SHEAR BUCKLING BEHAVIOR OF RECTANGULAR THIN PLATE WITH VARIABLE THICKNESS

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Abstract

A simplified computational procedure for shear buckling problems of rectangular thin plates with constant and variable thickness is presented. The discretization of the problem is carried out by means of finite differences. Geometric and material non-linearity are neglected. The effect of boundary conditions, aspect ratios, and tapering ratios on the shear buckling behavior is considered. The plate was analyzed with different tapering ratios (t_a/t_o) (1.0, 1.25, 1.5, 1.75, and 2.0). It is concluded that the shear buckling behavior of thin plate is very sensitive to the magnitude of tapering ratio.

Keyword: Finite differences, Eigen-value problem, Buckling behavior, Rectangular plates, Tapered plates

تحليل الإزاحة الكبيرة المرن-اللدن للصفائح غير متماثلة الخواص تحت حمل ضغط في المستوي

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الخلاصة

يقدم البحث طريقة حسابية مبسطة لتحليل الانبعاج المرن لصفائح رقيقة مستطيلة ذات سُمك مُتغير المقطع .تم تبسيط المسألة باستخدام طريقة الفروقات المحددة. اللاخطية الهندسية والمادية تم اهمالها. تم تمثيل مؤثرات اختلاف السُمك ونسب الأبعاد وظروف الإسناد على حمل الانبعاج القصي. تم تحليل الصفيحة مع نسب اختلاف السمك (١.٠,١.٢٥,١.٥,١.٧٥،و ٢.٠). تَبين من ذلك أن حمل الانبعاج القصي للصفائح الرقيقة حساس جدا الى قيمة التغير في اختلاف السمك.

Notations

$\boldsymbol{A} = 6\boldsymbol{h}_x^4 + 6\boldsymbol{h}_y^4 + 8\boldsymbol{h}_x^2\boldsymbol{h}_y^2$

$\boldsymbol{a},\boldsymbol{b}$ = Plate dimension in $\boldsymbol{x}$ and $\boldsymbol{y}$- directions respectively.

[$\boldsymbol{B}$] = In-plane stiffness matrix.

$\boldsymbol{B_x} = -4\boldsymbol{h}_y^2(\boldsymbol{h}_x^2 + \boldsymbol{h}_y^2)$

$\boldsymbol{B_y} = -4\boldsymbol{h}_x^2(\boldsymbol{h}_x^2 + \boldsymbol{h}_y^2)$

$\boldsymbol{C} = 2\boldsymbol{h}_x^2\boldsymbol{h}_y^2$

c = clamped edge.

$\boldsymbol{c_t} = (\boldsymbol{t_a} - \boldsymbol{t_o})/\boldsymbol{a}\boldsymbol{t_o}$ =Slope coefficient in- the plate.

$\boldsymbol{D} = \boldsymbol{E}\boldsymbol{t}_o^3/12(1-\nu^2)$ Modulus of Rigidity.

$\boldsymbol{D_{av}} = \boldsymbol{E}\boldsymbol{t}_{av}^3/12(1-\nu^2)$

$\boldsymbol{D_x} = \boldsymbol{h}_y^4$

$\boldsymbol{D_y} = \boldsymbol{h}_x^4$

$\boldsymbol{E}$ = Modulus of Elasticity.

$\boldsymbol{h_x},\boldsymbol{h_y}$ = Mesh size at $\boldsymbol{x}$ and $\boldsymbol{y}$-direction, respectively.

[*K*] = Bending stiffness matrix.

$\boldsymbol{N_x}$, $\boldsymbol{N_y}$, $\boldsymbol{N_{xy}}$ = In-plane forces.

$\boldsymbol{q}$= Transverse load.

s = Simple supported edge.

$\boldsymbol{t}$ **=** Plate thickness.

$\boldsymbol{t_a}$ **=** Thickness at the side x=$\boldsymbol{a}$.

$\boldsymbol{t_{av}}$ = Average thickness (($\boldsymbol{t_a}$+$\boldsymbol{t_o}$)/2).

$\boldsymbol{t_o}$ **=** Thickness at the side x=0.

$\boldsymbol{\nu}$ = Poisson's ratio.

$\boldsymbol{w}$ **=** Out-of-plane displacement.

Introduction

Thin plate elements used in naval and aeronautical structures are often subjected to normal **(Figure 1)** and shearing forces acting in the plane of the plate. If such in-plane forces are sufficiently small, the equilibrium is stable and the resulting deformations are characterized by the absence of the lateral displacements. As the magnitude of these in-plane forces increases, at certain load intensity, a marked change in the character of the deformation pattern takes place. That is, simultaneously with the in-plane deformations, lateral displacements are introduced. In this condition, the originally stable equilibrium becomes unstable and the plate is said to have buckled. The load producing this condition is called the critical load. The importance of the critical load is the initiation of a deflection pattern, which, if the load is further increased rapidly leads to very large deflections and eventually to complete failure of the plate. This is a dangerous condition, which must be avoided.

Though considerable amount of information is available on the buckling of isotropic and orthotropic plates of constant thickness with different boundary conditions and subjected to various types of loading [e.g. Salvadori (1949), Timoshenko and Gere (1961)], there are limited number of research works on rectangular plates with variable thickness. Chehil and Dua (1973) investigated the buckling behavior of simply supported rectangular plates with a linear thickness variation in one direction. They employed perturbation method to solve the governing equation of rectangular plate with variable thickness. Kobayashi and Sonoda (1990) used a power series method with the used of a coordinate transformation to solve analytically the buckling problem of uniaxially compressed rectangular plates with linearly tapered thickness. Chin, *et al* (1993) used finite element method to predict the buckling capacity of arbitrary shaped thin-walled members under any general load and boundary conditions. Ohga, et al (1995) used analytical procedure for the elastic buckling problems of thin-walled members with variable thickness cross section by using the transfer matrix method. More recently, Hussain, *et al* (2002) used the finite difference method to estimate buckling factor of rectangular plate with variable thickness cross section. The present paper is concerned with the elastic buckling behavior of rectangular thin plate with variable thickness under shear load with types of boundary conditions. In this study, a simplified finite difference method used to solve the governing differential equation of plates with variable thickness. The influences of thickness variation, plate aspect ratios, and boundary conditions on the buckling load are examined.

Governing Equation and Solution

The buckling of isotropic rectangular plates with linearly tapered thickness in the x-direction is considered as shown in Figure 2. The plate is subjected to uniform compressive load in y-direction. The thickness $t(x)$ and moment of inertia $I(x)$ are expressed as: -

$$t(x) = t_o(1 + c_t x) \qquad (1)$$

$$I(x) = I_o(1 + c_t x)^3 \qquad (2)$$

in which $c_t = (t_a - t_o)/at_o$; t_o and t_a denote the thickness at the sides $x = 0$ and $x = a$, respectively; $I_o = t_o^3/12$ is the second moment of area(per unit width) for the plate cross section at the side $x = 0$.

Within the classical small deflection theory of thin plates, the differential equation for the rectangular plate under consideration can be written in the form [Hussain, et al (2002)]: -

$$\nabla^4 w + \frac{2I'_x}{I_x}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) + \frac{I''_x}{I_x}\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) = \frac{1}{D_{(x)}}\left(N_{xy}\frac{\partial^2 w}{\partial x \partial y}\right) \qquad (3)$$

in which

$D_{(x)} = Et^3_{(x)} \Big/ 12\left(1-\nu^2\right)$ = is the flexural (or bending) rigidity of the section of the plate (and this is varying with respect to x).

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

$$I_x = I_o(1+c_t x)^3$$

$$I'_x = 3c_t I_o(1+c_t x)^2$$

$$I''_x = 6c_t^2 I_o(1+c_t x)$$

The solution of Equation (3) may be achieved by finite difference method as shown in Figure 3. By applying the finite difference molecules at the interior nodes of the subdivided plate, the following system of simultaneous linear equations in matrices will be obtained: -

$$[K]\{w\} + \lambda[B]\{w\} = 0 \qquad \textbf{(4)}$$

where the matrices $[K]$ and $[B]$ may be named as follows:

$[K]$: is the stiffness matrix for the plate

$[B]$: is the geometry matrix for the plate

Notice that Equation (4) is an Eigen-value problem. For a given thickness (t_o, t_a) and plate–aspect ratio a/b, the Eigen-value (λ) can be determined numerically by using any relevant technique. The smallest Eigen-value gives the most (fundamental) buckling load.

Numerical Results

Since accuracy of the buckling load depends on the mesh size and on the order finite difference approximation. The mesh effect has been investigated for a square plate. **Table (1)** gives a measure of convergence as a function of mesh size. It can be seen that a (16 × 16) mesh for this problem that gives results to within (2%) of the exact results (9.350)[()].

Comparison with Other Theoretical Studies

Figure (4) shows a comparison of the buckling coefficient, which are obtained by the finite difference method and theoretical results by [Timoshenko and Gere (1961)] for two boundary conditions of all simply supported edges and all clamped edges. Good agreement with theoretical results is achieved for every boundary condition.

Parametric Study

The effects of boundary condition and tapering ratio on the buckling behavior under shear load are studied.

Figures (5-10) present the relation between buckling coefficient ($k_{sh} = 2N_{xy}h_x^2h_y^2 \big/ D_{av}$) and aspect ratio ($a/b$) of a rectangular thin plate with different boundary conditions and different tapering ratios. The plate is analyzed with different tapering ratios (t_a/t_o) (1.0, 1.25, 1.5, 1.75, and 2.0). The aspect ratio (a/b) is taken to be in range (0.5 and 3.0). the boundary conditions are taken to be as [all simply supported edges (ssss), three edges simply supported and other edge clamped (sssc), two edges simply supported and other clamped edges (sscc), three edges clamped and other edge simply supported (sccc), all edges clamped (cccc), and two edges simply supported and other edges clamped (scsc)] .

From these figures, it can be seen that: -

1. The buckling coefficients decrease when the aspect ratio increases.
2. The buckling coefficients decrease when the tapering ratio increases (for the same volume of the plate).
3. The decreasing in the buckling coefficients when the aspect ratio (a/b) <1.0 less than the decreasing in the buckling coefficients when the aspect ratio (a/b) >1.0.
4. The decreasing in the buckling coefficients in plate with boundary condition (sscc) more than the decreasing in the buckling coefficient in plate with boundary condition (scsc).

Conclusions

A finite difference method has been employed to solve numerically the buckling problem of rectangular plates with linearly tapered thickness under shear load. The effect of plate aspect ratio, boundary condition, and tapering ratio on the buckling behavior are considered. The values of buckling coefficients decrease with an increase in the tapering ratio (for the same volume of the plate).

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Table (1): Convergence of buckling coefficient (k_{sh}) for a square simply supported with constant thickness (a/b=1.0)

Mesh size	Shear Buckling coefficient
8×8	10.137
10×10	9.824
12×12	9.661
14×14	9.570
16×16	9.511
18×18	9.470
20×20	9.440

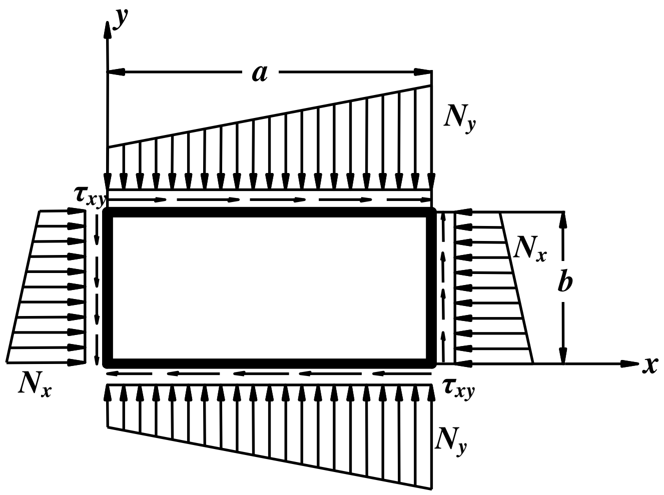

Figure (1): Plate under a general pattern of combined external loads

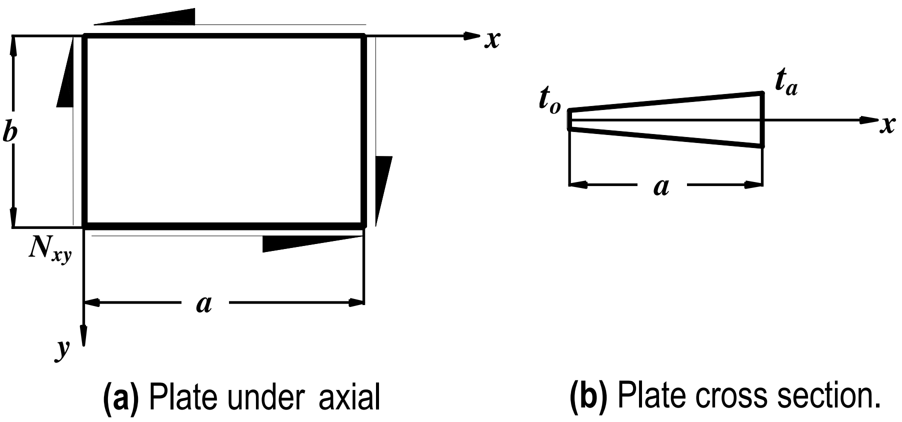

Figure (2): Rectangular thin plate with variable thickness under axial shear load

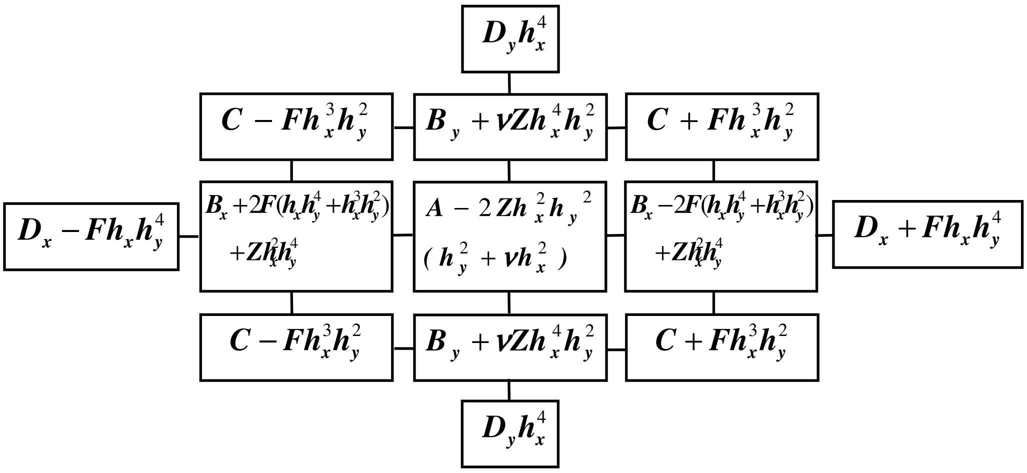

Figure (3): Plate equation in finite difference molecule form

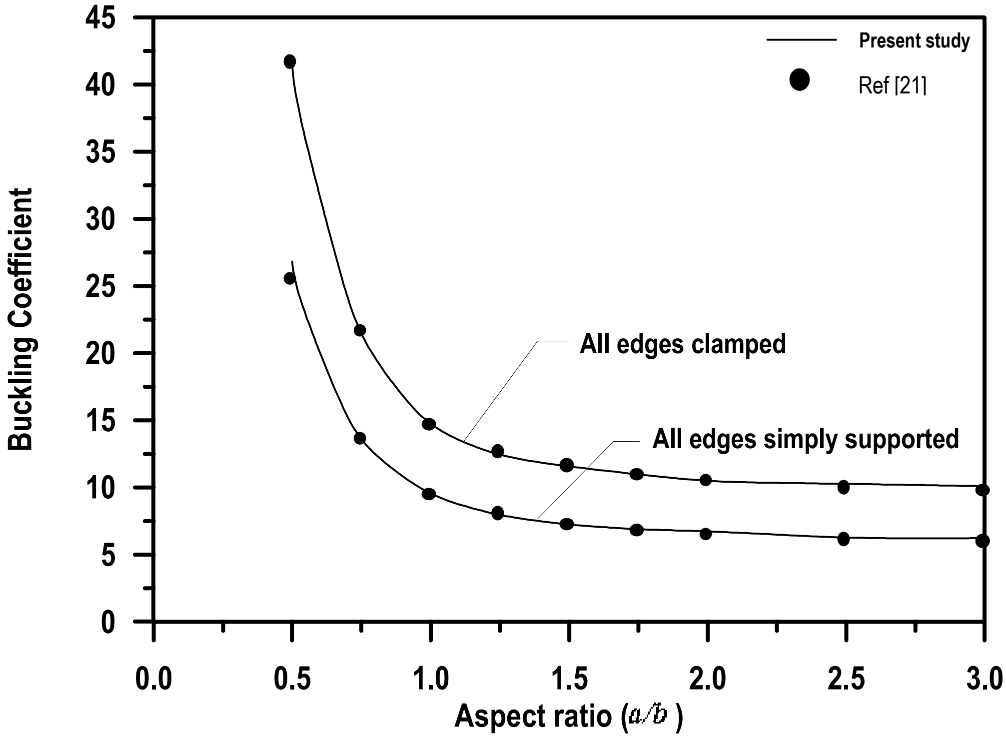

Figure (4): Comparison of buckling coefficient (k_{sh}) of rectangular thin plate with constant thickness

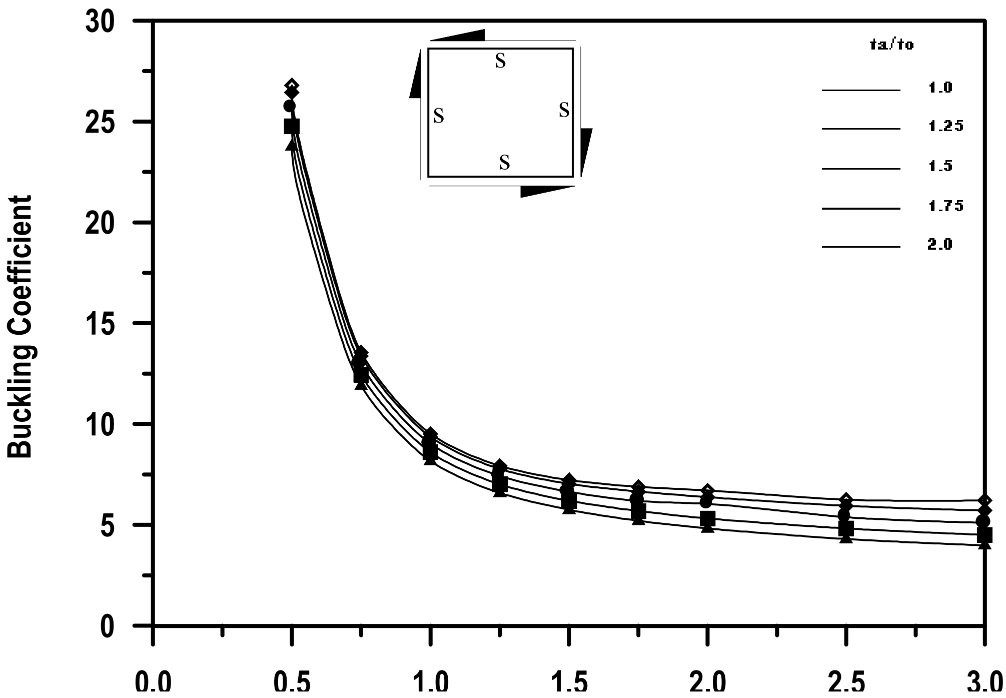

Figure (5): Comparison of buckling coefficient (k_{sh}) of rectangular thin plate with all edges simply supported

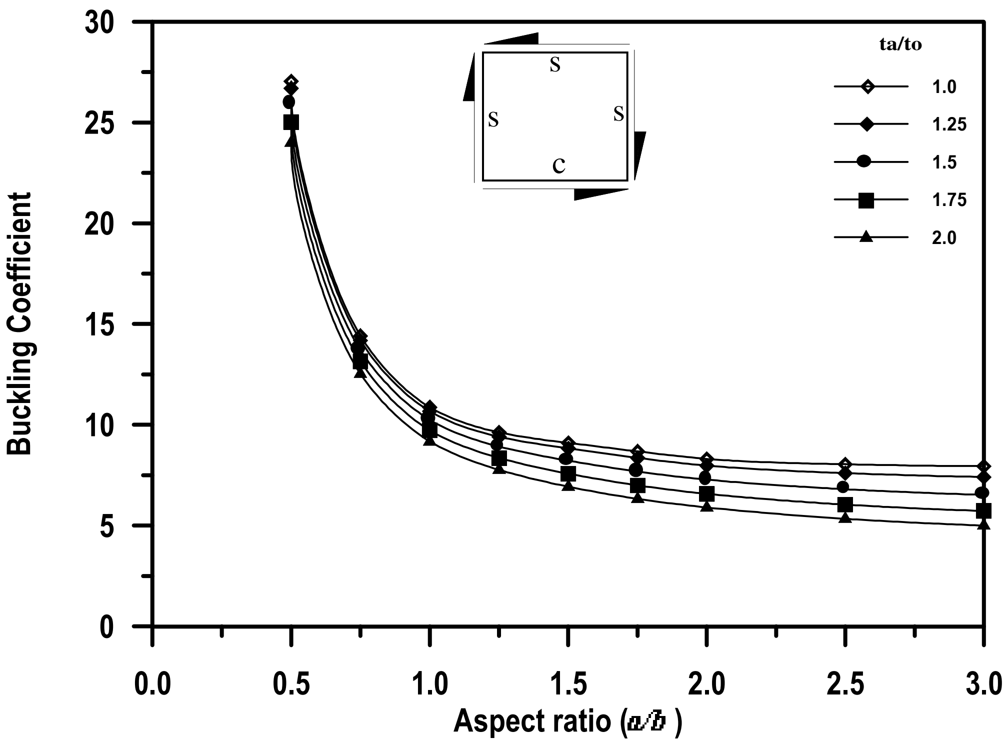

Figure (6): Comparison of buckling coefficient (k_{sh}) of rectangular thin plate with three edges simply supported and other edge clamped

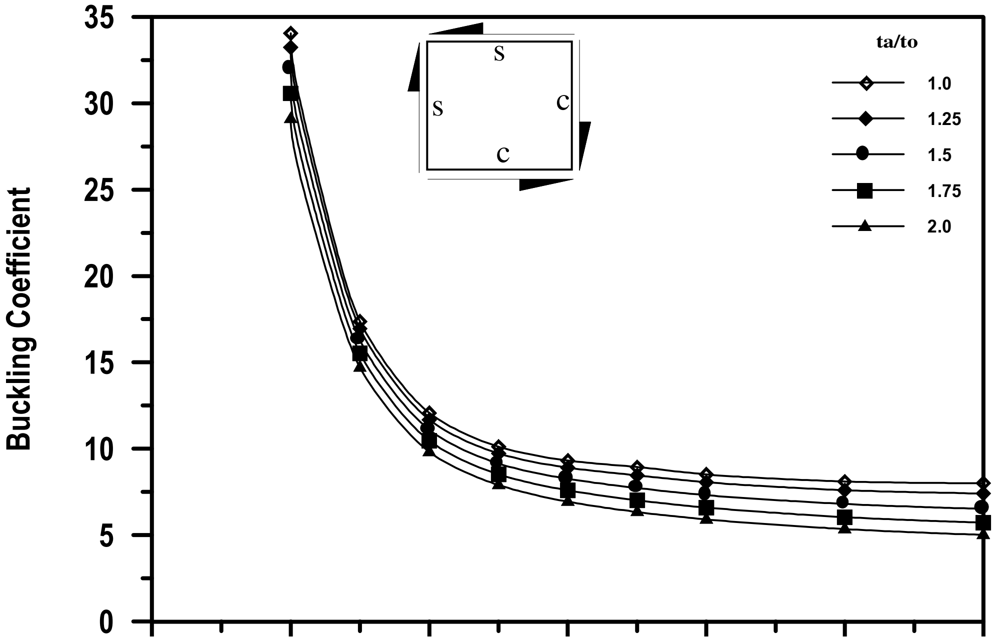

Figure (7): Comparison of buckling coefficient (k_{sh}) of rectangular thin plate with two edges simply supported and other edges clamped

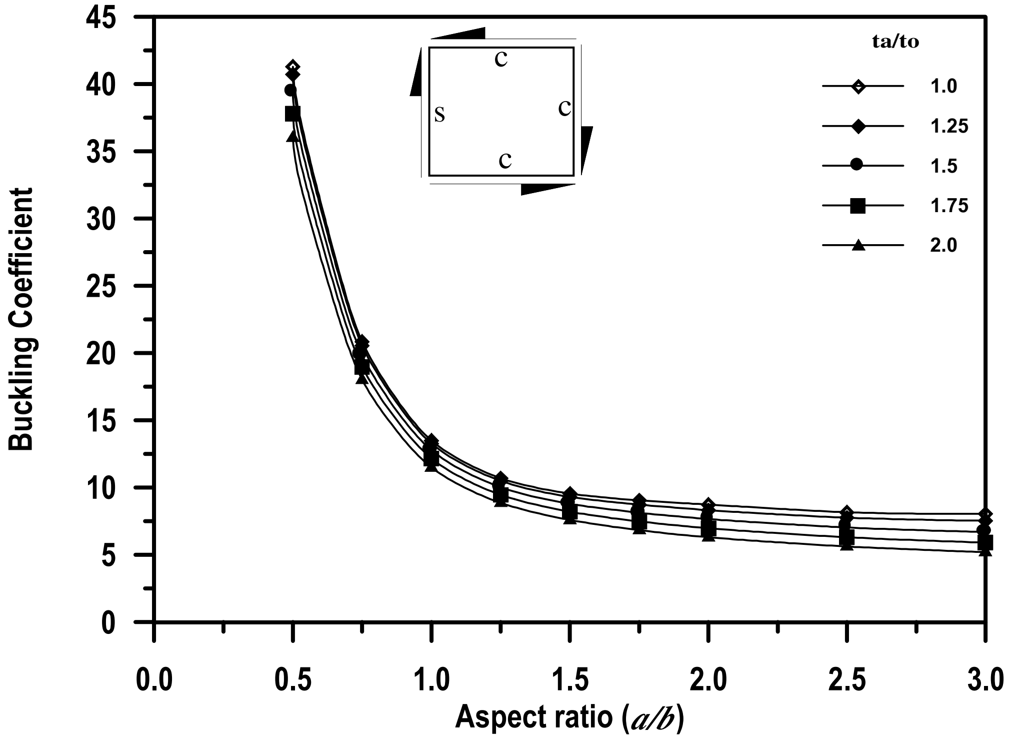

Figure (8): Comparison of buckling coefficient (k_{sh}) of rectangular thin plate with three edges clamped and other edge simply supported

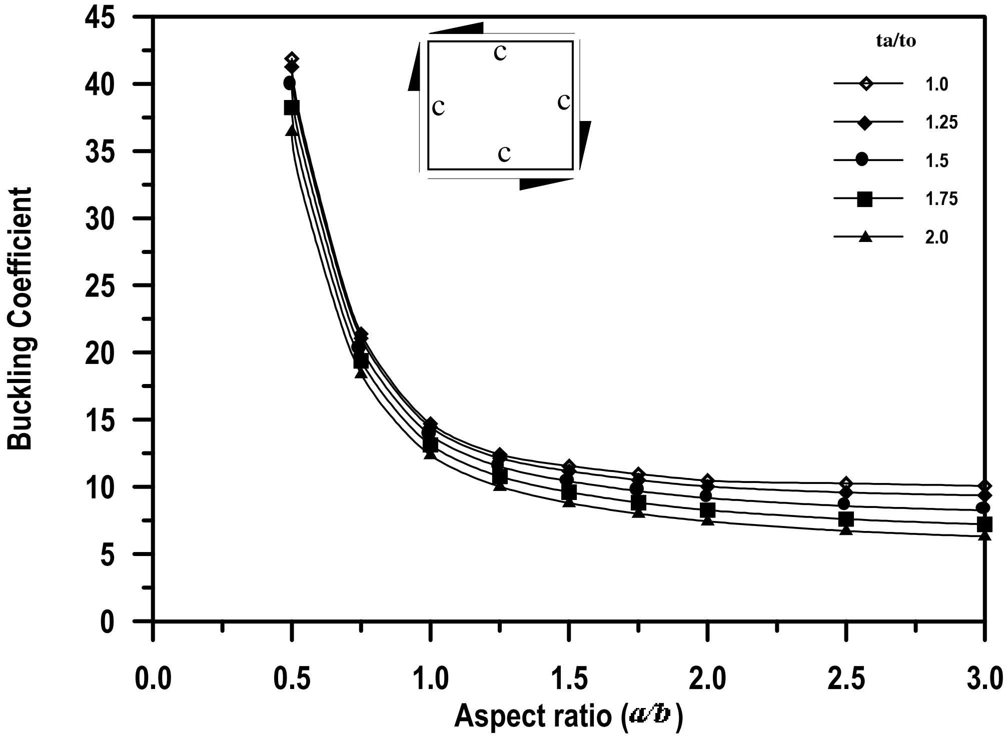

Figure (9): Comparison of buckling coefficient (k_{sh}) of rectangular thin plate with all edges clamped

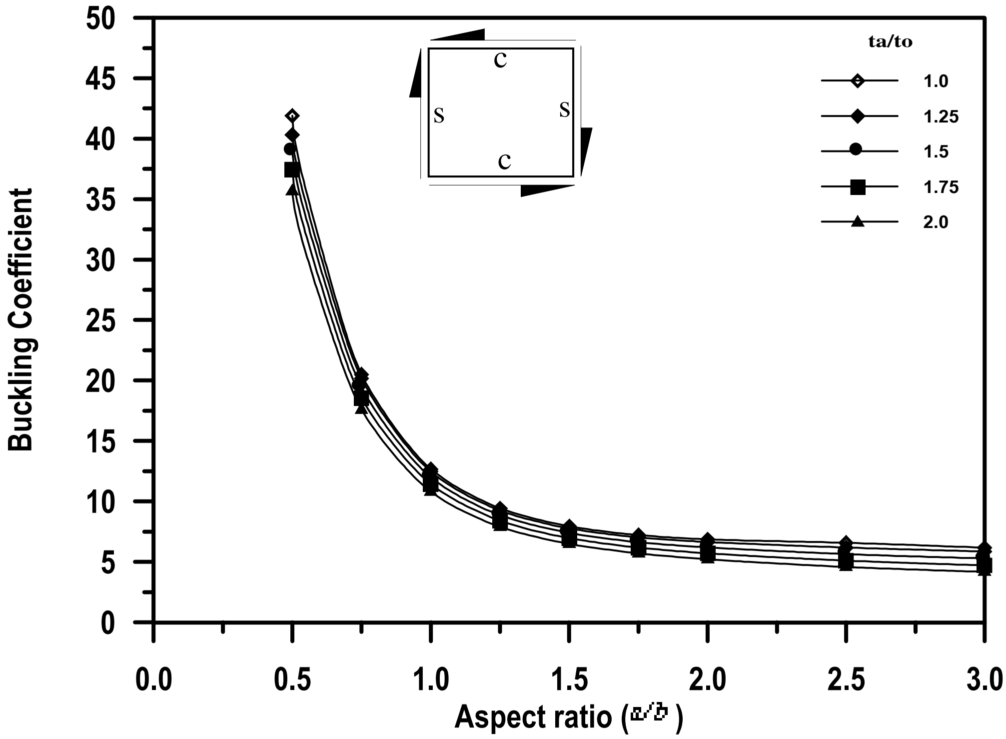

Figure (10): Comparison of buckling coefficient (k_{sh}) of rectangular thin plate with two edges simply supported and other edges clamped