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Simulation Study on Evolutionary Game Model between Technological Small and Medium Enterprises and Banks under Verification System

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Abstract

On the basis of evolutionary game theory, this paper adds risk compensation funds and establishes an evolutionary game model for Technological Small and Medium Enterprises (SMEs) and bank loans under the verification mechanism. And the stability analysis and simulation study of the game results of the four evolutionary strategies of the model are carried out. Finally, we have put forward some suggestions on the strategy selection and the development of the loan system.

Keywords: Evolutionary Game Model, Technological Small and Medium Enterprises (SMEs), Risk Compensation Funds, Verification System

1. Introduction

As an important source of improving overall national strength and national competitive advantage, the development of technological SMEs relates to the future of the country's economy. Compared with the ordinary SMEs, the biggest difference of technological SMEs is its research and development, production and sales of high-tech products. It has strong innovation ability and strong development potential. The Chinese government has been actively exploring how to give play to the role of the government in macro-control and enhance the capability of independent innovation in our country through policy orientation, so that scientific and technological innovation can become a new impetus to leverage China's economic growth. Therefore, promoting the rapid development of technological SMEs will enhance China's science and technology innovation ability and promote the rapid development of our economy. However, the technological SMEs have the characteristics of high risk, small size, less physical assets, more intangible assets and low credit grade, which make it very difficult to finance. Many scholars have their own shortcomings in SMEs Have done some research.

For example, banks, as the main lending institutions, face a series of characteristics of SMEs in science and technology, as well as the information asymmetry and moral hazard in the lending process, which makes the banks have "reluctant loans" and make the financing of science and technology SMEs difficult. Yu Minggui, Pan Hongbo (2008) found that enterprises with political relations get more bank loans and longer loan terms than non-political enterprises[1]. Moreover, as the financial development is lagging behind, the lower the level of the legal system and the government, the more serious the damage infringement property rights, the more obvious the effect of this loan of political relations. Ji Huiying (2011) found that due to the issuance process of financial institutions loans, handling aspects of roughly the same, and each SMEs average loan size is significantly lower than that of large enterprises, so each SME operating costs on each loan financial institutions are relatively high[2]. Zhang Mu, Zhou Zongfang (2012) used k-means clustering algorithm to get the final clustering centers from the initial cluster centers, then partitioned the credit levels to achieve the corporate credit rating[3]. Liu Jinwen (2012) found that the problem of "adverse selection" and "moral hazard" existing in the SME credit market further aggravated the capital shortage of SMEs under the circumstances that the external financing of SMEs relies mainly on bank loans. The difficulty of lending has become a constraint Small and medium-sized enterprise development "bottleneck"[4]. Zhang Xiaomei and Zhong Zhen (2013) conducted an empirical study on the relationship between bank size and SME loans in China based on the data of bank loans of listed SMEs in 2008. It concludes that: in China, small banks should not allocate high-quality listed SMEs as a loan object[5]. Yan Bailu(2015) used factor analysis to establish a non-equilibrium model to analyze the supply and demand of SME financing, and used Fisher's discriminant to study the influencing factors of SMEs bank loan defaults. The study found that, whether it is private lending or bank loans all tend to provide financing to larger enterprises. SMEs with high private lending and high private lending rates are more likely to default on bank loans[6].

To sum up, scholars at home and abroad have succeeded in researching the financing problem of SMEs abundantly, but there are few researches on the financing problems of small and medium-sized technology enterprises. This paper discusses the application of evolutionary game theory in science and technology type under the premise of bounded rationality on the game between SMEs and banks, this paper establishes an evolutionary game model of technological SMEs and bank loans under the verification system, and analyzes the interaction mechanism between the choice of corporate behavior strategies and the choice of bank behavior strategies, which aims to promote the banks to issue loans, technological SMEs can repay on time so as to solve the financing difficulties of technological SMEs.

2. Model Establishment

Evolution game theory formation and development generally go through three stages: First of all, biologists get enlightenment from game theory and use game theory to construct various biological competition evolution models, including animal competition, gender distribution and plant growth and development. Then, according to the law of biological evolution, biologists reconstruct the traditional game theory, including converting the traditional game theory's payment function into a biological fitness function, introducing a mutation mechanism to refine the traditional Nash equilibrium into an stable equilibrium and introduce a choice mechanism to construct replicator dynamic model. Subsequently, in view of the evolution of the traditional game of evolutionary game, economists in turn borrow the idea of biologists, the evolutionary game applied to economics, which further promote the development of evolutionary game[7].

2.1. *Loading Evolutionary Game Model of SMEs and Bank under Verification System*

Evolutionary Game Model Assumptions:

①There are only two participants in the game: technological SMEs and banks, both sides of the game are limited rationality.

②Behavioral strategy: SMEs' strategy selection set as S_1={overdue default, repay on time}; the bank's strategy selection set is S_2={verification, not verification}.

③Proportion of behavioral strategies taken: in the initial stage of the game of science and technology SMEs and banks, assuming the probability of overdue breach of contract for technological SMEs is 1-q, take the probability of repaying on time is q; The probability of a bank taking a loan is p, the probability of taking a non-loan is 1-p.

④Parameter assumptions and basic explanations: L: technological SMEs need to borrow money from the bank for project investment; K: own funds for the project investment; a: the rate of return of a firm upon successful investment in technological SMEs, 0 at failure; r: bank lending rates; T: loan period; i: bank cost of capital costs; r_0: government departments set up risk compensation funds, when the company defaults to give lending bank compensation ratio; p_i: average probability of successful investment in SMEs, and $0 \leq pi \leq 1$; C: in the process of bank supervision, it is verified that there is a fraudulent act of credit for technological SMEs after the loan and penalties will be imposed on technological SMEs; vL: bank checks costs, and assumes vL<C. Assuming the bank's post-lending regulatory verification is 100% effective, as long as banks choose to monitor the verification, it means that banks can accurately know whether SMEs have defaulted on their loans.

Combined with the above earnings analysis of banks and technological SMEs in various situations, a payment matrix is constructed as shown in Table 1.

Table 1. Payment matrix

		Technology SMEs	
		Overdue Default	Repay on Time
Banks	Verification	p_i(C-vL)-(1-p_i)vL-L(1+i-r_0) (banks)	pi(Lr(T)-Li)-vL (banks)
		pi((L+K)a-C) (SMEs)	Pi((L+K)a-Lr(T)) (SMEs)
	not Verification	-L(1+i-r_0) (banks)	pi(Lr(T)-Li) (banks)
		Pi(L+K)a (SMEs)	Pi((L+K)a-Lr(T)) (SMEs)

The expected earnings from verification by banks, the expected earnings from non-verification by banks, and the average expected earnings from banks for verification and non-verification strategies are:

$$U_{11} = (1-q)(p_i(C-vL)-(1-p_i)vL-L(1+i-r_0)) + q(p_i(Lr(T)-Li)-vL)$$

$$U_{12} = (1-q)(-L(1+i-r_0)) + q(p_i(Lr(T)-Li))$$

$$U_1 = pU_{11} + (1-p)U_{12}$$

The expected return of overdue breach of contract, the expected return of enterprises adopting timely repayments, the average expected earnings of enterprises adopting overdue default and repayment on schedule policies are as follows:

$$U_{21} = p(p_i((L+K)a-C)) + (1-p)(p_i(L+K)a)$$

$$U_{22} = p(p_i(L+K)a-Lr(T)) + (1-p)(p_i((L+K)a-Lr(T)))$$

$$U_2 = (1-q)U_{21} + qU_{22}$$

And then to build a dynamic equation system for replicators of banks and corporate strategies:

$$\frac{dp}{dt} = p(U_{11}-U_1) = p(1-p)((1-q)(p_iC-vL-qvL))$$

$$\frac{dq}{dt} = q(U_{22}-U_2) = q(1-q)(pp_iC-p_iLr(T))$$

3. Model Stability Analysis

The replicator dynamic equations have five local equilibrium points:

$E_1(0, 0)$, $E_2(1, 0)$, $E_3(0, 1)$, $E_4(1, 1)$, $E_5(q^*, p^*)$

where $q^* = \dfrac{p_iC-vL}{p_iC}$, $p^* = \dfrac{Lr(T)}{C}$, and $0 < \dfrac{p_iC-vL}{p_iC}$, $\dfrac{Lr(T)}{C} < 1$.

In order to describe simply, we make

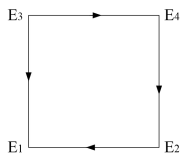
a=p_i(C-vL)-(1-p_i)vL-L(1+i-r_0),
b=p_i(Lr(T)-L_i)-vL,
c=-L(1+i-r_0),
d=p_i(Lr(T)-Li),
e=p_i(La-C),
f=p_iL(a-r(T)),
g=p_iLa,
h=p_iL(a-r(T)).

According to the relative sizes of a, b, c, d, e, f, g, and h, the linear stability of the replication system near the equilibrium point can be analyzed to obtain various equilibriums of evolutionary game models[8]. According to model assumptions are easy to know, b<d, g>h. Combined with the model assumptions, the analysis found that the replication system has four evolutionary conditions.

3.1. *Evolution 1*

Table 2. Evolution1

Parameter	Equilibrium Point	Stability	Phase Diagrams
$a < c$	$E_1(0,0)$	Stable node	
$b < d$	$E_2(1,0)$	Saddle Point	
$e < f$	$E_3(0,1)$	Unstable node	
$g > h$	$E_4(1,1)$	Saddle Point	



When the model parameters meet the conditions in Table 1, we know:

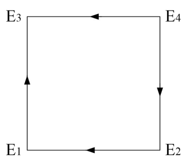
①The probability of successful enterprise investment multiplied by the penalties received by technology-based SMEs when defrauding loans is less than the verification costs of banks.

②The punishment imposed on technology-based SMEs when defrauding loans is greater than the interest paid by technology-based SMEs. In other words, in this case, SMEs will adopt the "overdue breach of contract" strategy, the bank will adopt a "do not check" strategy. According to the model and parameter hypothesis analysis, though the scientific and technological types of SMEs are punished with greater punishment than the SMEs to pay the loan interest, but because the bank's verification costs are higher, the company believes that the probability of bank verification is low.

Therefore, science and technology SMEs will adopt the "overdue breach of contract" strategy. And because banks have higher verification costs, banks will adopt a "non-verification" strategy. The final steady state is that technology-based SMEs will adopt the "overdue breach of contract" strategy, and banks will adopt a "non-verification" strategy.

3.2. *Evolution 2*

Table 3. Evolution2

Parameter	Equilibrium Point	Stability	Phase Diagrams
$a > c$	$E_1(0,0)$	Saddle Point	
$b < d$	$E_2(1,0)$	Saddle Point	
$e > f$	$E_3(0,1)$	Stable node	
$g > h$	$E_4(1,1)$	Unstable node	

When the model parameters meet the conditions in Table 2, we know:

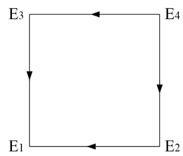
①The probability of successful enterprise investment multiplied by the penalties received by technology-based SMEs when defrauding loans is greater than the verification costs of banks.

②The punishment imposed on technology-based SMEs when defrauding loans is less than the interest paid by technology-based SMEs.In other words, in this case, SMEs will adopt the "overdue breach of contract" strategy, the bank will adopt a "verification" strategy. According to the model and parameter hypothesis analysis, the science and technology type of SMEs are punished by the penalty of lending less than the loan interest of technology-based SMEs to pay, the company's default cost is lower.

Therefore, technology-based SMEs have chosen to adopt the "overdue breach of contract" strategy. And because banks have lower verification costs, banks choose to adopt a "verification" strategy. The steady state finally reached is that the technology-based SMEs will adopt the "overdue breach of contract" strategy, and banks will adopt a "verification" strategy.

3.3. *Evolution 3*

Table 4. Evolution3

Parameter	Equilibrium Point	Stability	Phase Diagrams
$a < c$	$E_1(0,0)$	Stable node	
$b < d$	$E_2(1,0)$	Saddle Point	
$e > f$	$E_3(0,1)$	Saddle Point	
$g > h$	$E_4(1,1)$	Unstable node	

When the model parameters meet the conditions in Table 3, we know:

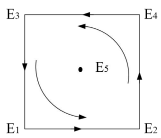
①The probability of successful enterprise investment multiplied by the penalties received by technology-based SMEs when defrauding loans is less than the verification costs of banks.

②The punishment imposed on technology-based SMEs when defrauding loans is less than the interest paid by technology-based SMEs. In other words, in this case, SMEs will adopt the "overdue breach of contract" strategy; the bank will adopt a "do not check" strategy. According to the model and parameter hypothesis analysis, because the scientific and technological type of SMEs are punished by the penalty of being less than the loan interest of science and technology SMEs, and the bank's verification costs are also high, the company believes that the probability of bank verification is low. The cost of default is also lower.

Therefore, technology-based SMEs have chosen to adopt the "overdue breach of contract" strategy. However, due to the higher verification costs of banks, banks chose to adopt a "non-verification" strategy. The final steady state is that technology-based SMEs will adopt the "overdue breach of contract" strategy, and banks will adopt a "non-verification" strategy.

3.4. *Evolution 4*

Table 5. Evolution4

Parameter	Equilibrium Point	Stability	Phase Diagrams
$a > c$	$E_1(0,0)$	Saddle Point	
$b < d$	$E_2(1,0)$	Saddle Point	
$e < f$	$E_3(0,1)$	Saddle Point	
$g > h$	$E_4(1,1)$	Saddle Point	
——	$E_5(q^*, p^*)$	Center	

When the model parameters meet the conditions in Table 4, we know:

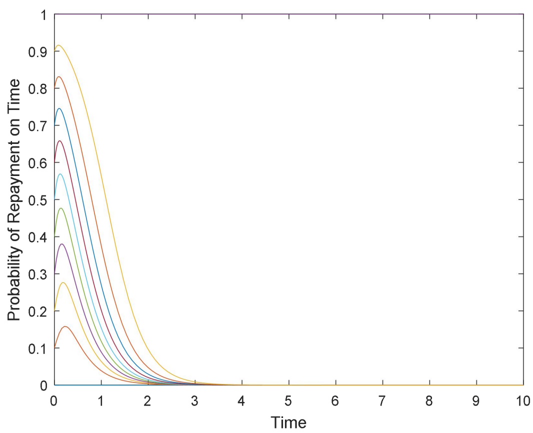
①The probability of successful enterprise investment multiplied by the penalties received by technology-based SMEs when defrauding loans is greater than the verification costs of banks.

②The punishment imposed on technology-based SMEs when defrauding loans is greater than the interest paid by technology-based SMEs. The probability of a stable game strategy for SMEs and banks in science and technology is oscillating near the midpoint $E_5(q^*, p^*)$, and the center of shock is. It can be found that the location of the center is determined by the average probability of successful investment projects for science and technology SMEs, penalties for science and technology SMEs, bank verification costs, and loan interest payable by technology-based SMEs.

Among them, it is inversely proportional to the probability of "overdue breaches of contract" of SMEs. When p_i becomes larger, the probability of “technetic breach of contract” of SMEs is reduced. It is inversely proportional to the probability of "overdue breach of contract" by technology-based SMEs, and inversely proportional to the probability of "checking" by banks. When it becomes larger, the probability of "overdue breaches of contract" for SMEs is reduced, and the probability of "checking" by banks is also reduced. It is proportional to the probability of "overdue breach of contract" of SMEs. When it becomes larger, the probability of overdue defaults by technology-based SMEs also increases. It is in direct proportion to the "verification" probability of the bank. When it becomes larger, the loan interest paid by technology-based SMEs will increase, and the probability of banks "checking" will increase.

4. Model Simulation Analysis

4.1. *Evolution 1*



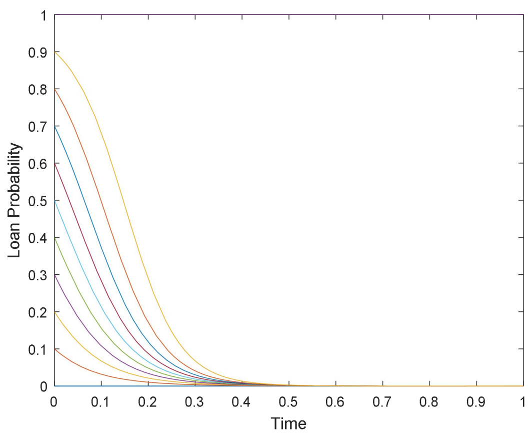


Fig. 1. Evolution 1(p_i=50%, C=30, vL=20, L=100, r(T)=5%)

When the model parameters satisfy a<c, b<d, e<f, g>h, $E_1(0, 0)$ is the stable node, $E_2(1, 0)$ and $E_4(1, 1)$ are the saddle points, and $E_3(0, 1)$ is the unstable node. Under such circumstances, SMEs will adopt the strategy of "default overdue" and the bank will adopt a strategy of "non-verification". Set the parameters shown in Figure 1. As shown in the figure, the bank strategy converges faster, and the "check" strategy in all games in 0.5 unit time has tended to zero. The convergence of corporate strategy is slower, and the strategy of "repayment on time" in all games tends to zero in 3.5 unit time. And q and p start game value set the smaller, it tends to stabilize faster. It can also be found that the probability that an enterprise "repays on time" in the initial stage of the game has a period of ascent, and then decreases to zero. It is possible that the bank strategy converged faster and converged to the "no verification" strategy within 0.5 unit of time, so that the enterprise chose to "overdue default" in order to obtain greater benefits.

4.2. *Evolution 2*

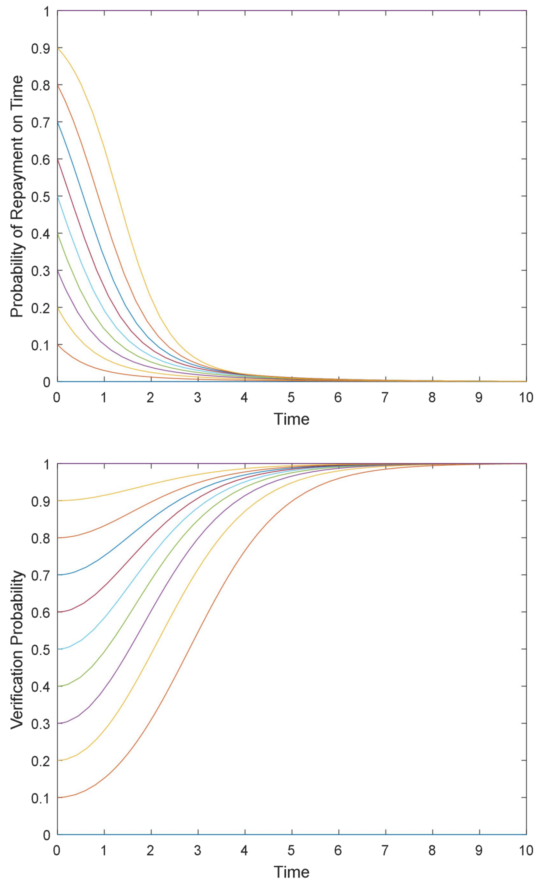


Fig. 2. Evolution 2(pi=50%, C=4, vL=1, L=100, r(T)=5%)

When the model parameters satisfy a>c, b<d, e>f, g>h, $E_3(0, 1)$is the stable node, $E_1(0, 0)$ and $E_2(1, 0)$ are the saddle points, and $E_4(1, 1)$ is the unstable node. Under such circumstances, S & T SMEs will adopt the strategy of "default overdue" and the bank will adopt a "verification" strategy. Set the parameters shown in Figure 2. As shown in the figure, the convergence time of the game strategy between the firm and the bank is similar, so there is no such process that the steady strategy probability moves to the opposite convergence direction in the initial stage of the game.

4.3. *Evolution 3*

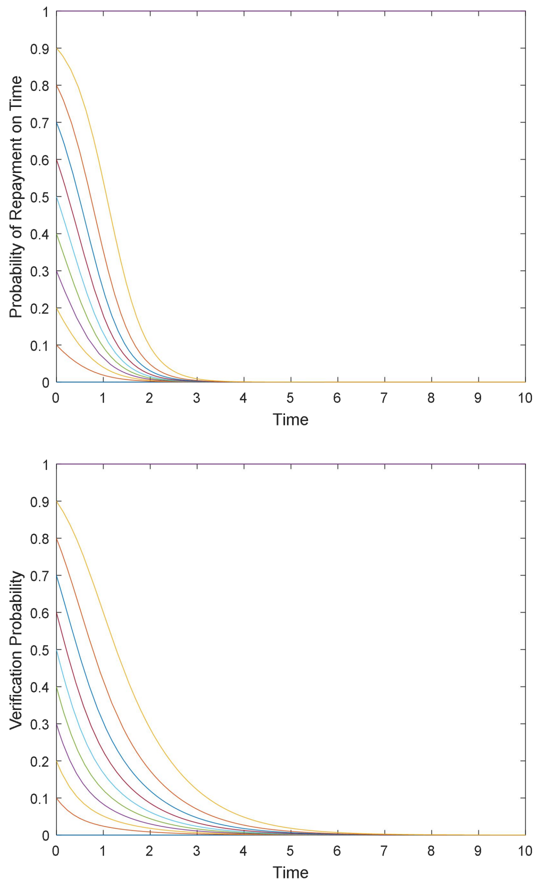


Fig. 3. Evolution 3(pi=50%, C=4, vL=3, L=100, r(T)=5%)

When the model parameters satisfy a<c, b<d, e>f, g>h, $E_1(0, 0)$is the stable node, $E_2(1, 0)$and $E_3(0, 1)$ are the saddle points, and $E_4(1, 1)$ is the unstable node. Under such circumstances, S & T SMEs will adopt the strategy of "default overdue" and the bank will adopt a strategy of "non-verification". Set the parameters shown in

Figure 3. As shown in the figure, the convergence rate of corporate strategy is faster, and the probability of "repayment on time" in all games in 3.5 unit time has tended to be zero. And the smaller initial game value of X is set, the faster it tends to be stable, the slower the convergence speed of banks, and the "check" strategy of all games tends to 0 in about 7 unit time. And the smaller initial game value is set, the faster it tends to be stable. With such parameter setting conditions, there is still not much difference between the technological SMEs and the bank converging to a stable time, but there are still a few.

4.4. *Evolution 4*

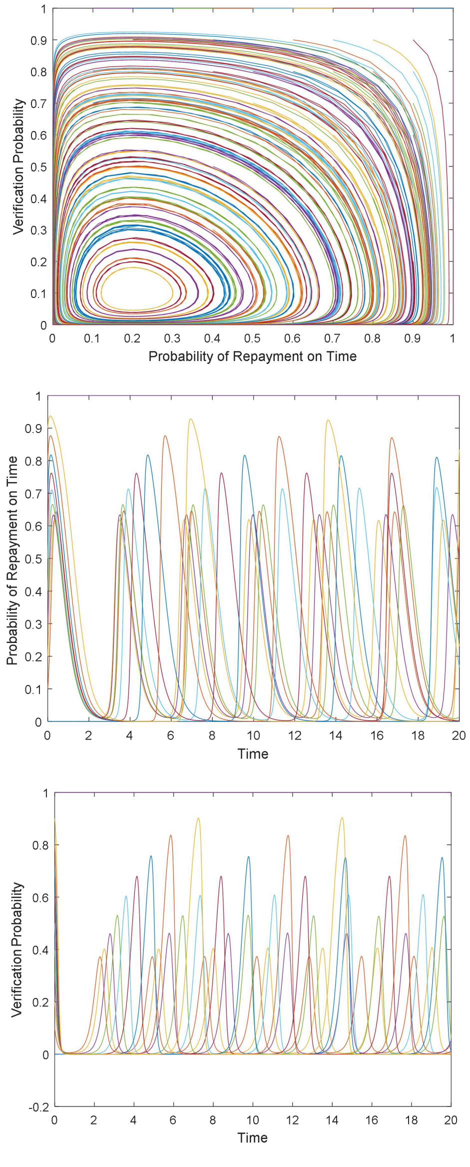


Fig. 4. Evolution 4(pi=50%, C=50, vL=20, L=100, r(T)=5%)

When the model parameters satisfy a>c, b<d, e<f, g>h, $E_1(0, 0)$ $E_2(1, 0)$, $E_3(0, 1)$, $E_4(1, 1)$ are saddle points and $E_5(q^*, p^*)$ is the center. The stable strategy of the game of the technological SMEs and banks has been shaking around the midpoint. Set the parameters shown in Figure 4. The game evolutionary process of q and p is circular, and there is no stable point. The change of p value will affect the change of q value; the change of q value will affect the change of p value. In order to maximize their own interests, the strategic choices of technological SMEs will be continuously adjusted according to the bank's strategic choices. The bank's strategic choices will also be constantly adjusted according to the strategic choices of the technological SMEs. The value of q increases first and then decreases gradually, and then continues to cycle. The value of p decreases first to 0, then keeps stable for a period of time and then gradually increases, and then it continues to cycle.

5. Conclusion

This article applies the theory of evolutionary game to analyze banks and technological SMEs, and analyzes the interaction mechanism between the choice of corporate behavior strategy and the choice of bank behavior strategy in the process of loan for technological SMEs. The results show that in the evolution of 1, 2, and 3, the strategic choices of SMEs are "overdue default". In Evolution 4, the strategic options for technological SMEs and banks fluctuate around center $E_5(q^*, p^*)$ and gradually approach the center. And found that the average probability (p_i) of successful investment projects for technological SMEs and the punishment (C) for technological SMEs are inversely proportional to the probability of "overdue default" for technological SMEs under such evolution. The cost of bank verification (vL) is proportional to the probability of overdue breach of contract for technological SMEs. The interest on bank loans r(T), is proportional to the probability of bank "verification".

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Appendix A

Fig.1. code

```
function dxdt=differential(t,x)
dxdt=[x(1)*(1-x(1))*(x(2)*15-2.5);x(2)*(1-x(2))*((1-
x(1))*(-5)-x(1)*20)];
end

for i=0:0.1:1
   for j=0.5
      [T,Y]=ode45('differential',[0 10],[i j]);
      figure(2)
      plot(T,Y(:,1));
      xlabel('Time');
      ylabel('Probability of Repayment on Time');
      hold on
   end
end

for i=0.5
   for j=0:0.1:1
      [T,Y]=ode45('differential',[0 1],[i j]);
      figure(3)
      plot(T,Y(:,2));
      xlabel('Time');
      ylabel('Verification Probability');
      hold on
   end
end
```

Fig.2. code

```
function dxdt=differential(t,x)
dxdt=[x(1)*(1-x(1))*(x(2)*2-2.5);x(2)*(1-x(2))*((1-
x(1))*(1)-x(1)*1)];
End

for i=0:0.1:1
   for j=0.5
      [T,Y]=ode45('differential',[0 10],[i j]);
      figure(2)
      plot(T,Y(:,1));
      xlabel('Time');
      ylabel('Probability of Repayment on Time');
      hold on
   end
end

for i=0.5
   for j=0:0.1:1
```

```
    [T,Y]=ode45('differential',[0 10],[i j]);
    figure(3)
    plot(T,Y(:,2));
    xlabel('Time');
    ylabel('Verification Probability');
    hold on
  end
end
```

Fig.3. code

```
function dxdt=differential(t,x)
dxdt=[x(1)*(1-x(1))*(x(2)*2-2.5);x(2)*(1-x(2))*((1-
x(1))*(-1)-x(1)*3)];
end
for i=0:0.1:1
  for j=0.5
    [T,Y]=ode45('differential',[0 10],[i j]);
    figure(2)
    plot(T,Y(:,1));
    xlabel('Time');
    ylabel('Probability of Repayment on Time');
    hold on
  end
end
for i=0.5
  for j=0:0.1:1
    [T,Y]=ode45('differential',[0 10],[i j]);
    figure(3)
    plot(T,Y(:,2));
    xlabel('Time');
    ylabel('Verification Probability');
    hold on
  end
end
```

Fig.4. code

```
function dxdt=differential(t,x)
dxdt=[x(1)*(1-x(1))*(x(2)*25-2.5);x(2)*(1-x(2))*((1-
x(1))*(5)-x(1)*20)];
End

clear
for i=0.1:0.1:1
  for j=0:0.1:1
    [T,Y]=ode45('differential',[0 10],[i j]);
    figure(1)
    plot(Y(:,1),Y(:,2));
    xlim([0 1]);
    ylim([0 1]);
    xlabel('Probability of Repayment on Time');
    ylabel('Verification Probability');
    hold on
  end
end

for i=0:0.1:1
  for j=0.5
    [T,Y]=ode45('differential',[0 20],[i j]);
    figure(2)
    plot(T,Y(:,1));
    xlabel('Time');
    ylabel('Probability of Repayment on Time');
    hold on
  end
end
for i=0.5
  for j=0:0.1:1
    [T,Y]=ode45('differential',[0 20],[i j]);
    figure(3)
    plot(T,Y(:,2));
    xlabel('Time');
    ylabel('Loan Probability');
    hold on
  end
end
```