-14-18	2.85
0-19-21	2.29
0-22-23	0.75
0-4-16	10.8
0-7-9	-1.2
0-12-1	10
0-6-24	-1.5
total	40,8

In this research, we present mixed integer linear programming (MILP) model of CVRP. The objective function of the ILP model of CVRP minimizes the distance of travel (Yuliza et al., 2019). Now, instead of minimizing the distance of travel, will be maximize the total saving of the distance of travel. The CVRP model for LPG gas distribution is can be stated as:

Max $0.4x_{511}+0.4x_{011}$ subject to

$x_{05}+x_{011}=1$

$x_{05}+x_{115}=1$

$x_{011}+x_{511}=1$

$x_{511}<=1$

$x_{115}<=1$

$y_5+1050x_{511}+4000x_{511}-y_{11}<=4000$

Table 3. Compared the ILP model from CVRP and the MILP model from CVRP

The ILP Model Distribution Route from CVRP	Distance Traveled (km)	The MILP Model Distribution Route from CVRP	Distance Traveled (km)
0-5-11-0	4.8	0-5-11	0.4
0-3-13-0	21.9	0-3-13	7.9
0-17-15-0	31.5	0-17-15	5,5
0-2-20-0	9.1	0-2-20	0.7
0-8-10-0	7.2	0-8-10	2.4
0-14-18-0	7.25	0-14-18	2.85
0-19-21-0	4.19	0-19-21	2.29
0-22-23-0	1.65	0-22-23	0.75
0-4-16-0	17,8	0-4-16	10.8
0-7-9-0	5.8	0-7-9	-1.2
0-12-1-0	23.2	0-12-1	10
0-24-0	6.6	0-6-24	-1.5
0-6-0	5		
Total Distance	145.99		40.8

$y_{11}+1200x_{115}+4000x_{115}-y_5<=4000$
$y_5>=1200$
$y_5<=4000$
$y_{11}=1050$
$y_{11}<=4000$

the function value of the CVRP model is 0.4 and the value x_{115} = 0.792, x_{011} = 0.792, x_{511} = 0.207, x_{05} = 0.207, y_{11} = 1050 and y_5 = 4000. The decision variables are non-integer values so branching is done. After branching, the CVRP model is obtained :

Max 0.4x511+0.4x115 subject to
$x_{05}+x_{011}=1$
$x_{05}+x_{115}=1$
$x_{011}+x_{511}=1$
$x_{511}<=1$
$x_{511}<=1$
$y_5+1050x_{511}+4000x_{511}-y_{11}<=4000$
$y_{11}+1200x_{511}+4000x_{115}-y_5<=4000$
$y_5>=1200$
$y_5<=4000$
$y_{11}>=1050$
$y_{11}<=4000$
$x_{511}<=0$

The function value of the CVRP model is 0.4 and the value x_{115} = x_{011} = 1, x_{511} = x_{05} = 0, y_{11} = 1050 dan y_5 = 4000. Path 0⟶11⟶5⟶0 represents the route 0⟶11⟶5 where gas base 11 has demand 1050 kg, gas base 5 has demand 1200 kg, the value of the vehicle load is 1050 kg the value of the vehicle load is 4000 kg. MILP model solutions from CVRP with the branch and cut method are shown in the Table 2.

3.2 Computational Results

Both of the models, the ILP model distribution route from CVRP and the MILP model distribution route from CVRP, were solved using LINGO 13.0. From Table 3 compared the ILP model from CVRP and the MILP model from CVRP to get the optimal solution. ILP model from CVRP and MILP model from CVRP were solved by branch and cut method.

From the Table 3., the optimal route of the ILP model from CVRP is 0-5-11-0 with travel distance 4.8 km. the optimal route of the MILP model from CVRP is 0-5-11 with travel distance 0.4 km. The feasible route, 0⟶5⟶11⟶0 is replaced by a path from node 0 to node 11, 0⟶5⟶11. Ratio of this travel distace from the ILP model from the CVRP and the MILP model from the CVRP is 12 km.

4. CONCLUSIONS

From the result and discussion, it can be concluded the optimal solution of the CVRP model using branch and cut method is the route with optimum distance obtained as follows: 0-5-11 with an optimal distance of 12 km, 0-3-13 with an optimal distance of 7.9 km, 0-7-15 with an optimal distance of 5.5 km, 0-2-20 with an optimal distance of 0.7 km, 0-8-10 with an optimal distance of 2.4 km, 0-14-18 with an optimal distance of 2.85 km, 0-19-21 with an optimal distance of 2.29 km, 0-22-23 with an optimal distance of 0.75 km, 0-4-16 with an optimal distance of 10.8 km, 0-7-9 with an optimal distance of -1.2 km, 0-12-1 with an optimal distance of 10 km and 0-6-24 with an optimal distance of -1.5 km. Percentage comparison of flow formulation from CVRP and modified assignment formulation from CVRP is 3.578 or 357.8 %.

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