Non-Diversifiable Risk in Investment Portfolios --- an Aid to Investment Decision Making

Emma Anyika

Department of Accounting and Finance, Mount Kenya P.O. Box 342-01000 Thika, Kenya

E-mail: mmnk55378@gmail.com

Abstract

Modeling Non - Diversifiable risk in investment portfolios is undertaken in this paper together with redefinition of estimators of diversifiable risk and portfolio expected returns to reflect normal market conditions. GARCH (General Auto - Regressive Conditional Heteroskedasticity) models are then used to make forecasts of given time series, from which future predictions of Non - Diversifiable risk, Diversifiable risk and portfolio expected returns are made. The required investment decisions are then made. In making investment decisions several factors are considered. These include profits, dividend yield, price earning ratios, and expected future performance of financial institutions. This paper has considered expected future performance of financial institutions. In particular the paper derives a method of determining non - diversifiable risk in investment portfolios that enables investors and investment managers make viable investment decisions. This study is expected to improve the accuracy of predicting future expected performance of financial institutions. Investment analysts can now rely on the predictions to make good investment decisions.

1. Introduction

Risk underlies most investment decisions. This is because it is not possible to anticipate the occurrence of possible future events with certainty and hence, making any correct predictions about the cash flow sequence is not possible. The simplest expression for risk in investment is the standard deviation of returns on investments. For single investments other risk determining expressions include coefficient of variation and the Beta (B) factor method, Wilkes (1996) and Value at Risk, Vlaar (2000). When dealing with investment portfolios, return on investment is a weighted average of the expected returns of the individual investment. This alone will not allow one to make investment decisions since one needs to determine the best portfolio by looking at that which will give maximum returns at the lowest risk. This necessitates determination of portfolio risk, which is divided into non–diversifiable and diversifiable risk.

Diversifiable risk is that which the investor can eliminate if he held an efficient portfolio. The non–diversifiable risk on the other hand is that risk which still exists in all well diversified efficient portfolios. The investor therefore seeks to eliminate the diversifiable risk. Nevertheless non–diversifiable risk permeates most areas of investment. These are particularly those characterized by unstable rates of returns due to the erratic nature of market forces. These include interest related portfolios and also exchange rates related ones. For new, short term, and long term players in investment markets there is need for proper determination of non diversifiable risk as a basis for investment decision making. This paper focuses on the problem of limited tools for estimation and prediction of non–diversifiable risk in investment portfolios that has lead to indecisiveness by investors and incorrect investment decisions. Non–diversifiable risk is not normally determined since most estimators present are

used to determine diversifiable risk which can on the onset eliminate some risk by combining several investment portfolios until the best one is realized i.e. one with the highest returns at the lowest risk. Thus new and long–term investors who want to enter risky businesses, i.e. risk seekers, lack risk benchmarks on which to base their investment decisions. Many investors congest short–term investment markets believing that they have high returns and are less risky. This eventually brings down the earnings from these investments. For example short–term interest earning government bills and bonds will have their interest earning rates decreased if we have many investors in the market. Thus investors are left with only long term investments to consider. The situation could at least be made bearable if they knew precisely how much risk they are bound to encounter and how it will affect their rates of return.

The aim of this research was to determine a method of estimating non–diversifiable risk to enable investment managers make viable investment decisions. This estimator is that which accounts for the normal market conditions. This will decrease losses made by financial organizations due to inability to predict non–diversifiable risk accurately and reduce drastically the large provisions set aside to hedge and manage risk. The financial environment is full of risk hedging techniques and risk management measures for systematic risk. These are mostly done on speculative terms Jennifer (2003). There are no real concrete and long lasting solutions to the problems of non–diversifiable risk. Most financial market players blame it on the volatility of financial markets, i.e. that one cannot determine and predict non–diversifiable risk since the forces determining it keep on fluctuating.

The proper determination of non–diversifiable risk in this paper is expected to, prevent the making of wrong investment decisions by, for example, relying only on an investment managers belief or experience in risk management, boost the confidence of new, short - term and long–term investors since they would know the level of risk to be encountered. It would also enable investors have a broad spectrum of portfolios to invest in since most of them just engage in short term investments where non–diversifiable risk can be speculated easily, facilitate development of suitable risk managing and hedging techniques to minimize non–

diversifiable risk and set free colossal amounts of money from elaborate schemes hatched to hedge and manage portfolio risk thus enabling reinvestment to improve or increase the profitability of financial organizations. The uncertainty of non–diversifiable risk, i.e., the fact that it is believed not to be exactly known Robert (1993), always makes investment markets so jittery such that any small force acting on these markets sends them tumbling. The proper determination of non–diversifiable risk will go a long way into stabilizing the investment markets.

Basic concepts applied include, diversifiable risk (also known as non –systematic or specific risk) which is the risk that can be eliminated by diversification. It is unique to the company or it's industry and includes management competence and shifts in demand for company products, non–Diversifiable risk (also known as systematic or market risk) which is related to fluctuation of the market as a whole and cannot be eliminated by diversification, Portfolio expected returns a weighted average of the expected returns of each asset held in isolation and Investment portfolio which is a combination of more than one investment with a belief that risk shared is risk less.

2. Literature Review

Originally portfolio expected return was the only criterion for making investment decisions. This has been lacking particularly in areas where the future of the expected returns was uncertain. Thus other methods which incorporated determination of future uncertainties have to be sought. Markowitz (1952) identified statistical measures of dispersion as measures of risk to be a way of indicating future uncertainties in investment decision making. Later on he modified his risk estimator to include correlation coefficient. This was significant since for example if investments are perfectly negatively correlated i.e. their correlation coefficient is negative one, and then holding them in a portfolio greatly reduces their risk. This is synonymous to the idea of not carrying all your eggs in one basket, i.e. in different baskets risk is shared thus less. Researchers then began to examine the implications of all investors using this approach.

This led to the development of the Capital Asset Pricing Model (CAPM), Sharpe, *et al* (1964), which

gave a simple relationship between expected returns and risk in a competitive market. It also provided further insight into the process of diversification. Although there is a great deal of evidence which supports CAPM, there are major difficulties in testing the model. First it is stated in terms of investors' expectations rather than historic returns. Secondly the market portfolio (M) should include all risky investments, whereas most market indices contain only a sample of shares. Furthermore research in the 1980's mainly on U.S.A stocks revealed a number of stock market anomalies which are inconsistent with CAPM. These include "the small firm effect" and "calendar effects" Dimson (1988). The model should therefore not be relied upon for more than general indications of the market pricing mechanism. Currently, research in this area is centered on how to use the Markowitz estimator of risk and the CAPM in investment decision making. Less emphasis is being placed on further development of better risk estimators and models. Stulz (1999) argues that total risk is often costly and discusses how taking total risk into account in capital budgeting is necessary to make capital budgeting and capital structure decisions consistent. Harper (2003) discusses risk and returns in balanced portfolio and views the efficacy of cash and cash equivalents in investor portfolios. All the above researchers do not account for non–diversifiable risk as properly defined in their risk estimation thus rendering their estimators inaccurate. This paper seeks to remedy this by determining total risk which accounts for diversifiable and non–diversifiable risk according to its definition.

2.1. *Research methodology*

The population for this study was "Turnover of shares in Kenya shillings (Ksh) on the Nairobi Stock Exchange (NSE) from the year 2009 to 2013". NSE is a Kenyan Capital Market. Currently there are fifty six financial institutions listed on the NSE. Every day the total turnover of shares in terms of volume and Ksh. is recorded for each one of the fifty six financial institutions. Each month a bulletin is released showing the ten or twenty leading companies in terms of turnover in Ksh. These companies differ from one period to another. Thus data from NSE is first analyzed.

Returns for shares are derived by taking a base month and the weighted average share price of an investment stock for that month and considering it as a buying price Subsequent share prices are taken as the selling prices and the returns are thus calculated. Simple random sampling is then used to pick the companies with the best returns over the five year period. This entails ranking the companies in terms of one with the highest turnover in Ksh for the five year period. It also involves ensuring that the companies selected are those that are making positive returns frequently. This is due to the fact that this study determines methods that will enable investors make the best investment decisions, thus picking a financial institution with the highest and most frequent positive returns is the first step towards making good investment decision.

Turnover from shares is derived by multiplying the weighted average share price per month by the turnover of shares in volume per month. Simple random sampling is then used to pick a number of companies from those with the highest turnover over the five year period. Assumptions made include the following; the sample data size is representative of the population, the values of weights range from negative infinity to positive infinity. It is important to note that the estimators derived and tested are used to predict future performance of certain financial institutions from the rest of the population. These estimators are expected to be used for prediction of future performance of financial institutions situated in any location, the only limitation to the use of the estimators being the accuracy of the forecasting techniques. S–Plus version 2000 and Mat lab version 6.1 software are used for mathematical and statistical calculations, problem solving, graphing and forecasting.

3. Results

3.1. *Derivation of True Functions of Diversifiable and Non–Diversifiable Risk*

Let the Markowitz portfolio expected returns and diversifiable variance be given by (1) and (2) respectively. See Andrew (1995) and Alexander (1974).

$$E(R_u) = \sum_{i=1}^{\infty} x_i E(R_i) \qquad (1)$$

$$P_m^2 = \sum_{i=1}^{\infty} x_i^2 \sigma_i^2 + 2\sum_{i=1}^{\infty} \sum_{j}^{\infty} x_i x_j \sigma_{ij} \qquad (2)$$

Non–diversifiable variance is computed as

$$P_q^2 = \sum_{i=1}^{\infty} x_i^2 \sigma_i^2 \qquad (3)$$

where, $-\infty < x_i < \infty$, $0 < \sigma_i^2 < \infty$, and

x_i = Weight of an investment i

R_i = Return of investment i

$E(R_u)$ = Expected returns

σ_i^2 = Variance of investment i

σ_{ij} = Covariance of investments i and j

and u, m & q are arbitrary symbols differentiating i & u expected returns and non–diversifiable & diversifiable variances respectively.

To find the weight of investment i that will maximize expected returns and minimize total risk we apply the classical optimization method with no constraints See Rao (1994). We thus differentiate the expression;

$$E(R_u) - \sigma_m^2 = E\left(\sum_{i=1}^{\infty} x_i R_i\right) - \sum_{i=1}^{\infty} x_i^2 \sigma_i^2 - 2\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} x_i x_j \sigma_{ij} , i \neq j$$

with respect to x_i i.e.

$$\frac{\partial \{E(R_u) - P_m^2\}}{\partial x_i} = E(R_i) - 2x_i\sigma_i^2 - 2\sum_{j=1}^{\infty} x_j \sigma_{ij} = 0, i \neq j \qquad (4)$$

and differentiate

$$P_m^2 + P_q^2 = \sum_{i=1}^{\infty} x_i^2 \sigma_i^2 + 2\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} x_i x_j \sigma_{ij} + \sum_{i=1}^{\infty} x_i^2 \sigma_i^2 , i \neq j$$

with respect to x_i, i.e.

$$\frac{\partial \{P_m^2 + P_q^2\}}{\partial x_i} = 4x_i\sigma_i^2 + 2\sum_{j=1}^{\infty} x_j \sigma_{ij} = 0, i \neq j \qquad (5)$$

where $E(R_u) - P_m^2$ = maximum returns (derived by subtracting diversifiable variance from expected returns)

$P_m^2 + P_q^2$ = total variance (derived by adding diversifiable variance to non–diversifiable variance)

Note:

i) The second derivative of (4) is equal to $-2\sigma_i^2$ implying that x_i obtained will always maximize returns.

ii) The second derivative of (5) is equal to $4\sigma_i^2$ implying that x_i obtained will always minimize risk. See Rao (1994).

Equate (4) to (5) to get, the expected returns of investment i.

$$E(R_i) - 2x_i\sigma_i^2 - 2\sum_{j=1}^{\infty} x_j \sigma_{ij} = 4x_i\sigma_i^2 + 2\sum_{j=1}^{\infty} x_j \sigma_{ij}, i \neq j$$

$$E(R_i) = 6x_i\sigma_i^2 + 4\sum_{j=1}^{\infty} x_j \sigma_{ij}, i \neq j$$

$$\frac{1}{2}E(R_i) = 3x_i\sigma_i^2 + 2\sum_{j=1}^{\infty} x_j \sigma_{ij}, i \neq j \qquad (6)$$

Hence for infinite investments, values of x_i are given by the expression

$$\begin{pmatrix} 3\sigma_1^2 & 2\sigma_{12} & \dots & 2\sigma_{1\infty} \\ 2\sigma_{21} & 3\sigma_2^2 & \dots & 2\sigma_{2\infty} \\ \vdots & & \ddots & \vdots \\ 2\sigma_{\infty 1} & \dots & & 3\sigma_{\infty}^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_\infty \end{pmatrix} = \frac{1}{2}\begin{pmatrix} E(R_1) \\ E(R_2) \\ \vdots \\ E(R_\infty) \end{pmatrix} \qquad (7)$$

Therefore, it follows from the derived values of x_i in equation (7) and substituting them in the square root of equations (2) & (3) that the true function for portfolio diversifiable risk is given by

$$\left(\sum_{i=1}^{\infty} x_i^2 \sigma_i^2 + \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} x_i x_j \sigma_{ij}\right)^{\frac{1}{2}}, i \neq j \qquad (8)$$

and that for portfolio non-diversifiable risk is

$$\sum_{i=1}^{\infty} \left(x_i^2 \sigma_i^2\right)^{\frac{1}{2}} \qquad (9)$$

3.2 Empirical Study Using Sampled Data

In this section, an empirical study is done to check if the true functions developed in the previous section work as estimators.

i) The data used was turnover of shares for Kenya Commercial Bank (KCB) given in Appendix 5 and East African Breweries Limited given in Appendix 6

First we normalize /standardize the data by taking logarithms and getting the first difference i.e. log

$\left(\dfrac{y_t - y_{t-1}}{y_{t-1} - y_{t-2}}\right)$, whereby the sum of the ratios of the values of the difference between consecutive periods and the earlier of these periods will give the trend line,
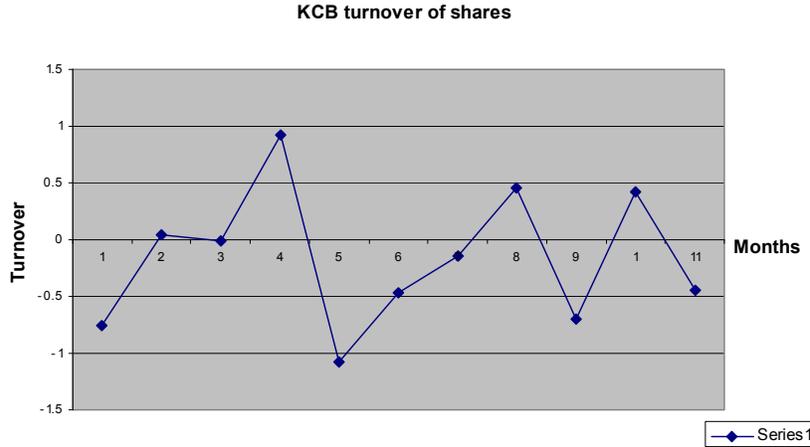
KCB turnover of shares

Figure 1: A plot of normalized turnover per month.

see Figure 1 given byMurray in1961 .

Final values for finding the risks are given by the weights,

$$\binom{x_1}{x_2} = 0.552 \begin{pmatrix} 3(0.3621844) & 2(-0.167223) \\ 2(-0.167223) & 3(0.590005) \end{pmatrix} \begin{pmatrix} \frac{1}{2}(-0.158996) \\ \frac{1}{2}(-0.142371) \end{pmatrix}$$

where,

variance of KCB = 0.3621844
variance of EABL = 0.590005
covariance of KCB & EABL = 0.167223
expected returns of KCB = - 0.158996
expected returns of EABL = - 0.142371

Therefore Diversifiable risk of a portfolio of KCB & EABL = 0.082687821 or 8.268% and its non – diversifiable risk = 8.88%.

ii) If we assume that $x_i \geq 0$ implying that sales are always positive and $\sum_{i=1}^{\infty} x_i = 1$ implying perfect market conditions for the same data as in i) above (Markowitz assumptions) then

$$P_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \sigma_{12}$$

where P_p^2 = portfolio variance.

To find the value of x_1 that minimizes portfolio risk we differentiate P_p^2 with respect to x_1 and equate it to zero

$$\frac{\partial P_p^2}{\partial x_1} = 2x_1 \sigma_1^2 + 2x_2 \sigma_{12}$$

$$2x_1 \sigma_1^2 - 2(1 - x_1)\sigma_{12} = 0$$

$$2x_1 \sigma_1^2 + 2\sigma_{12} - 2x_1 \sigma_{12} = 0$$

$$0.724369\, x_1 + 0.33444 - 0.33444\, x_1 = 0$$

$$0.389929\, x_1 = -0.33444$$

$$x_1 = \frac{-0.33444}{0.389929}$$

Note: The negative numerator value x_1 indicates short selling. (Act of selling securities you do not own)

since $x_i \geq 0$, x_1 becomes

$$x_1 = \frac{0.33444}{0.389929} = 0.85769$$
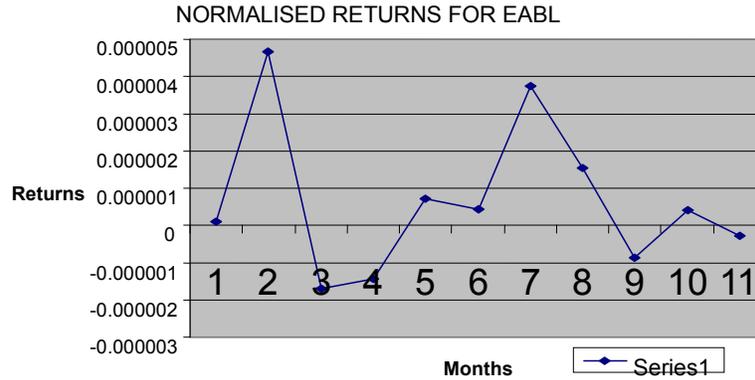
NORMALISED RETURNS FOR EABL

Figure 2: Normalized / Standardized Returns of EABL (derived by taking the first differences of the returns of shares, then the reciprocal of these differences).

since $\sum_{i=1}^{\infty} x_i = 1$, $x_2 = 1 - x_1 = 0.1423053$ and

Diversifiable risk = 85.769 % while Non–diversifiable risk = 14.23 %

iii) When we use returns of shares of KCB from Appendix 1 and EABL (Figure 2) from Appendix 4 we are not making any of the above assumptions

The final results are as follows, the weights

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 7.20592 \times 10^{23} \begin{pmatrix} 3.3922 \times 10^{-13} & 3.2582 \times 10^{-14} \\ 3.2582 \times 10^{-14} & 1.8238 \times 10^{-12} \end{pmatrix} \begin{pmatrix} -6.8657 \times 10^{-09} \\ -1.5087 \times 10^{-07} \end{pmatrix}$$

where

3 (variance of KCB) = 3.3922×10^{-13}

3 (variance of EABL) = 1.8238×10^{-12}

2 (covariance of KCB & EABL) = 3.2582×10^{-14}

½ (value for expected returns of KCB) = -6.8657×10^{-09}

½ (value for expected returns of EABL) = -1.5087×10^{-07}

$x_1 = -5220$, $x_2 = -198436.9026$

Therefore diversifiable risk = 5.310 and non–diversifiable risk = 17.215 %

THEOREM 1: Non-diversifiable risk, $\left(x_i^2 \sigma_i^2 \right)^{\frac{1}{2}}$ of a given investment i remains unchanged for i =1, 2, 3, …, ∞, investment of a given portfolio.

We make the following assumptions;

i) $-\infty < x_i < \infty$

ii) For diversifiability, $0 \le x_i^2 \sigma_i^2 < \infty$ and for non - diversifiability, $x_i^2 \sigma_i^2 = z$, where $0 \le z < \infty$.

iii) Sampled data is normally distributed and representative of the population.

iv) Variances of investments $i = 1, 2, 3, …, \infty$. are uncorrelated for non–diversifiability.

Proof:

From equation (5), the value of x_1 that minimizes risk for a two sampled investment portfolios is given by,

$$\frac{\partial (P_m^2 + P_n^2)}{\partial x_1} = 4x_1 s_1^2 + 2x_2 s_{12}$$

$$4x_1 s_1^2 + 2x_2 s_{12} = 0$$

$$4x_1 s_1^2 = -2x_2 s_{12}$$

$$\frac{4x_1 s_1^2}{4s_1^2} = \frac{-2x_2 s_{12}}{4s_1^2}$$

$$x_1 = -\frac{1}{2} \frac{x_2 s_{12}}{s_1^2} \qquad (10)$$

For three investments the value of x_1 that minimizes risk is given by,

$$\frac{\partial (P_m^2 + P_n^2)}{\partial x_1} = 4x_1 s_1^2 + 2x_2 s_{21} + 2x_3 s_{31} = 0$$

which gives

$$4x_1 s_1^2 = -2x_2 s_{21} - 2x_3 s_{31}$$

so that

$$x_1 = -\frac{1}{2}\frac{x_2 s_{21}}{s_1^2} - \frac{1}{2}\frac{x_3 s_{31}}{s_1^2} \qquad (11)$$

For four investments the value of x_1 that minimizes risk is given by,

$$x_1 = -\frac{1}{2}\frac{x_2 s_{21}}{s_1^2} - \frac{1}{2}\frac{x_3 s_{31}}{s_1^2} - \frac{1}{2}\frac{x_4 s_{41}}{s_1^2} \qquad (12)$$

continuing in the same manner we see that for infinite investments the value of x_1 that minimizes risk is given by,

$$x_1 = -\frac{1}{2}\frac{x_2 s_{21}}{s_1^2} - \frac{1}{2}\frac{x_3 s_{31}}{s_1^2} - \frac{1}{2}\frac{x_4 s_{41}}{s_1^2} \cdots \frac{1}{2}\frac{x_\infty s_{\infty 1}}{s_1^2} \qquad (13)$$

Similarly $x_2, x_3, ..., x_\infty$ can be obtained.

Taking expectations of equations (10), (11), (12), (13), to remove bias in the sample variance and considering the assumption (iv) we obtain

$$E\left(x_1 s_1^2\right) = E\left(-\tfrac{1}{2}x_2 s_{21}\right)$$

$$x_1 E\left(s_1^2\right) = -\tfrac{1}{2}x_2 E\left(s_{21}\right)$$

$$x_1\left(\frac{n-1}{n}\right)\sigma_1^2 = \tfrac{1}{2}x_2\left(0\right)$$

$$as \; n \to \infty$$

$$x_1 \sigma_1^2 = 0$$

where n is the sample size and n>1. NOTE: $E\left(\sigma_{ij}\right) = 0$ from assumption (v) . For equation (11),

$$E\left(x_1 s_1^2\right) = E\left(-\tfrac{1}{2}x_2 s_{21}\right) + E\left(-\tfrac{1}{2}x_3 s_{31}\right),$$

$$x_1\left(\frac{n-1}{n}\right)\sigma_1^2 = -\tfrac{1}{2}x_2\left(0\right) - \tfrac{1}{2}x_3\left(0\right),$$

$$as \; n \to \infty$$

$$x_1 \sigma_1^2 = 0 .$$

Similarly for investments 4, 5, 6,…, ∞, $x_i\sigma_i^2 = 0$ .Thus

non–diversifiable risk estimator (since we are using sampled data) is $P_q = \left(x_i^2 \sigma_i^2\right)^{\frac{1}{2}}$ of investment $i = 1, 2,$ …, ∞, for a given portfolio remains unchanged.

iv) Analysis of returns of shares of three investments (see Appendix 1, 2 and 3) gives the following results,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 15097 \times 10^{21} \begin{pmatrix} -1.14589 \times 10^{-18} & 2.82915 \times 10^{-20} & -9.10521 \times 10^{-15} \\ 2.82915 \times 10^{-20} & -9.10521 \times 10^{-15} & 6.83994 \times 10^{-21} \\ -9.10521 \times 10^{-15} & 6.83994 \times 10^{-21} & -4.85532 \times 10^{-20} \end{pmatrix}$$

$$\begin{pmatrix} -6.86872 \times 10^{-08} \\ -1.50874 \times 10^{-07} \\ 3.32536 \times 10^{-07} \end{pmatrix}$$

$$(14)$$

where the right hand side is the product of the reciprocal of determinant of variance covariance matrix, inverse of variance covariance matrix and values representing means of the three investment returns.

$$x_1 = -3.484843067$$
$$x_2 = 1.581127467$$
$$x_3 = 0.719814553$$

Thus diversifiable risk of a portfolio of KCB, EABL &STAN CHART $= 2.54382 \times 10^{-06}$ %, and its non–diversifiable risk $= 8.93518 \times 10^{-06}$ %.

For a two investment portfolio of KCB and EABL diversifiable risk for KCB is 0.01755 and that of EABL is 0.15460. For an investment portfolio of KCB, EABL and STANCHART diversifiable risk for KCB is 2.15374×10^{-09}, EABL is 8.71066×10^{-08} and that of STAN CHART is 9.14787×10^{-11} .

Clearly portfolio risk has been diversified but non–diversifiable risk for KCB and EABL is not the same as that one for two investments as per the definition of non–diversifiable risk and the findings of theorem one. This is also true for turnover of a three and four investment portfolio. Thus there is some white noise in the data analyzed. We therefore remedy our estimator for non–diversifiable risk to include the random error.

This is done by adding $\sum_{i=1}^{\infty}\left(s_{e_i}^2\right)^{\frac{1}{2}}$ to the non–diversifiable risk estimator. We thus denote this model estimator as,

$$p_G = \sum_{i=1}^{\infty}\left(x_i^2 s_i^2\right)^{\frac{1}{2}} + \sum_{i=1}^{\infty}\left(s_{e_i}^2\right)^{\frac{1}{2}}$$

where e_i is an independent random variable with mean zero and variance $\dfrac{s_{ei}^2}{n-1}$ (i.e. sample variance)

THEOREM 2: The non–diversifiable risk model estimator P_G is a consistent estimator of non–diversifiable risk P_q.

We make the following assumptions;

i) e_i is an independent random variable with mean zero

i.e. $E\left(\sigma_{e_i}^2\right) = 0$ and variance $\dfrac{\sigma_{e_i}^2}{n-1}$

ii) Variances of investments $i = 1, 2, 3, \ldots, \infty$. are uncorrelated.

iii) Sampled data is representative of the population and is normally distributed.

Where G & q are arbitrary symbols.

Proof:

By definition we note that

$$P_G = \sum_{i=1}^{\infty}\left(x_i^2 s_i^2\right)^{\frac{1}{2}} + \sum_{i=1}^{\infty}\left(s_{e_i}^2\right)^{\frac{1}{2}}$$

Thus

$$s_G^2 = \sum_{i=1}^{\infty}x_i^2 s_i^2 + 2\sum_{i=1}^{\infty}\left(x_i^2 s_i^2\right)^{\frac{1}{2}}\sum_{i=1}^{\infty}\left(s_{e_i}^2\right)^{\frac{1}{2}} + \sum_{i=1}^{\infty}s_{e_i}^2 \qquad (15)$$

The expectation of equation (15) is

$$E\left(P_G^2\right) = E\left(\sum_{i=1}^{\infty}x_i^2 s_i^2\right) + E\left[2\sum_{i=1}^{\infty}(x_i s_i)(s_{e_i})\right] + E\left(\sum_{i=1}^{\infty}s_{e_i}^2\right)$$

Hence

$$E\left(P_G^2\right) = \frac{n-1}{n}\left(\sum_{i=1}^{\infty}x_i^2\sigma_i^2\right)$$

as $n \to \infty$

$$E\left(P_G^2\right) = (n-0)\left(\sum_{i=1}^{\infty}x_i^2\sigma_i^2\right)$$

Note : Since s_i^2 is the sample variance of sampled data of investment i which is normally distributed thus there is bias in the unmodified form of the sample variance.

Therefore

$$(P_G) = \sum_{i=1}^{\infty}\left(x_i^2\sigma_i^2\right)^{\frac{1}{2}} = \sigma_q$$

This proves that P_G is an unbiased estimator of P_q.

From (15), observe that

$$\operatorname{var}\left(P_G^2\right) = \operatorname{var}\left(\sum_{i=1}^{\infty}x_i^2 s_i^2\right) + \operatorname{var}\left[\sum_{i=1}^{\infty}\left(x_i^2 s_i^2\right)^{\frac{1}{2}}\left(s_{e_i}^2\right)^{\frac{1}{2}}\right] + \operatorname{var}\left(\sum_{i=1}^{\infty}s_{e_i}^2\right)$$

$$= \left(\frac{2}{n-1}\sum_{i=1}^{\infty}x_i^4\sigma_i^4\right) + \left(\frac{2}{n-1}\right)\left(\frac{2}{n-1}\right)\left(\sum_{i=1}^{\infty}x_i^4\sigma_i^4\right)\left(\sum_{i=1}^{\infty}\sigma_{e_i}^4\right)$$

$$\operatorname{var}\left[\left(\sum_{i=1}^{\infty}x_i^2\sigma_i^2\right)^{-\frac{1}{2}}\left(\sum_{i=1}^{\infty}\sigma_{e_i}^2\right)^{-\frac{1}{2}}\right] + \left(\frac{2}{n-1}\right)\left(\sum_{i=1}^{\infty}\sigma_{e_i}^4\right)$$

$$= \frac{2}{n-1}\left[\left(\sum_{i=1}^{\infty}x_i^4\sigma_i^4\right) + \left(\frac{2}{n-1}\right)\left(\sum_{i=1}^{\infty}x_i^4\sigma_i^4\right)\left(\sum_{i=1}^{\infty}\sigma_{e_i}^4\right)\operatorname{var}\left[\left(\sum_{i=1}^{\infty}x_i^2\sigma_i^2\right)^{-\frac{1}{2}}\left(\sum_{i=1}^{\infty}\sigma_{e_i}^2\right)^{-\frac{1}{2}}\right] + \left(\sum_{i=1}^{\infty}\sigma_{e_i}^4\right)\right]$$

$$(16)$$

Note: In equation (16), $\left(x_i^2\sigma_i^2\right)^{\frac{1}{2}}$ is rewritten as $\left(x_i^2\sigma_i^2\right)\left(x_i^2\sigma_i^2\right)^{-\frac{1}{2}}$ and $\left(\sigma_{e_i}^2\right)^{\frac{1}{2}}$ as $\left(\sigma_{e_i}^2\right)\left(\sigma_{e_i}^2\right)^{-\frac{1}{2}}$ to assist in simplification. Also $\operatorname{var}\left(s^2\right) = \dfrac{2}{n-1}\sigma^4$ as proved by Tobago (2010)

Recalling that as $n \to \infty$ $E\left(\sigma_G\right) = \sigma_q$, it follows from this result (i.e. $\operatorname{var}\left(\sigma_G\right) \to 0$) that P_G is a consistent estimator of Non–diversifiable risk.

4. Discussion

4.1. *Presentation of Results*

4.1.1 Forecasts of Time Series

In forecasting we used GARCH (Generalized Auto-Regressive Conditional Heteroskedastic) models. This is because the time series values have low valued correlations. In GARCH modeling the true GARCH (A, B) model parameters of the time series are entered. These parameters correspond to a given GARCH (A, B) model for the conditional variance F (t) and innovations Y (t), sequences

$$F(t) = L(t) + S(1)*F(t-1) + S(2)*F(t-2) + \cdots +$$
$$S(P)*F(t-P) + W(1)Y^2(t-1) + W(2)Y^2(t-2) +$$
$$\cdots + W(Q)Y^2(t-Q)$$

for time steps t = 1,2, … N, where

  S = order of AR

  W = order of MA

  A & B = model order determined by the number of
      elements of *S & W*

  S, W = coefficient.

  t = the current time index.

  Y (*t*) = square root *F* (*t*) *L* (*t*) where *L* (t) is an
      identical and independent sequence *N* (0, 1)
      and *Y* and *F* are related.

  The forecasts for the investment time series from
Appendix 1 through to Appendix 3 are given in Table I
and Table II.

*4.1.2 Prediction of Expected Returns Diversifiable Risk
and Non–Diversifiable Risk*
Using the forecasts from Section 4.1.1 above and
equations 7, 8 and 9 the predictions for expected
returns, diversifiable risk and non–diversifiable risk are
made as Table III and Table IV.

4.2. *Result Discussion*

4.2.1 Derivation of Forecasts
In forecasting using GARCH models we let t be the
current time index, {F (t)} be the return series of
interest, {Y (t)} the innovations noise process and L(t)
be an identical and independent sequence. The input
coefficient vectors AR (Autoregressive) and MA
(Moving Average) are specified exactly as they would

TABLE I: The table below represents forecasted returns of shares of the given companies in Kenyan shillings (ksh).

COMPANY	EABL	ICDC	KCB	STAN CHART
1	55660000	3881300	38460000	32670000
2	42710000	3590200	29520000	25070000
3	36010000	3320800	24880000	21130000
4	31720000	3071700	21920000	18620000
5	28680000	2841300	19810000	16840000
6	26370000	2628200	18230000	15480000
7	24550000	2431000	16960000	14400000
8	23050000	2248600	15930000	13540000
9	21810000	2080000	15070000	12790000
10	20740000	1923900	14330000	12180000
11	19820000	1779600	13690000	11630000
12	19000000	1646100	13130000	11160000
13	18290000	1522600	12640000	10730000
14	17650000	1408400	12200000	10360000
15	17070000	1302800	11790000	10020000
16	16540000	1205000	11430000	9710000
17	16060000	1146000	11100000	9420000

TABLE II: The table below represents forecasts of turnover of shares of the given companies in ksh.

COMPANY	EABL	ICDC	KCB	STAN/CHART
1	105230000	8609700	406500000	69829000
2	99830000	7718800	385640000	66245000
3	94710000	6920100	365850000	62846000
4	89850000	6204100	347080000	59621000
5	85240000	5562100	329270000	56561000
6	80860000	4986600	312370000	53659000

be read from the ARMA (Autoregressive Moving Average) (R,M) model equation when solved for F (t): F (t) = L (t) + AR (1) F (t-1) +… + AR(R) F (t-R) + Y (t) MA (1) Y (t-1) +… + MA (M) Y (t-m)

series. To estimate, and make forecasts from the GARCH (A, B) parameters from the equation: Y (t) = square root (F (t)*L (t)) where Y (t) represents innovations noise process, you simulate (i.e. , to

TABLE III: Seventeen month predictions of returns of shares of the given companies

Portfolio	Expected returns	Diversifiable risk	Non–diversifiable risk
EABL & ICDC	0.559 or 55.9 %	0.219 or 22 %	0.223 or 22.32 %
ICDC & STANCHART	1.27 or 127 %	0.503 or 50.3 %	0.5104 or 51.04 %
EABL & STANCHART	1.14 or 114 %	0.478 or 47.8 %	0.478 or 47.84 %
EABL, ICDC & STANCHART	1.34 or 134 %	0.532 or 53.2 %	0.552 or 55.21 %

TABLE IV: Six month Predictions of turnover of shares of the given companies

Portfolio	Expected returns	Diversifiable risk	Non–diversifiable risk
EABL & ICDC	1822 %	150.82 %	255 %
ICDC & STANCHART	1821 %	150.81 %	262 %
EABL & STANCHART	2063 %	203 %	203.2 %
EABL, ICDC & STANCHART	474 %	83.79 %	95.3 %

Note that the coefficients of F (t) and Y (t) are assumed to be 1, and are not part of the AR / MA input vectors.

For the following ARMA (2, 2) model, of ICDC

F (t) = 0.036Y (t-1) - 0.002 Y (t-2) + Y (t) + 0.402 Y (t-1) + - 0.107 Y (t-2)

AR = [0.036 – 0.002] and MA = [0.402 – 0.107]. The first 20 weights of the infinite order AR approximation may be found as follows:

ZI = garchar ([0.036 – 0.002], [0.402 – 0.107], 20);

where ZI represents weights of the polynomials generated and garchar converts j–th lag of the return series and innovations processes F (t-j) and Y (t-j), respectively. To maintain consistency, the j-th element of the truncated infinite–order auto- regressive output vector, ZI (j), is the coefficient of the j–th lag of the observed return series, F (t-j), in the infinite order representation of the input ARMA (R, M) process. The AR and MA input vectors differ from the corresponding AR and MA 'polynomials' formally presented in time

reverse–engineer the process for comparison) (see Bollersler (1986), Box, Jenkins (1994), Engle and Robert (1982), Hamilton 1994). Figures 3 and 4 show that the forecasted time series is normally distributed thus there is no need for its standardization before making predictions. This also explains the downward trend of the forecasts i.e. to maintain the bell like shape of the curve indicative of normally distributed data. (Note that the plots show values for half the distribution because both halves are identical). See Lucey (2000). Returns of shares are forecasted in the long term since these data were prepared for long term investors who are not ready to sell their shares in the near future. Short term forecasts are meant for speculative buyers who are risk averse and can sell their shares anytime the prices go up. The estimators used to predict future expected returns, diversifiable risk and non–diversifiable risk are applicable within the normal ranges of negative infinity to positive infinity. This is reflected in Tables III and IV where values range from the tens to hundreds to thousands etc, compared to Markowitz estimator of diversifiable risk whose weight values x_i must be greater than or equal to zero i.e. $x_i \geq 0$. The sum of these weights for any number of investments in a portfolio must be equal to one i.e.

$\sum_{i=1}^{\infty} x_i = 1$. The first assumption limits the data set being used since if it is one with negative returns the estimator collapses. This means that one assumes positive returns always and yet many a time financial institutions experience losses or negative sales. The second assumption implies perfect market conditions, i.e. the portfolio is efficient with no systematic risk present. Under normal financial environments this is not true since fluctuations in returns do occur frequently. The development of an estimator for non–diversifiable risk in this study is an indication of the presence of market risk.

For R_k the return of k CAPM is given by

$$E(R_k) = E(R_f) + \frac{\left(E(R_m) - E(R_f)\right)\sigma_{km}}{\sigma_m^{\,2}}$$

where
  m= market portfolio
  $E(R_k)$ = expected returns of the risky security k
  $E(R_f)$ = risk free rate of return
  $E(R_m)$ = expected returns of the market portfolio m
  $\sigma_m^{\,2}$ = variance of the market portfolio m
  σ_{kn} = covariance between the single risky security k and the market portfolio m

This model works under the following assumptions, See Andrew (1993):

i) All investors are risk–averse and measure risk in terms of standard deviation of portfolio return (as for the Markowitz model).
ii) All investors have a common time horizon for investment decision making (e.g. one month or two years).
iii) All investors have identical subjective estimates of future returns and risks for all securities.
There exists a risk–free asset and all investors may borrow or lend unlimited amounts at the risk–free nominal rate of interest.
iv) All securities are completely divisible, there are no transaction costs or differential taxes, and there are no restrictions on short–selling.

Information is freely and simultaneously available to all investors.

Although there is a great deal of evidence which supports CAPM, much courage is required to develop a model on the basis of these assumptions. Many of them are clearly unrealistic, for example the assumption that there exists a risk–free asset and all investors may borrow or lend unlimited amounts at the risk–free nominal rate of interest. The model should therefore not be relied upon for more than general indications of the market pricing mechanism as seen in section 2.1. The estimators developed in this study have addressed the above short comings such that none of the above assumptions are made in deriving these estimators.

Furthermore the Beta factor i.e. $B_k = \frac{\sigma_{km}}{\sigma_m^{\,2}}$ is an indicator of non–diversifiable risk. This is not a good indicator since from the definition of non–diversifiable risk, there should not be any correlation between the various investments, yet the expression σ_{km} in the beta factor formulae represents covariance between single risky security k and the market portfolio M which counterfeits the definition of non–diversifiable risk, thus giving weight to the doubts cast on the CAPM practical applicability by researchers in the 1980's. The predictions in tables III and IV indicate that the higher the risks the higher the expected returns. This is expected since high risks imply venturing into unknown areas. Thus one can only be motivated to do so if he/she expects big rewards for taking these risks. This is experienced in financial markets where monopolists venture into new business areas. If their ventures are successful they expect to reap maximum benefit since there is no competition from other financial institutions. If they fail in their ventures they stand to incur heavy losses since they will not be able to recover their base capital among other costs incurred.

There would be no other financial institution trading in the same items thus they cannot sell their business to them or merge with these financial institutions in order to diversify their risk thus minimizing it and reaping the benefits of economies of scale.

The predictions show a clear distinction between diversifiable risk and non–diversifiable risk, whereby non–diversifiable risk is always the higher of the two risks. Table IV which has six month predictions of turnover of shares shows this observation clearly. Unlike the data for returns of shares, the data for

turnover of shares is not derived using a common reference price. Variances in the time series tend to be huge leading to higher risks. The length of time the forecasts postulate, i.e. six month have an impact on risk. Whereby investors can sell their investments easily when the prices are high thus returns will be high. The high returns will thus be accompanied by high risks as experienced by monopolists.

5. Conclusion and Recommendations

5.1. *Conclusion*

In this study a non–diversifiable risk model estimator, diversifiable risk and expected return estimators have been developed. In doing so we have done away with the many theoretical like assumptions that are usually employed in similar studies as seen in Section 4.3.1 The presence of white noise in non–diversifiable risk has been established which confirms the fact that such a risk is independent i.e. cannot be diversified.

This paper confirms that the risks are high as it should be in the stock market due to the high volatility experienced. Moreover, our study indicates that diversification reduces diversifiable risk but not non–diversifiable risk. Investment analysts have relied on Markowitz estimator of risk Markowitz (1952) , the market model and the CAPM to analyze the effect of risk in investment decision making. These have had various shortcomings as evidenced by earlier researchers (Section 2.0) and this study's findings (Section 4.3). Development of risk estimators and expected return estimators that have addressed these shortcomings in this study is expected to improve the accuracy of predicting future expected performance of financial institutions. Investment analysts can now rely on the predictions to make good investment decisions. Financial institutions can now venture into the unknown future financial environment knowing what to expect. Predictability of the future of financial institutions is expected to spill over to investment markets. Overtime investment markets have been known to be unstable, particularly stock, credit, bond and foreign exchange markets. It is hoped that a systematic method will be established which will ensure frequent determination and publication of non–diversifiable risk for respective financial institutions, i.e., those on NSE and other capital markets with a view to reducing their future

uncertainties. Stabilized investment markets mean increased investor confidence. We should therefore expect to see an influx of investors in the investment markets.

A thriving investment market bolsters the economy. Such that we have increased levels of foreign exchange which strengthens the value of the Kenyan shilling. Imports and exports then become cheap thus influencing the economy positively. New financial institutions will be started and the existing ones sustained. Employment opportunities will be created leading to increased purchasing power, money supply and Gross Domestic Product. All these improve the economy and eventually the way of life of individuals in this economy. Lending and borrowing of funds is hoped to increase due to stable interest rates. Small scale businesses will thrive, improving the standards of living of individuals in these business environments.

Determination of non–diversifiable risk and prediction of future performance of financial institutions does not eliminate risk. Nevertheless their proper estimation enables financial analysts make accurate current and future business plans (Paul 2003). Expected future cash flows are made accurately by incorporating this determined risk in their discounting methods. Correct capital investment decision making is facilitated. Required levels of cash and cash equivalents will always be maintained leading to high profitability of the financial institutions. Capital allocation will also be made accurately, since the optimum portfolio will be accurately identified. Despite the good investment decision making and proper financial planning anticipated portfolio risk has to be addressed as an entity. If left unchecked it can cause unexpected losses. This is particularly for financial institutions which trade in interest related portfolios and foreign exchange trading portfolios. With the determination of total risk, risk management techniques are expected to be properly initiated and established, Jennifer (2003). Hedging schemes such as derivatives, options and futures are hoped to cost less, karithi (2003). Savings are anticipated in risk management departments which should be used to increase the wealth of owners of the financial institutions. These will then motivate them to continually invest in them thus sustaining their growth and development.

Having discussed the positive impact this study is expected to have on the general environment, unstable or risky environments should be a thing of the past, while stabilized environments should be the norm.

5.2. *Recommendations*

From the above conclusive results this paper recommends the use of all derived estimators in both capital allocation in investment portfolios and net present value investment decision making criterion.

Risk is not only experienced in financial environments. It is also encountered in such areas as medical practices, whereby there is some risk attached to various curative methods. Notably are the radiotherapy and chemotherapy methods of curing cancer. The environment also experiences some health hazards due to environmental pollution or poor sanitation, i.e. environmental risk. It would be challenging to adapt the risk determining methods developed in this study in determining medical and environmental risks with a view to reducing them.

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APPENDICES

Appendix 1:

Table V: Returns For KCB Shares from January 2009 – December 2013 Base Price April 2009 in Kenyan Shillings (Ksh)

YEAR / MONTH	2009	2010	2011	2012	2013
JAN	-254977217	-4909732	-48293377	-28599388	-176773446
FEB	-11858496	-18114517	-35537228	-86093014	-27392971
MAR	-20843003	-11145914	-20663935	-235975275	-235975275
APR	-35789720	-20446319	-16526970	-249196506	-201979585
MAY	-12178766	-72691432	-44283975	-12896377	-72325367
JUN	-31529069	-69476411	-12607319	-24309319	-3479830
JUL	-8193042	-15160040	-94872136	-28713343	-80326198
AUG	-8925414	-78123604	-73423079	-16496101	-30548808
SEP	-27908155	-35911665	-7891213	-48476514	-44356839
OCT	-6450228	-7641496	-23904507	-60275745	-31303040
NOV	-19916397	-19712658	-10867080	-140518771	-39819260
DEC	-6620513	-2055670	-4412974	-16975041	-61658147

Appendix 2:

Table VI: Returns For Standard Chartered Shares from January 2009 – December 2013 Base Price April 2009 in Ksh

YEAR / MONTH	2009	2010	2011	2012	2013
JAN	-10356582	2465938	529514	1111462	7044060
FEB	114576	6572177	3748395	1485758	7846250
MAR	328887	938150	4481874	393326	21271177
APR	-260376	811697	6695297	-82382	21024330
MAY	-951995	473787	910638	713692	40984696
JUN	435601	-239449	2715433	1376041	53320718
JUL	2712442	698182	3454344	2466946	14738214
AUG	2979260	170156	2664586	3125082	45759692
SEP	3627526	11828563	788886	789498	30389636
OCT	2478381	6078253	2120709	2634216	250200283
NOV	4948147	1699087	618373	4069588	83728614
DEC	1474898	-832158	543333	6367901	15751289

Stan chart is one of the three companies with the highest returns and least negative sales thus analyzed for investment purposes.

Appendix 3:

Table VII: Returns for ICDC (Industrial Commercial Development Corporation) Shares From January 2009 – December 2013
Base Price April 2009 1n Ksh.

YEAR / MONTH	2009	2010	2011	2012	2013
JAN	11935779	834948	3910778	-661320	-4407558
FEB	418644	1115537	2724842	-379153	-1360183
MAR	1346134	748676	3267626	-614656	-137580
APR	3239774	130815	486736	-2644600*	89436
MAY	971928	936465	3403992	-980303	1238786
JUN	731335	-356349	9535859	-1628149	3385594
JUL	850178	458268	185853	-10295746	-510603
AUG	1439235	344772	961124	-32399844	15592897
SEP	4163159	4570032	-652724	-236345	5216308
OCT	2460198	945932	73658	-552725	6314293
NOV	774004	2910810	-666640	-572520	3275379
DEC	1459217	507676	-96840	-148838	13410155

* This value is calculated from Table XV as 20.875 – 38.375 = - 17.5 X 151120 = - 2644600

Appendix 4:

Table VIII: Returns For East African Breweries Shares From January 2009 – December 2013 Base Price April 2009 in Ksh.

YEAR / MONTH	2009	2010	2011	2012	2013
JAN	-10106177	-173378	4433440	980372	22434220
FEB	2715705	-1470663	17493087	1298971	49769005
MAR	-9821081	-830676	5217513	1314180	50841507
APR	-283444	-914474	0*	856305	90741122
MAY	1748910	-2648273	2786088	3520085	137930125
JUN	948375	-7199135	459345	2908110	199612113
JUL	489416	1106750	676910	13447207	22265976
AUG	16667673	-3985070	-756480	14598393	323570569
SEP	4112532	1861435	322465	22556215	155506703
OCT	386565	3300665	4528066	8768165	162636994
NOV	-321409	-9450660	1236078	29758680	177642340
DEC	503414	1054648	4122929	60118940	82052882

* is a rare value which indicates that the share price of April 2011 is at par with that of April 2009.

Appendix 5:

TABLE IX: Turnover for KCB from shares for the years 2009 – 2013 IN KSH

YEAR MONTH	2009	2010	2011	2012	2013
JAN	327531709	4107488	22019778	8954014	77310126
FEB	57563116	18114517	16772678	25391488	11980046
MAR	64039370	6958143	10979475	63794401	31424618
APR	6232688	14910085	32051257	67655160	148367720
MAY	518649381	43367048	21730863	3217234	3331709520
JUN	43872364	43372557	13403376	4093669	72749417
JUL	15061956	9443960	29123540	4765700	97403422
AUG	10745499	43918134	20571842	2858406	54024740
SEP	31055691	20150434	3966706	6897065	85512669
OCT	6136675	4387974	6767486	10770750	80214040
NOV	16424561	12306168	3605193	30860459	137787282
DEC	5838395	12496811	1241589	4357636	79203149

All values are greater than zero indicating presence of trading for the shares of KCB within this period.

APPENDIX 6:

TABLE X: Turnover for east african breweries from shares for the year 2009 – 2013 in KSH.

YEAR MONTH	2009	2010	2011	2012	2013
JAN	456462305	4629179	57827475*	15273156	60558630
FEB	14000968	9845272	228170700	18433014	96149482
MAR	108458889	32040360	55053761	11714976	86315305
APR	78230475	30749188	44099577	8507807	171045565
MAY	42119583	40502995	33655937	33067468	66678263
JUN	14083369	39176690	5991450	18107100	295773078
JUL	10172851	7581238	7888605	108469564	37608452
AUG	121598251	59206758	3609040	93887704	429389988
SEP	34900943	26414646	2949602	108146238	205652636
OCT	3955849	46838001	30125497	36685194	201641710
NOV	12397185	45087524	11018748	98166600	212222050
DEC	20424215	26285067	71302424	153152160	233933799

This value is derived as follows: From Appendix 10 the row for EABL we have 771033 X 75 = 57827475