BEHAVIOR OF THICK SPHERICAL VESSELS UNDER HIGH PRESSURE

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Abstract

The aim of this study was to investigate elasto-plastic thermal stresses in a thermoplastic of Thick Spherical vessels under high pressure The present study deals with spherical shells analysis, The elastic and plastic theory of spherical shell is consider in this search, with a thick sphere subjected to different types of loading such as internal pressure, external pressure and thermal loading have been studies. The Tresca yield condition used in this study. When the applied pressure exceeds the minimum pressure required to initiate the yielding at the inner radius, plastic zone starts to be formed. The residual stress components also were calculated using elastic and elasto-plastic solution result

سلوك الخزانات السميكة الكروية تحت ألضغوط العالية

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ألخلاصة:
الهدف من هذا البحث كان تحري الاجهادات ألحرارية في طور اللدونة-المرونة للخزانات الكروية السميكة وتحت الضغوط ألعالية.اعتمدت نظريات اللدونة والمرونة للقشريات ألكروية مع تسليط أنواع مختلفة من الأحمال كالضغوط الداخلية والضغوط الخارجية والأحمال الحرارية جميعها أخذت بنظر الاعتبار.تم استخدام نظرية ترسكا للخضوع في هذه الدراسة.عندما الضغط المسلط يتجاوز الحد الأدنى للضغط المطلوب لبدء الخضوع في نصف القطر الداخلي ينتج عنه تشكل منطقة التشوه اللدن.كذلك تم دراسة الاجهادات المخزونة أو المتبقية وتأثيرها على التشوه اللدن والمرن

Nomenclature

σy: Yield strength of material of uniaxial tension test
σ_r: Radial Stress
σ_θ: Hoop stress
A and B: Constant
r : Radius of thick sphere
a: inner Radius
b: outer Radius
p_e :pressure at plastic Zone
$(\sigma_r)_c$: The radial stress at r=c
C: Radius of sphere at plastic zone
P_i: inner radius
P_o: outer radius
T_b and Ta: temperatures at the outer and inner radius respectively
α : linear coefficient of expansion.
β: Dimensionless quantity

Introduction

Cylindrical or spherical pressure vessels (e.g., hydraulic cylinders, sewage treatment plants, gun barrels, pipes, boilers and tanks) are commonly used in industry to carry both liquid and gases under pressure. When the pressure vessel is exposed to this pressure, the material comprising the vessel is subjected to pressure loading, and hence stresses, from all directions. The normal stresses resulting from this pressure are functions of the radius of the element under consideration; the shape of the pressure vessel (i.e., open ended cylinder, closed end cylinder, or sphere) as well as the applied pressure. A detailed study of stress analysis of spherical pressure vessel with internal pressure and external pressure are to be considered. Spherical pressure vessels are classified into thick sphere if the outer radius is larger then the inner radius by 10 %, otherwise the sphere can be considered as thin sphere. in the thin spheres the radial and tangential stresses are assumed to be constant along the thickness of the sphere but in thick spheres the radial and tangential stresses very across the thickness of the sphere according to the type of loading such as internal pressure and external pressure . An effective numerical iterative method is proposed for the pressure loading analysis. Elastic stress behavior is considered first, followed by some cases of elasto-plastic stress distributions, and finally plastic deformation. for the spherical pressure vessel, the hoop and axial stresses are equal and are one half of the hoop stress in the cylindrical pressure vessel as shown in **Figure** (1). This makes the spherical pressure vessel a more “efficient” pressure vessel geometry [1-4]

Failure Criteria

The purpose of failure criteria is to predict or estimate the failure/yield of structural members subjected biaxial or triaxial states of stress. There are more one theory for Failure Criteria, dependent on the nature of the material (i.e. brittle or ductile), as shown below:

- Maximum shear stress criterion, Von Mises criterion for Ductile material.
- Maximum Shear Stress Criterion, Tresca's criterion for Ductile material.
- Maximum normal stress criterion, Mohr’s theory for Brittle material.

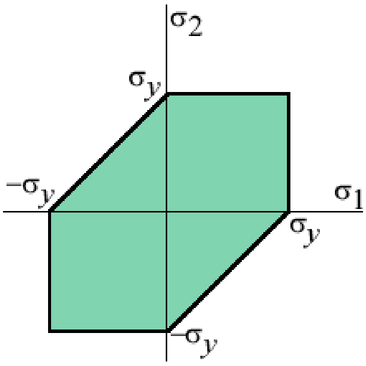
Whether a material is *brittle* or *ductile* could be a subjective guess, and often depends on temperature, strain levels, and other environmental conditions. However, a *5% elongation* criterion at break is a reasonable dividing line. Materials with a larger elongation can be considered ductile and those with a lower value brittle. Another distinction is a brittle material's compression strength is usually significantly larger than its tensile strength.

All popular failure criteria rely on only a handful of basic tests (such as uniaxial tensile and/or compression strength), even though most machine parts and structural members are typically subjected to multi-axial loading. This disparity is usually driven by cost, since complete multi-axial failure testing requires extensive, complicated, and expensive tests. The success of all machine parts and structural members are not necessarily determined by their strength. Whether a part succeeds or fails may depend on other factors, such as stiffness, vibrational characteristics, fatigue resistance, and/or creep resistance. The maximum shear stress criterion, also known as Tresca's or Guest's criterion [4] , is often used to predict the yielding of ductile materials. Yield in ductile materials is usually caused by the *slippage* of crystal planes along the maximum shear stress surface. Therefore, a given point in the body is considered safe as long as the maximum shear stress at that point is under the yield shear stress obtained from a uniaxial tensile test. With respect to plane stress, the maximum shear stress is related to the difference in the two principal stresses. Therefore, the criterion requires the principal stress difference, along with the principal stresses themselves, to be less than the yield shear stress, this criterion has a good agreement with experimented results obtained for ductile material and due to simplicity of this criteria will be used in this study.

$$|\sigma 1| < \sigma y,\ |\sigma 2| < \sigma y,\ |\sigma 1 - \sigma 2| < \sigma y$$

Where , σy = Yield strength of material of uniaxial tension test

As shown below, the maximum shear stress criterion requires that the two principal stresses .



Mathematical Formulation

The condition of spherical symmetry is assumed for the thick sphere with stress-strain relations and incompressibility conditions for plastic deformation. The equation is solved in the spherical Coordinates. The governing equation (equilibrium equation) for the thick spherical vessel is:

$$\sigma_r (rd\theta)^2 - (\sigma_r + d\sigma_r)(r+dr)^2 (d\theta)^2 + 4\sigma_\theta rd\theta dr \sin(d\theta/2) = 0$$
$$\sigma_r r^2 - (\sigma_r + d\sigma_r)(r^2 + 2rdr) + 2\sigma_\theta rdr = 0$$
$$\sigma_r r^2 - (r^2\sigma_r + r^2 d\sigma_r + 2r\sigma_r dr) + 2\sigma_\theta edr = 0$$
$$simplified - and - dividingby(r^2 dr) \rightarrow$$
$$\frac{d\sigma_r}{dr} - \frac{2\sigma_r}{dr} + \frac{2\sigma_\theta}{r} = 0$$
$$\therefore \frac{d\sigma_r}{dr} = \frac{2(\sigma_\theta - \sigma_r)}{r}$$

Formulation Of Tresca's

This theory predicts that yielding will start when the maximum shear stress in the material becomes equal to the maximum shear stress at yielding in a simple tension test:

$$\frac{1}{2}|\sigma_1 - \sigma_2| = Y \Rightarrow \frac{\sigma_1}{Y} - \frac{\sigma_2}{Y} = \pm 1 \qquad \text{……..(1)}$$

For thick spheres the principle stress are $\sigma_r and \sigma_\theta$,Where

σ_r is compressive and σ_θ is tension so that:

$$\sigma_\theta - \sigma_r = Y \qquad \text{…………(2)}$$

Case 1: Internal Pressure Only

Elastic Analysis:
In the analysis of elastic and elastic-plastic case we must use some fundamental equations, such as equilibrium and compatibility equations.
From Equilibrium equation for spherical coordinates has been derived in last section, and it is given as follows:

$$\frac{d\sigma_r}{dr} = \frac{2(\sigma_\theta - \sigma_r)}{r} \qquad \text{……………(3)}$$

Compatibility equation in terms of stresses is given by:

$$\frac{d}{dr}(\sigma_r + 2\sigma_\theta) = 0 \qquad \text{……………(4)}$$

In the elastic solution for this case and for other, Lames equation has been used to find the solution, from equation (4), it can be seen that the amount $(\sigma_r + 2\sigma_\theta)$ is independent of the radius r, there fore:
$(\sigma_r + 2\sigma_\theta)$=A
Where A is a constant.

By substitute Eq. (4) into the Eq. (3) and integrating, will be obtained

$$\sigma_r = A + \frac{B}{r^3} \qquad \text{………(5)}$$

$$\sigma_\theta = A - \frac{B}{2r^3} \qquad \text{………..(6)}$$

$\sigma_{r=-p}$ at r=a
$\sigma_r = 0$ at r=b
By substituting the above boundary condition in Eq.(5) and (6), we obtain:

$$A = \frac{P}{\frac{b^3}{a^3} - 1}$$

$$B = \frac{-Pb^3}{\frac{b^3}{a^3} - 1}$$

Thus Eq. (5) and (6) can be written as:

$$\sigma_r = \frac{-p(\frac{b^3}{r^3} - 1)}{(\frac{b^3}{r^3} - 1)} \qquad \text{……….(7)}$$

$$\sigma_\theta = \frac{p(\frac{b^3}{2r^3}+1)}{(\frac{b^3}{a^3}-1)} \qquad \text{……………….(8)}$$

These Equations express the radial and tangential elastic stress distribution across the thickness of the wall and it is noted that the redial stress is compressive while the tangential stress is tensile.

Plastic Analysis

If the internal pressure is increased to a critical value (p_e), plastic yielding begins at the radius, where the yield criterion (Tresca) is first satisfied.

By substituting Eq. (7) and (8) into Eq. (2):

$$\frac{p}{\left(\frac{b^3}{a^3}-1\right)}\left(\frac{b^3}{2r^3}+1\right)+\frac{p}{\left(\frac{b^3}{a^3}-1\right)}\left(\frac{b^3}{r^3}-1\right)=Y$$

This equation reduces to

$$P_e = \frac{2}{3}Y\left(\frac{b^3}{a^3}-1\right)\frac{r^3}{b^3}$$

It has note that has minimum value at r=a (i.e) the difference ($(\sigma_\theta - \sigma_r)$ has the greatest value at r=a, this means that the initiation of the plastic zone will begin-from the inner radius when $p=p_e$

$$P_e = \frac{2}{3}Y(1-\frac{a^3}{b^3})$$

With further increase in the internal pressure, the plastic zone spreads out wards and the elastic-plastic boundary is a spherical trace at each stage.

To find the stress distribution at the plastic zone (at each point) in this zone the yield criterion will be satisfied as will as the equilibrium equation therefore by substituting equation.

From Eq (1) and (2) we get:

$\frac{d\sigma_r}{dr} = \frac{2}{r}Y$ then integrating

$\sigma_r = 2Y\ln r + A$

Where A is a constant to be evaluated from the boundary condition,

At r=a, $\sigma_r = -p$ then A= (-P-2Yln a),

And $\sigma_r = 2Y\ln\left(\frac{r}{a}\right) - P$ …………… (9)

To obtain the expression for σ_θ Eq. (9) is substituted into the Tresca's criterion Eq. (2) we get.

$$\sigma_\theta = Y(1+2\ln(\frac{r}{a})) - P \qquad \text{......... (10)}$$

To find the stresses in the elastic zone, Lames equation are also applied as for the elastic case, but the constant will be a function of the plastic radius (c), there for the sphere is assumed to be consisted from two spheres ,the inner sphere is completely plastic and the outer sphere is completely elastic, thus the boundary condition become:

$\sigma_r = 0$ at r=b

$\sigma_r = (\sigma_r)_c$ at r=c

Where

$(\sigma_r)_c$ is the radial stress at r=c, and it is equal to:

-($(\sigma_r)_c = \frac{2}{3}Y(1-\frac{c^3}{b^3})$ from the first boundary condition:

A=-B/b^3

And from second boundary condition, $B=\frac{2}{3}Y\left(\frac{1-\frac{C^3}{b^3}}{\frac{1}{b^3}(\frac{b^3}{c^3}-1)}\right)$

Then Lames equations become

$$\sigma_r = -\frac{2}{3}Y\frac{C^3}{b^3}(\frac{b^3}{r^3}-1) \qquad \text{.......... (11)}$$

and

$$\sigma_\theta = \frac{2}{3}Y\frac{C^3}{b^3}(\frac{b^3}{2r^3}+1) \qquad \text{......... (12)}$$

At the elastic-plastic boundary the stresses must satisfy the continuity condition therefore equalizing Eqs. (11) and (12) at r= c, will yield

$$\frac{P}{2Y} = \ln\frac{c}{a} + \frac{(1-\frac{c^3}{b^3})}{3}$$

This equation is used to determine the radius of plastic phase.

The solution is accomplished at any value of pressure (P), by solving it by using suitable numerical method

Residual Stress Analysis

The residual stresses can be found by subtracting Eqs. (7) and (8) from Eqs (9) and (10).but in the elastic zone the residual stress can be found by subtracting Eqs. (7) and (8) from Eqs (11) and (12).

$$\sigma_r = -\frac{2Y}{3}(\frac{C^3}{a^3}-\frac{P}{P_e})(\frac{a^3}{r^3}-\frac{a^3}{b^3}) \quad ,c\le r\le b$$

$$\sigma_\theta = -\frac{2Y}{3}(\frac{C^3}{a^3}-\frac{P}{P_e})(\frac{a^3}{r^3}+\frac{a^3}{b^3}), \ c\le r\le b$$

$$\sigma_r = -\frac{2Y}{3}(\frac{p}{p_e}(1-\frac{a^3}{r^3}) - \ln\frac{r^3}{a^3}, \ a\le r\le c$$

$$\sigma_\theta = -\frac{2Y}{3}(\frac{p}{p_e}(1+\frac{a^3}{r^3}) - \ln\frac{r^3}{a^3}, \quad a \le r \le c$$

Case 2: External Pressure Only

There are some case in which a sphere is subjected to external pressure, the analysis for such a case is similar to for the internal pressure case, but boundary condition are varied:

Elastic Analysis

The Lames equations after applying the boundary condition becomes :

$$\sigma_r = \frac{P_O}{(1-\frac{a^3}{b^3})}(1-\frac{a^3}{b^3}) \text{ and } \sigma_\theta = \frac{P_O}{(1-\frac{a^3}{b^3})}(1+\frac{a^3}{2r^3})$$

It is noted that the radial stress and as well as the tangential stress are both compressive while they are tensile for the case of internal pressure only

Plastic Analysis

To find the stress distribution at the plastic zone (r<c) Tresca yield criteria should be satisfied as well as the equilibrium equation.
Thus the stresses will be:

$$\sigma_r = -P - \frac{2}{3}\frac{c^3}{b^3}Y(1-\frac{b3}{R^3})$$

$$\sigma_\theta = -P - \frac{2}{3}\frac{c^3}{b^3}Y(1+\frac{b3}{2R^3})$$,Since these stresses must be continuous across the elastic-plastic boundary:

So R=C , $[\sigma_r]_{elastic} = [\sigma_r]_{plastic}$ then

$$-P-\frac{2}{3}\frac{c^3}{b^3}Y(1-\frac{b^3}{c^3}) = 2Y\ln\frac{a}{c}$$

So P $= P = 2Y\ln\frac{a}{b} + \frac{2}{3}Y(\frac{C^3}{b^3}-1)$

By solving the above equation numerically using Newton Raphson method the value of the radius C can be obtain for any value of applied pressure.

Case 3: Internal and External Pressure

The sphere in this case is subjected to internal and external pressure applied simultaneously. By superposition of the two elastic stresses for the internal and external pressure case and by applying the following boundary condition in to lames equation:

$\sigma_r = -P_i$ at r=a

σ_r =-P_O at r=b

Then

$$\sigma_r = -\frac{P_i}{\left(\frac{b^3}{a^3}-1\right)}(\frac{b^3}{r^3}-1) - \frac{P_O}{\left(1-\frac{a^3}{b^3}\right)}(1-\frac{a^3}{r^3}) \text{ and } \sigma_\theta = -\frac{P_i}{\left(\frac{b^3}{a^3}-1\right)}(\frac{b^3}{2r^3}+1) - \frac{P_O}{\left(1-\frac{a^3}{b^3}\right)}(1+\frac{a^3}{2r^3})$$

To obtain the critical of the difference in pressures that will case the initiation of plastic zone at the inner radius (r=a),sub, $\sigma_r and \sigma_\theta$ in Tresca criterion($(\sigma_\theta - \sigma_r) = Y$

Case 4: Thermal Loading

The thermal loading, has significant effect on the value of stresses due to temperature difference that may occurs between the inner and the outer radius of the sphere, this will cause a difference in the expansion or contraction between the layers of the thick sphere, there for this will give rise-to the radial and tangential stresses. The steady state temperature distribution in the case of spherical symmetry is give by:

$$T = T_b + (T_a - T_b)\times\frac{\frac{b}{r}-1}{\frac{b}{a}-1}$$

Where T_b &T_a are temperatures at the outer and inner radius respectively.
The strain in the sphere is the superposition of that due to pressure loading and thermal loading, then the elastic stress-strain equation is:

$$\varepsilon_r = \frac{1}{E}(\sigma_r - 2\nu\sigma_\theta) + \alpha T$$

$$\varepsilon_\theta = \frac{1}{E}((1-\nu)\sigma_\theta - \nu\sigma_r) + \alpha T$$

where α is the linear coefficient of expansion.

After Appling compatibility equation and substitution T with integrating we can get the dimensionless quantity given as:

$$\beta = \frac{\alpha(T_a - T_b)}{(1-\nu)}$$

The radial stress and as well as the tangential stress are given by the Equations below:

$$\sigma_r = -\beta E\left(\frac{\frac{b}{r}-1}{\frac{b}{a}-1} - \frac{\frac{b^3}{r^3}-1}{\frac{b^3}{a^3}-1}\right)$$

$$\sigma_\theta = -\beta E\left(\frac{\frac{b}{2r}-1}{\frac{b}{a}-1} - \frac{\frac{b^3}{2r^3}+1}{\frac{b^3}{a^3}-1}\right)$$

Result And Discussion

The Calculation show that the internal pressure required to initiate the plastic zone from the inner radius is P_i=14 Mpa and the state of total plasticity is reached when P_i=33.2Mpa, because of increasing the loading of vessel, the volume increase, which leads to plastic deformations take place. At internal pressure P_i=8Mpa, the whole sphere is in fully elastic state. This pressure is lower than the pressure needed to initiate the plastic deformation at the inner surface, **Figure(1)** shows the non-dimensional radial and tangential stress distribution along the thickness of the sphere, we conclude from that when the external pressure increased beyond the critical value of the pressure the plastic zone will propagate outward that's leads to increasing the radial stress . **Figure (4)** shows the variation of the dimensional residual radial and tangential stresses across the thickness of the sphere when it is loaded with pressure P=30 Map. we see that the hoop stress increase during the thickness of the sphere because of the Residual stress distribution increase due to overloads. **Figure (5)** show the non-dimensional radial and tangential stress distributions due to thermal loading ,we see that the hoop stress is larger then radial stress because of the Coefficient of thermal expansion mismatch between different phases. **Figure (6)** show the elasto-plastic state of stresses for external pressures of 20Mpa this figures also show the propagation of plastic zone as the pressure increases. due to the plastic area increase leads to yielding take place.

Conclusions

For the internal pressure case it can be observed that the plastic zone starts at the inner radius and spreads outward as the pressure increase, also there is a direct proportionality between elastic boundary radius and internal pressure. The residual stresses increases as the amount of plastic deformation increases. There is a direct proportionality between the internal pressure and the value of the residual tangential. For thermal loading it is noted that the radial stress is negative and reaches a maximum value between the inner and the outer radius. This maximum stress will be at R/B=0.64.it also observed those values of stresses increases as temperature difference increases. Effects of residual stress may be either beneficial or detrimental, depending upon the magnitude, sign, and distribution of the stress with respect to the load-induced stresses. Very commonly, the residual stresses are detrimental, and there are many documented cases in which these stresses were the predominant factor contributing to fatigue and other structural failures when the service stresses were superimposed on the already present residual stresses.

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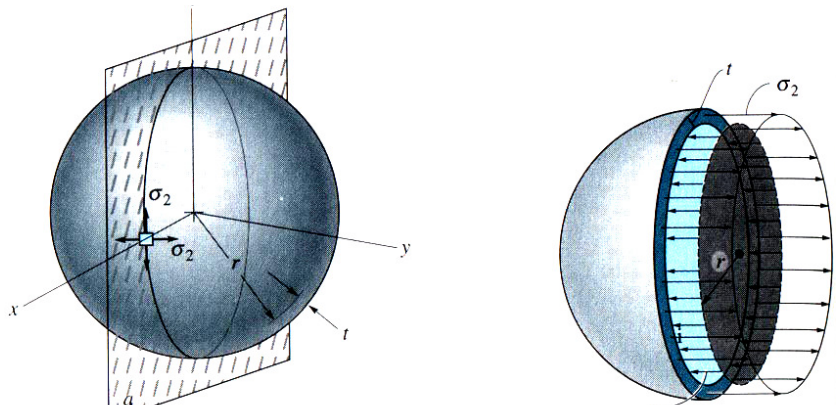

Figure (1): Pressure and Internal Hoop and Axial Stresses.

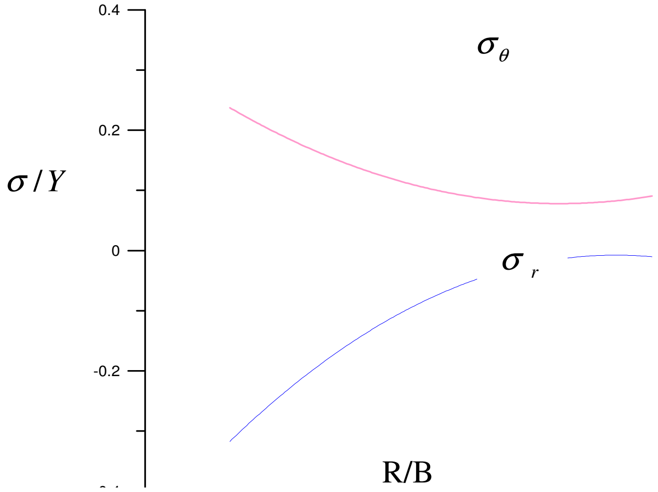

Figure (1): Elastic stress distribution for internal pressure(p_i=8 Mpa)

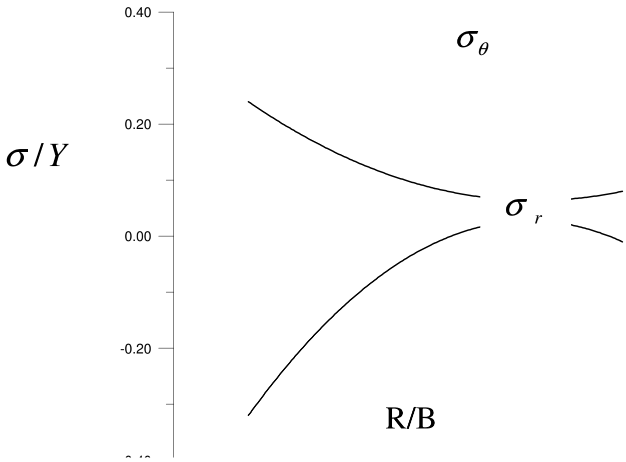

Figure (2): Elastic stress distribution for internal pressure(p_i= 18 Mpa)

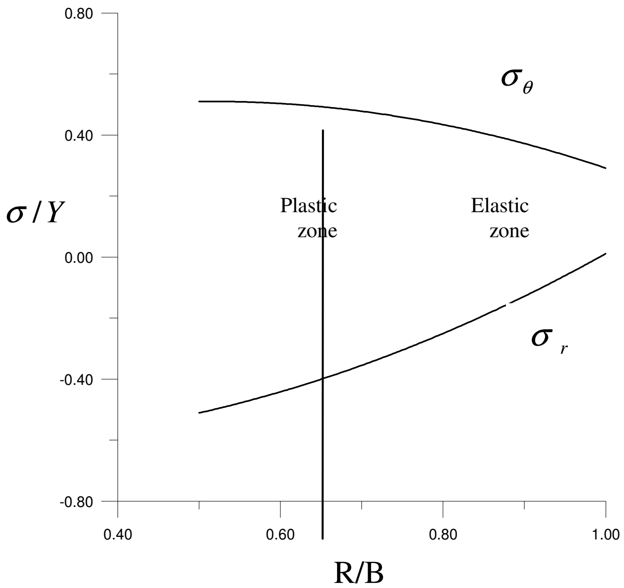

Figure (3):Elastic-Plastic stress Distribution for(p_i=25 Mpa)

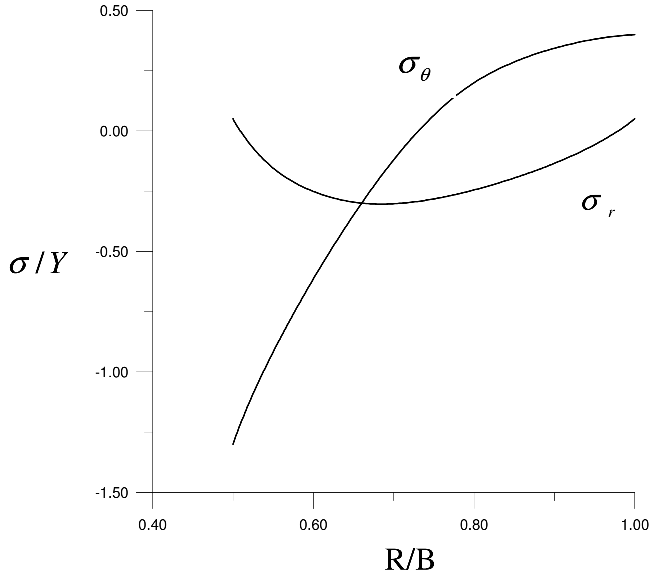

Figure (4):Residual stress distribution for internal pressure (p_i=30 Mpa)

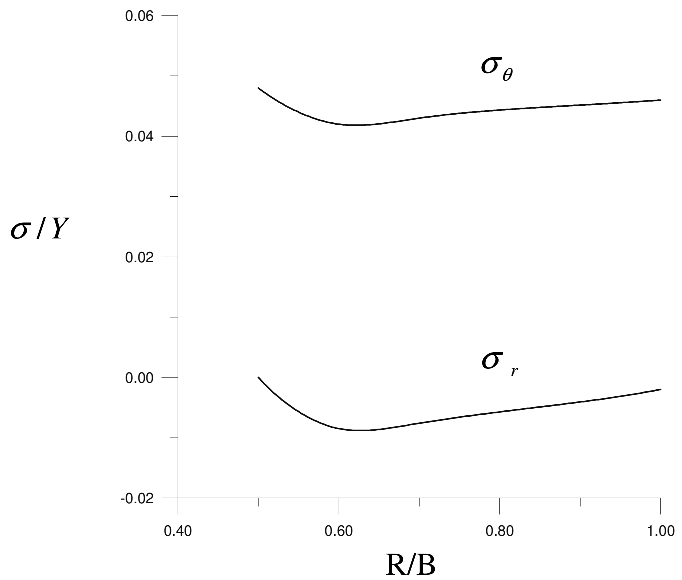

Figure (5):Radial and tangential stress Distribution for thermal loading (T_a=500,T_b=20)

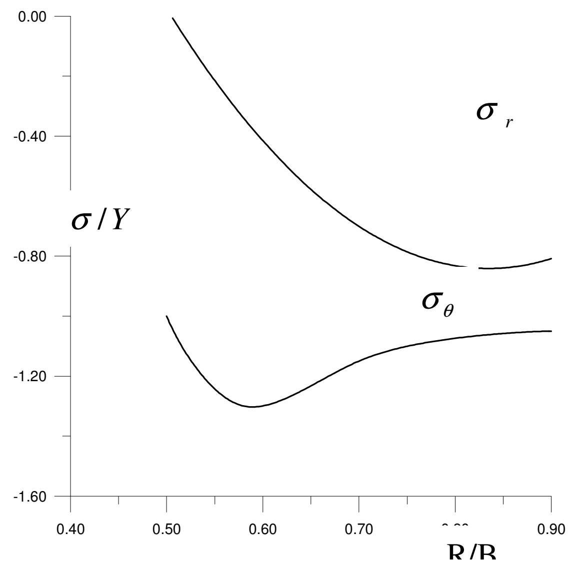

Figure (6):Elastic –plastic stress distribution for external pressure(p_o=20 Mpa)