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Genetically compatible graphs for planetary gear train synthesis

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ARTICLEINFO

ABSTRACT

Planetary gear trains (PGTs) are used in a wide range of mechanical applications. Graphs are used to represent PGTs. PGTs are classified into single-planet and multi-planet gear trains depending on the number of vertices in the second level of the graph. In this paper, a novel method for synthesizing PGTs up to the second level is developed. The first step in the synthesis process is the enumeration of spanning trees. A combinatorial analysis is then used to obtain the link assortment arrays and categorize them into families. Based on the parent graphs and spanning trees, the geared graphs are generated for each family. For simplicity, the steps of the newly developed genetically compatible graphing method are applied to 5-link, 1-DOF planetary gear trains. The synthesis steps are programmed in MATLAB.

1. Introduction

For many years, many studies have focused on planetary gear train synthesis and analysis [1-14]. Graph theory has aided PGT structural synthesis, resulting in the development of three distinct synthesis methods. The first method produces PGTs with N links from N-vertex parent graphs by assigning geared edges to the parent graph. This is known as the non-recursive method [1-3]. According to some researchers, this method is complicated to implement on a computer and requires a large number of parent graphs.

The second method, known as the recursive method, generates PGTs with N links by adding graph components to PGTs with $(N - 1)$ links. [4-6]. However, the PGTs enumerated by the recursive method are incomplete.

The last method employs the parent-bar-linkage to enumerate PGTs. However, it is not currently in use. [7, 8]. PGTs with up to nine links have been studied using the methods described above [1-8]. For the reasons stated above, a method that easily provides a complete solution to the enumerated PGTs is required. Isomorphic graphs must be deleted as part of the solution. The presence of pseudo-isomorphic graphs, on the other hand, further complicates the isomorphism challenging task [5]. The synthesis results of PGTs with more than six links are in disagreement [3, 6]. To avoid the problem of pseudo isomorphic graphs, Chatterjee and Tsai [9, 10] proposed the rooted graph of PGTs. Based on spanning trees of rooted graphs, a method for enumerating PGTs is also proposed. Hsu and Lam [11, 12] proposed a graph model that is identical to all pseudo isomorphic graphs. In addition, Hsu [13] proposed a method for detecting structural isomorphism. Despite the fact that Hsu [14] proposed a parent graph method for synthesizing PGTs, he believed that synthesizing PGTs with more than six links was not possible using an atlas of parent graphs. Then, Hsu [15] developed a new method based on acyclic graphs. Shanmukhasundaram et al. [16] also synthesized PGTs using acyclic graphs. Yang et al. [17, 18] contradicted [15 and 16] and used parent graphs to generate PGTs with up to nine links. Shanmukhasundaram and colleagues [19] examined the recursive, non-recursive, and acyclic graph-

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Nomenclature:			
A	The adjacency matrix	N	Number of Links
a_{ij}	The (i, j) elements of the adjacent matrix	o	Root or the ground vertex
DOF	Degree of Freedom	PGT	Planetary gear train
E	Edges or the joint between links.	V	Vertices
F	Number of degrees of Freedom	$V_2, V_3, \ldots, V_m$	Number of binaries, ternary…m-nary vertices
G	Graph	$[V_1, V_2, V_3, \ldots, V_m]$	Link assortment array
L	Number of independent loops	v	Number of vertices
m	The maximal degree of vertex		

based approaches in depth. Graph theory enabled the modeling of PGT kinematic structure as well as the advancement of structure synthesis methodologies [20-28]. In this paper, geared graphs for a given number of links and DOF are enumerated using a method that makes use of the correspondence between spanning-tree graphs and parent graphs. Because the transfer vertices in a spanning tree graph are visible, all feasible gear pair connections can be identified, and geared graphs can be generated. It is justified that generating geared graphs from the entire collection of spanning-tree graphs and parent graphs results in the enumeration of an exhaustive and combinatorial complete set of geared graphs. The purposes of the work are listed as follows:

 - To identify structural characteristics of PGTs and translate them into graph representation language.
 - To overcome the problem of pseudo-isomorphism.
 - To promote an effective method to build a structural synthesis for planetary gear trains with any number of links and degree of freedom.
 - To greatly prohibit the generation of isomorphic graphs.

For the sake of simplicity and clarity, the new method will be applied to PGTs with five links and 1-DOF. It applies to greater degrees of freedom and links. In section 2, the concepts of graph theory are defined. In section 3, the synthesis of planetary gear trains is discussed. Finally, in section 4, the conclusion about the synthesis is briefed and explained.

2. Conceptions

Graph: - A graph (G) contains a set of vertices (V), which represent the number of links, jointly with a set of edges (E) which represent the joint between links.

2.1. Rooted Graph

A rooted graph consists of many vertices. It has one vertex that is different from other vertices. This vertex is named the root. It is usually utilized to represent the basis of a mechanism or fixed link. Fig. 1 shows a functional representation of the well-known Simpson gear train.
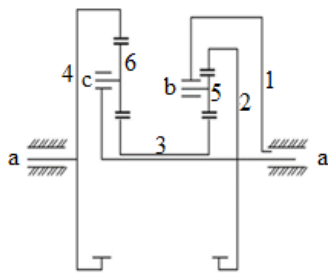

Figure 1. Functional representation of the simpson gear train.

To represent the well-known Simpson gear train, first, consider the ground vertex as vertex 0. Then the links are represented as vertices and distributed based on functional representation. After that, the dotted line is drawn for the revolute joint and a bold line for the gear joint. Fig. 2 denotes rooted graph representation of the Simpson gear train [22,23].
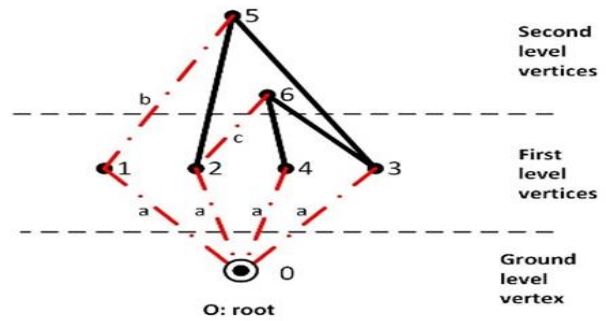

Figure 2. Rooted graph representation of the Simpson gear train.

2.2. Spanning Tree

A tree is a linked graph that doesn't have loops. Fig. 3 denotes the spanning-tree graph of the rooted graph that is shown in Fig. 2.
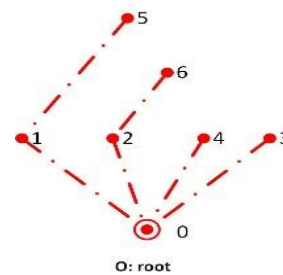
Figure 3. The spanning-tree graph of the rooted graph shown in Figure 2.

2.3. Parents Graph

If all the edges of a rooted graph of PGT are assumed to be revolute or have the same color, then the resulting graph is the parent graph. In the parent graph, there is no distinction between revolute and geared edges.
 Fig.4 shows the parent graph of the rooted graph which is shown in Fig. 2. By using the corresponding matrix equation, the parent graphs are synthesized.

Figure 4. parent graph of the rooted graph which is shown in Figure 2.

2.4. Adjacency Matrix

The following is a definition of an adjacency matrix element:

$$a_{ij} = \begin{cases} 1 \ if \ vertex \ i \ is \ adjacent \ to \ vertex \ j \\ 0 \ otherwise \ (inclde \ i = j) \end{cases}$$

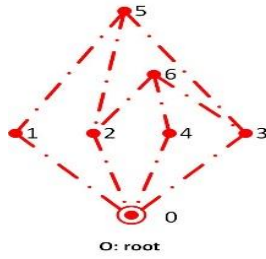Where a_{ij} indicates the (i, j) elements of the adjacent matrix. It is a $(v * v)$ symmetric matrix and its diagonal elements are zeros. The degree of vertex is the summation of the elements of a row or column. The adjacent matrix has unique determination [23]. The adjacency matrix for parent graph shown in Fig. 4 is:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

2.5. Fundamental Circuit

The addition of a chord to spanning tree results in the formation of exactly one circuit. A set of independent loops or fundamental circuits (FCs) are formed according to a combination of all circuits concerning a spanning tree. They serve as a foundation for the circuit space [27].
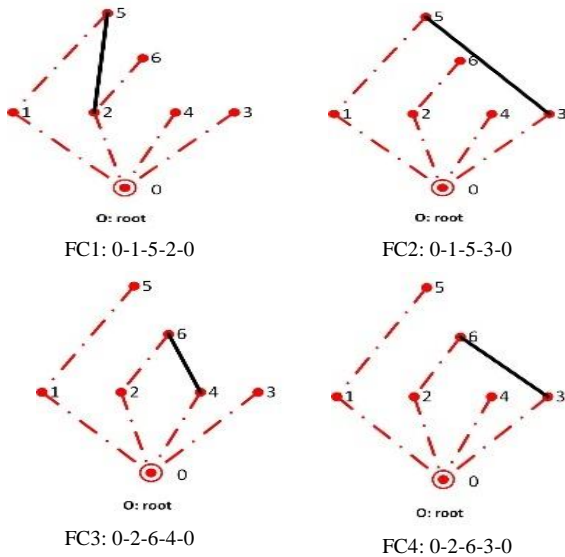

FC1: 0-1-5-2-0

FC2: 0-1-5-3-0

FC3: 0-2-6-4-0

FC4: 0-2-6-3-0

Figure 5. The four fundamental circuits of the rooted graph shown in Figure 2.

3. Synthesis and analysis of planetary gear train

The process of enumeration of PGTs is split into three steps. First step, trees that are suitable for the creation of rooted graphs are enumerated. Second step, parent graphs are enumerated. In third step, geared graphs are detected from parent graphs and spanning trees. The rooted graph of an N-link, F-DOF PGT comprises $(N + 1)$ vertices and $(2 N - F - 1)$ edges; hence, the number of independent loops can be obtained from $L = e - v + 1$ [25-28]. The link assortment arrays of a spanning tree and a parent graph are expressed as $[V_1, V_2, V_3, ..., V_m]$ and $[V_2, V_3, ..., V_m]$, respectively. $V_2, V_3, ..., V_m$ is the number of binary, ternary…m-nary vertices in the spanning tree or the parent graph, respectively. The maximal degree of a vertex m is obtained from $m = L + 1$.

3.1. Topological Features of Spanning Trees

It was shown, in a spanning tree, all geared edges are removed from the geared graph results. The spanning tree explains an open-loop kinematic chain that is consists of links which joined together via revolute joints. Generally, the parent graph doesn't have a unique spanning tree. Fig. 6 shows two spanning trees corresponding to the graph shown in Fig. 4. However, rooted-graph representation of PGTs rules some particular arrangement of the vertices. For this reason, a reliable enumeration method is required to synthesize the PGTs. So, a novel method to precisely enumerate spanning trees of N-link PGTs is presented in this paper.
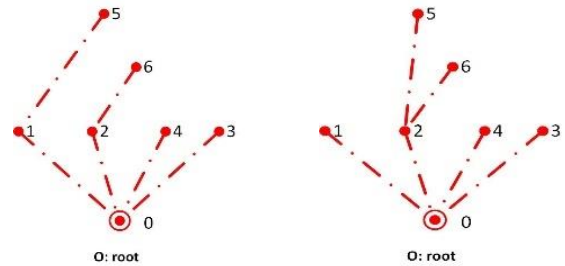
Figure 6. Two spanning trees corresponding to the graph shown in Figure 4.

Let v denotes the vertices, and e represents the number of edges in a graph. Also, let V_k refer to the number of vertices of degree k. So, V_1 denotes the number of vertices of degree one, V_2 the number of vertices of degree two, etc. as shown in Eq. (1).

$$V_1 + V_2 + V_3 + \cdots + V_m = v \tag{1}$$

Where m refer to the maximal degree of a vertex. Since a tree of v vertices contain v − 1 edges and every edge have two end vertices and each of the V_k vertices are incident by k edges, as shown in Eq. (2).

$$V_1 + 2V_2 + 3V_3 + + \cdots + mV_m = 2(v - 1) \tag{2}$$

For example, for a 5-link, single-DOF PGT, we have v = 6, e = 8, and m = 4, it follows that:

$$V_1 + V_2 + V_3 + V_4 = 6 \tag{3}$$

$$V_1 + 2V_2 + 3V_3 + 4V_4 = 10 \tag{4}$$

The flowchart for the enumeration of spanning tree graphs is shown in Fig. 7.

Figure 7. Flowchart for the enumeration of spanning tree graphs.

By using a MATLAB program to solve Eqs. (3) and (4), we obtain the following link assortment arrays for the spanning trees: [4 2 1 1 1 1], [3 2 2 1 1 1], [3 3 1 1 1 1], and [2 2 2 2 1 1]. See appendix B.

The link assortment arrays are classified into families according to maximum vertex degree; the vertex with the greatest degree is chosen as the root.

Family 1: (maximum vertex degree = 4): [4 2 1 1 1 1]

Family 2: (maximum vertex degree = 3): [3 2 2 1 1 1], [3 3 1 1 1 1]

Family 3: (maximum vertex degree = 2): [2 2 2 2 1 1]

Table 1. Spanning trees for the 5-link, single-DOF PGT.

Family 1	Family 3
[4 2 1 1 1 1]	[2 2 2 2 1 1]

Family 2	
[3 2 2 1 1 1]	[3 3 1 1 1 1]

3.2. Topological features of parent graphs

Based on equation (5) and (6), All conceivable link assortments for a given v-vertex graph can be derived.

$$V_2 + V_3 + \ldots + V_m = v \tag{5}$$

$$2V_2 + 3V_3 + \ldots + mV_m = 2e \tag{6}$$

Taking (N = 5) for instance, we have (v = 6), (e = 8) and (m = 4). The link assortment Eq. is shown in Eq. (7).

$$V_2 + V_3 + V_4 = 6 \tag{7}$$

$$2V_2 + 3V_3 + 4V_4 = 16 \tag{8}$$

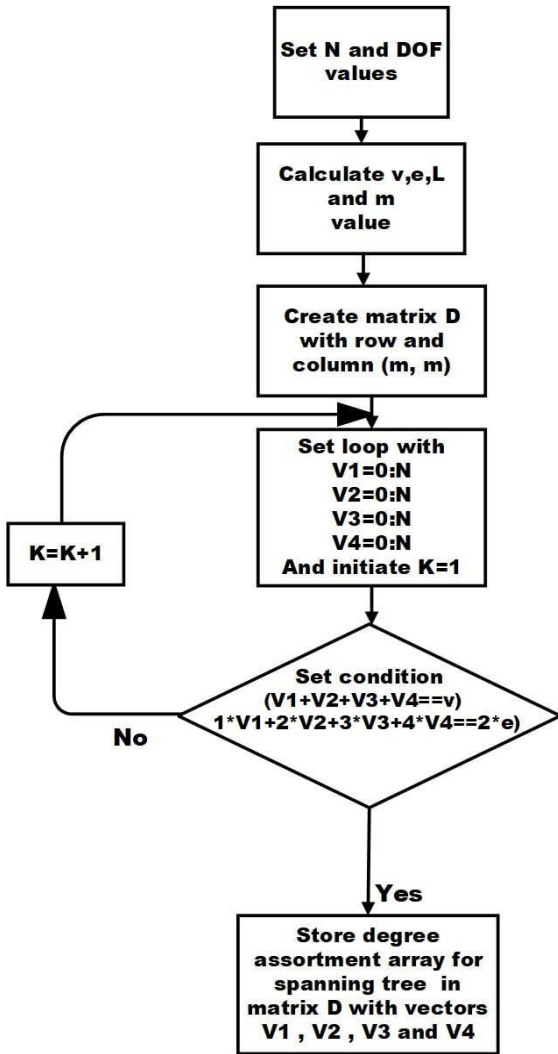The flowchart for the enumerate of parent graphs is shown in Fig. 8. Five-link assortment arrays are achieved by using a MATLAB program: [4 2 2 2 2 4], [3 3 3 2 3 2], and [3 3 2 2 3 3]. The link assortment arrays are also classified into families according to maximum vertex degree:

Family 1: (maximum vertex degree = 4): [4 2 2 2 2 4]

Family 2: (maximum vertex degree = 3): [3 3 3 2 3 2], and [3 3 2 2 3 3].

The second family contains two vertices in the second level, whereas the first family contains only one vertex.

Table 2. parent graphs for 5-link, single-DOF PGTs.

Family 1	Family 2	
[4 2 2 2 2 4]	[3 3 3 2 3 2]	[3 3 2 2 3 3]

3.3. Geared graphs

Now by comparing the parent graph and spanning trees for each family (1 and 2), we get the following geared graphs:

Table 3. Geared graphs for 5-link, single-DOF PGT.

Family 1	
[4 2 2 2 2 4]	
[4 2 1 1 1 1]	

Family 2	
[3 3 3 2 3 2]	[3 3 2 2 3 3]
[3 2 2 1 1 1]	[3 3 1 1 1 1]
	(b)

[3 3 3 2 3 2]	[3 3 3 2 3 2]
[3 2 2 1 1 1]	[3 3 1 1 1 1]

(c)	(d)
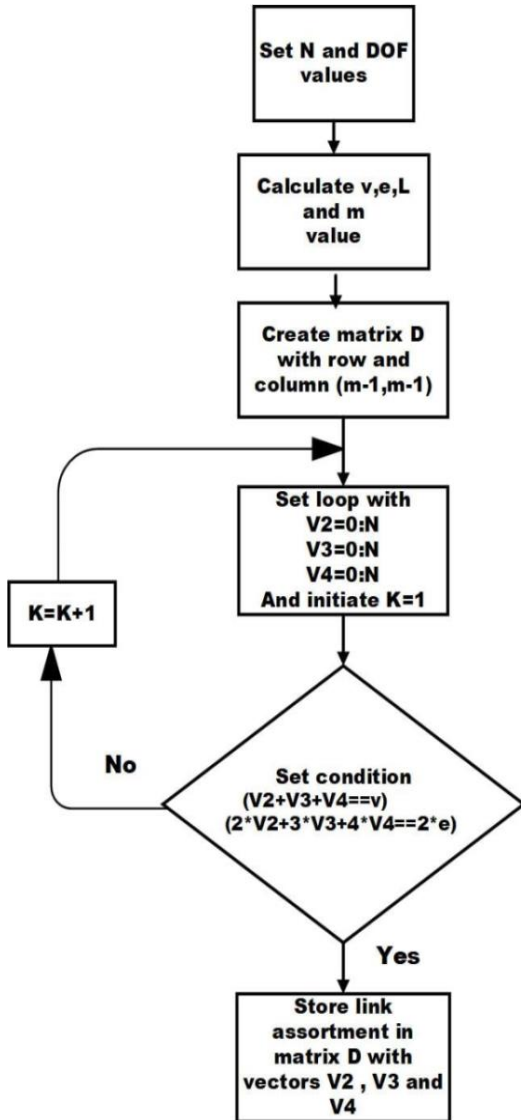
Figure 8. Flowchart for the enumeration of parent graphs.

The new method can be implemented on a computer using an adjacency matrix and a nested-do loops algorithm.

3.4. Isomorphism

Two graphs are isomorphic if their vertices and edges have a one-to-one correspondence, and their incidences are preserved. A link assortment array identifies each family. Isomorphism cannot exist between graphs from different families. In Table 3, for example, two of the four graphs in family 2 have the same spanning tree. The link assortment arrays of the parent graph and spanning tree of Figures (a) and (c) are the same. The weighted vertex degree d_w is calculated by assigning edge weights of one for revolute edges and two for geared edges. For example, the weighted vertex degree of vertices 0 through 5 of graph (a) shown in Table 3 are 3 4 4 3 5 3, respectively. The weighted link assortment array for the graph shown in Fig. (a) is [3 4 4 3 5 3], whereas the weighted link assortment array for the graph shown in Fig. (c) is [3 4 5 2 5 3]. As a result, they aren't isomorphic. It should be noted that only graphs having the same link assortment arrays can be isomorphic. As a result, only graphs with the same link assortment arrays must be checked for isomorphism, and there are often few of them. A new algorithm based on the trail and graph marking is developed. If the vertex-edge of two graphs are equivalent, the graphs are isomorphic. The vertices are to be identified by a three-digit number $d_t d_p d_w$, where d_t and d_p represent the degree of vertex at the spanning tree and the parent graphs, respectively. The new vertex identification method takes into account not only the number of edges incident to the vertex but also the type of edges. The graphs of two PGTs are shown in Fig. 9 (a) and (b) (Fig. 14 of ref. [29]).
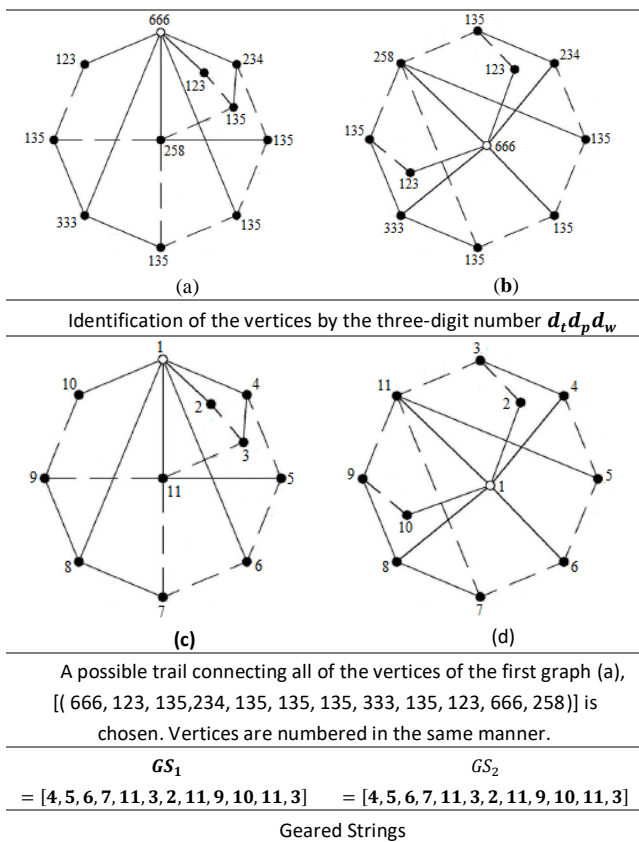

(a)   (b)

Identification of the vertices by the three-digit number $d_t d_p d_w$

(c)   (d)

A possible trail connecting all of the vertices of the first graph (a), [(666, 123, 135,234, 135, 135, 135, 333, 135, 123, 666, 258)] is chosen. Vertices are numbered in the same manner.

GS_1	GS_2
$= [4, 5, 6, 7, 11, 3, 2, 11, 9, 10, 11, 3]$	$= [4, 5, 6, 7, 11, 3, 2, 11, 9, 10, 11, 3]$

Geared Strings

Figure 9. The steps for identifying isomorphism using the trail and graph marking method

First, the vertices of the two graphs are identified by the three-digit number $d_t d_p d_w$. Second, a possible trail connecting all of the vertices of the first graph (a) is chosen. Vertices are numbered in the same manner as [(666, 123, 135, 234, 135, 135, 135, 333, 135, 123, 666, 258)]. Repetition of vertex and edge is allowed. Third, the two graphs are numbered according to the suggested trail. Finally, check the numbering of the geared strings (GSs) of both geared graphs. If they are not equivalent, then the two graphs are not isomorphic. Because $GS_1 = GS_2$, the two graphs are isomorphic.

3.5. Functional Representation

The geared graphs shown in Table 3 can be transformed to their corresponding functional diagrams as shown in Appendix A. To start, each edge-labeling possibility of a spanning tree must be determined. Fig. 10 shows all possibilities for labeling the edges of spanning trees. Figure A.1 shows the graphs of 5-link 1-DOF PGTs that have only single-planet PGTs. Column 1: Graphs showing one of the possible distributions of internal and external gear pairs. Column 2: Corresponding functional representation to column 1. Figure A-2 shows a graph of a double-planet 5-link PGT.
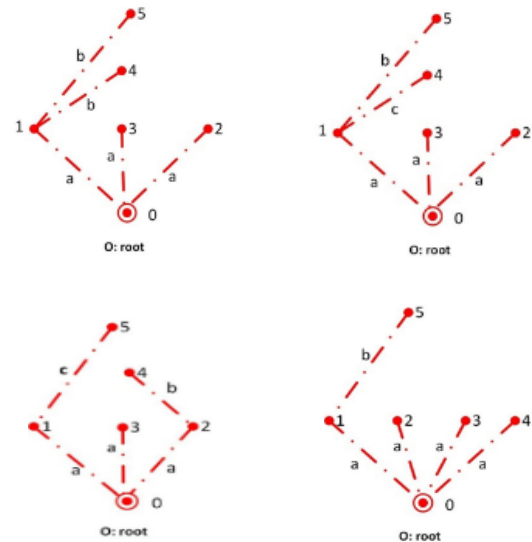
Figure 10. Possibilities for labeling the edges of spanning trees.

Fig. 11 shows the labelled graph representation and the functional representation of the first graph in Table 3 which belongs to family 1. An internal gear pair will be represented by an upper-case G and an external gear pair will be represented by a lower-case G.

4. Results

This paper describes a method for synthesizing 5-link, 1-DOF PGTs. Given a set of links, all possible link assortments are found, and the graphs associated with each link assortment are synthesized. Table 1 shows the spanning trees for the 5-link, single-DOF PGT.
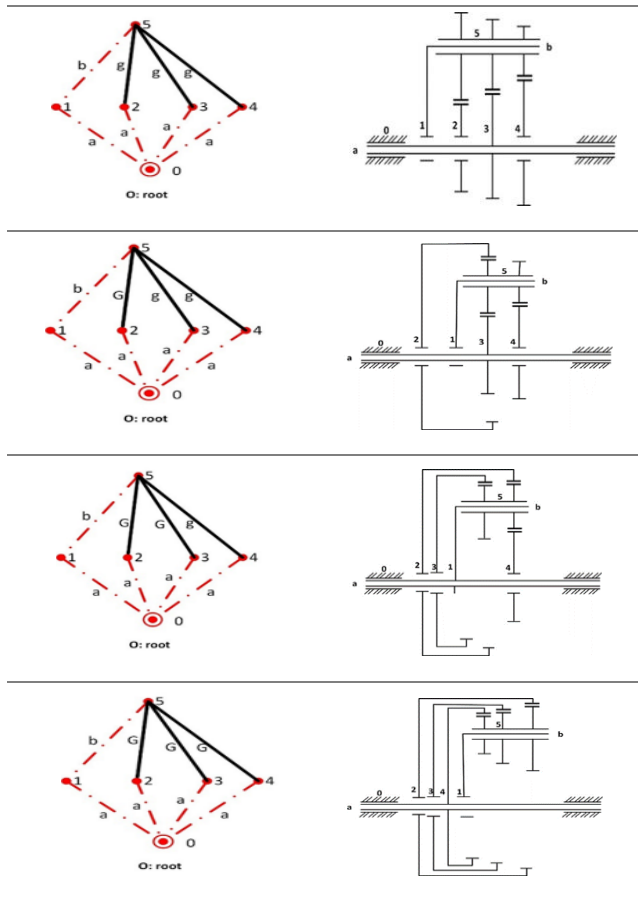
Figure 11. Five-link PGTs that have only single-planet FGEs

To reduce the number of graphs generated and concentrate on methodology, the spanning trees were built with a vertex distribution up to the second level and a ground vertex degree greater than two. Table 2 shows the parent graphs for the 5-link, single-DOF PGT. Table 3 displays the 5-link geared graphs for the link assortments [4 2 2 2 2 4], [3 3 3 2 3 2], and [3 3 2 2 3 3]. The spanning trees and parent graphs are used to generate geared graphs. Each geared graph represents a 5-link, single-DOF PGT with a distinct topology and function. Figure 10, for example, shows five-link PGTs that have only single-planet PGTs. The detailed results for the synthesis of graphs are shown in Appendix A.

The results are a test of the current method's utility as a new method added to the previously existing technique. To automate the method, a computer program is created using a nested-do loops algorithm.

5. Conclusion
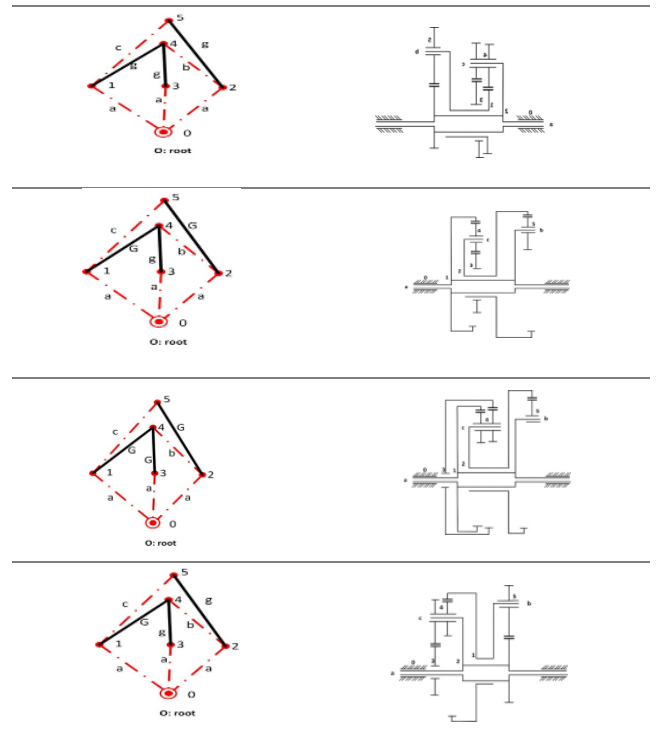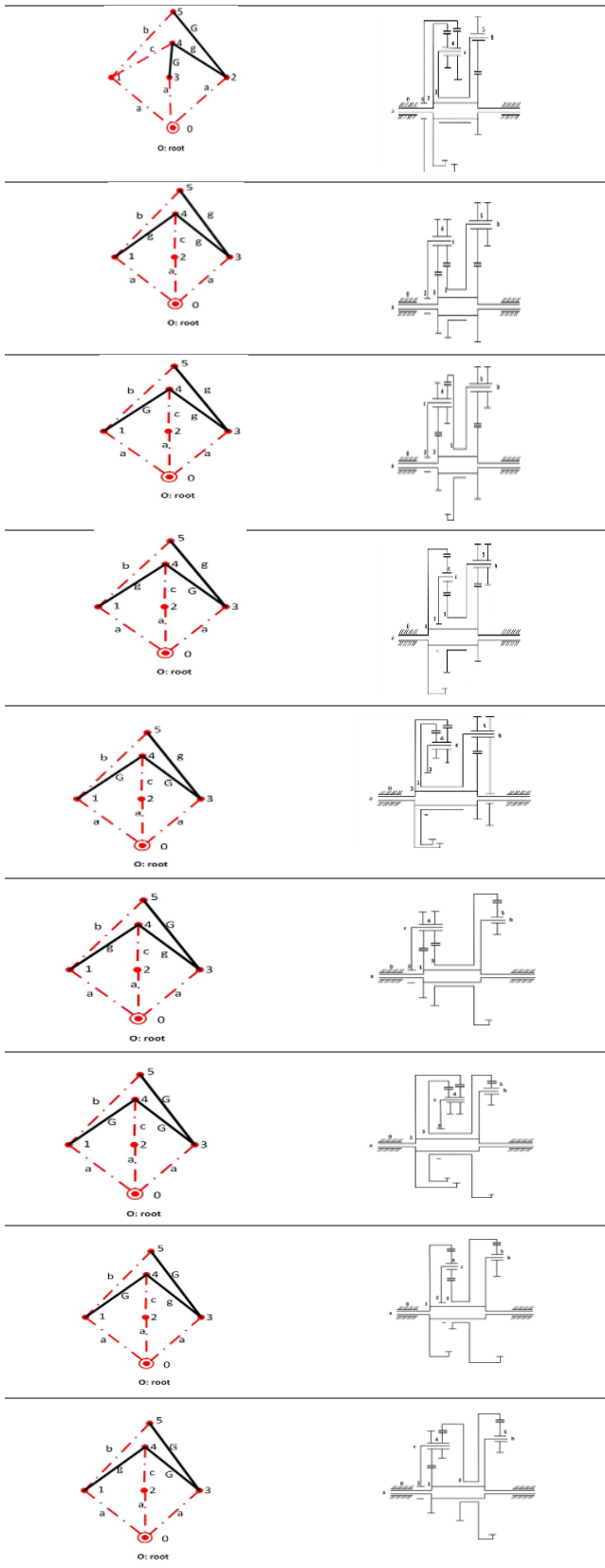
This paper presents a genetically compatible graph method to synthesize planetary gear trains. The new method is established on the link assortment arrays of spanning trees and parent graphs. Geared edges are determined by the fact that a parent graph contains several spanning trees. The presented method can be utilized for PGTs with any number of DOFs. Because this method makes use of a small number of spanning-tree graphs and parent graphs, it is possible to efficiently enumerate PGTs with any number of links. It is easy to program on a computer, can be used directly to perform PGT synthesis, and can generate an exhaustive and combinatorial complete set of geared graphs. The MATLAB algorithm is built to perform this method and the results show precise graphs without many similarities.

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Appendix A

Figure A.1 shows the graphs of 5-link 1-DOF PGTs that have only single-planet PGTs. Column 1: Graphs showing one of the possible distributions of internal and external gear pairs. Column 2: Corresponding functional representation to column 1. Figure A-2 shows a graph of a double-planet 5-link PGT.

Graph representation	Functional representation

Appendix B

A MATLAB program using an adjacency matrix and a nested-do loops algorithm to solve a system of 2 linear equations in 3 unknowns:

```
clear
clc
close all
N=5; Dof=1;
v=N+1;
e=2*v-Dof-3;
L=e-v+1;
m=L+1;
k=1;
D=zeros(m-1,m-1);
for V2=0:N
    for V3=0:N
        for V4=0:N
            if(V2+V3+V4==v)
                if (2*V2+3*V3+4*V4==2*e)
                    D(k,:)=[V2 V3 V4];
                k=k+1;
                end
            end
        end
    end
end
disp('Vertex Degree Listing  LA =')
disp('    V2   V3   V4 '),disp(D
```